Certifying Positive Almost Sure Termination of Probabilistic Pushdown Automata

Tobias Winkler, Joost-Pieter Katoen

Highlights 2024

Certificates in Verification

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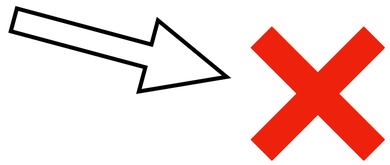
System

Property



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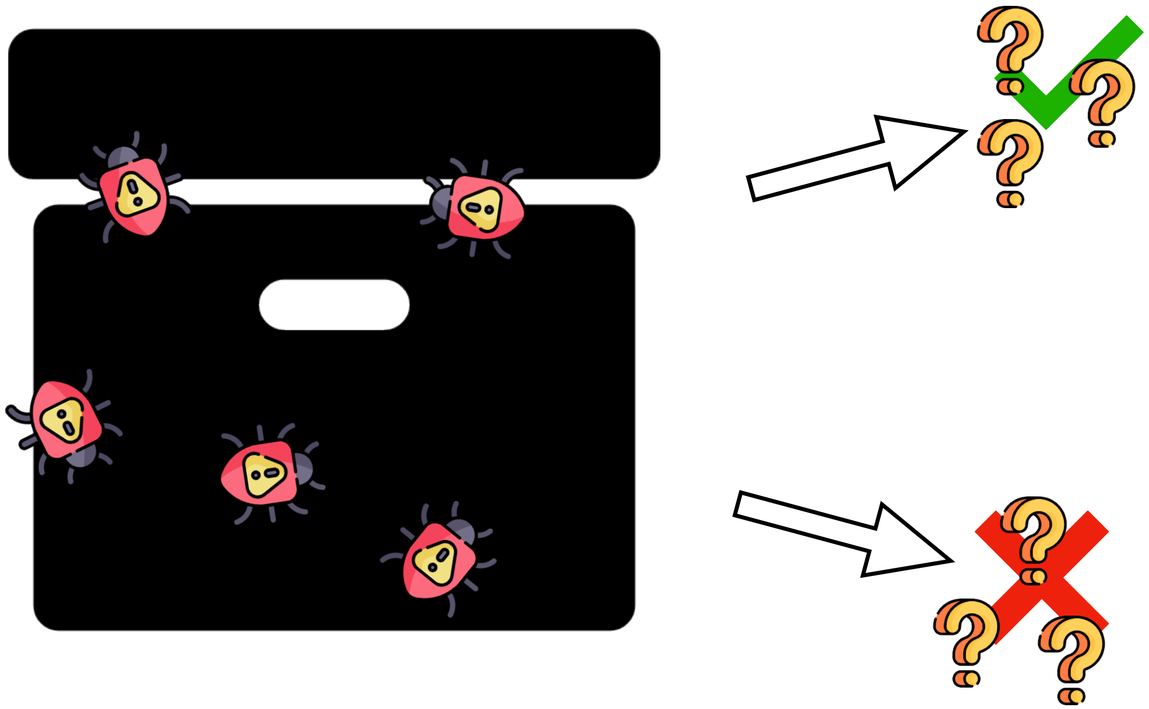

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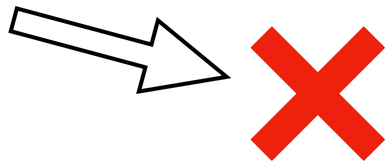

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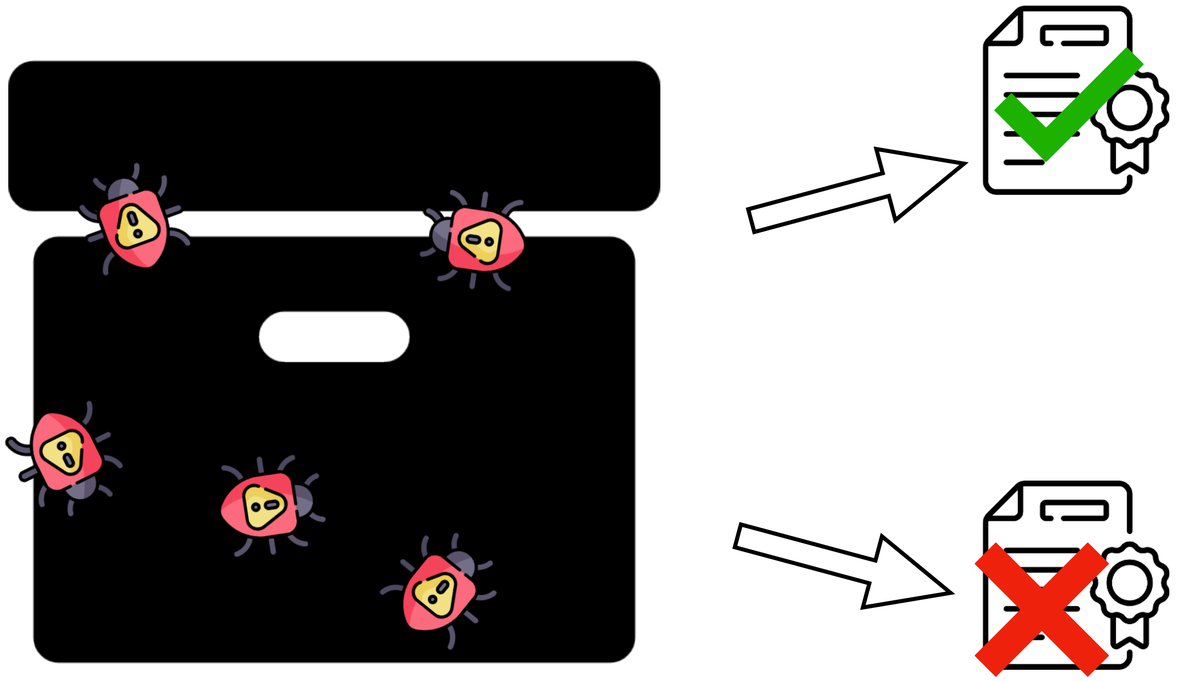

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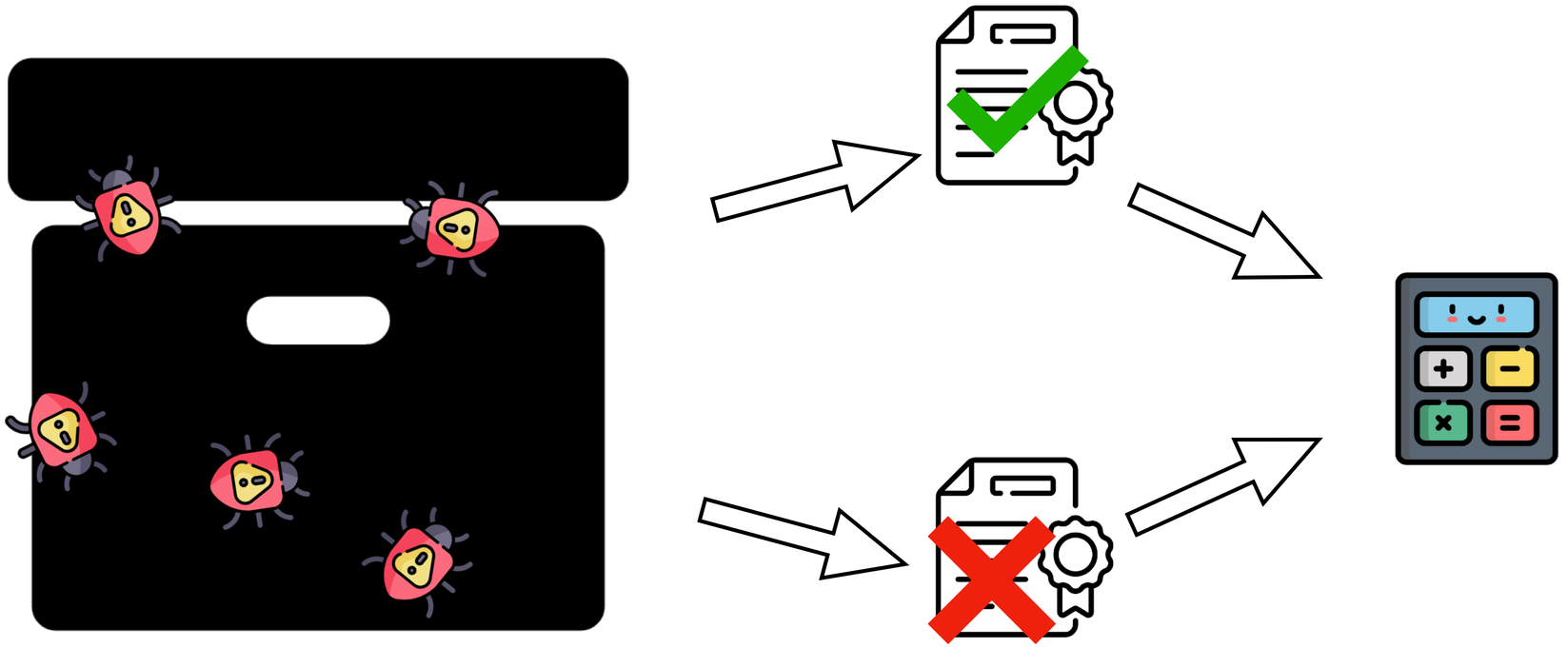

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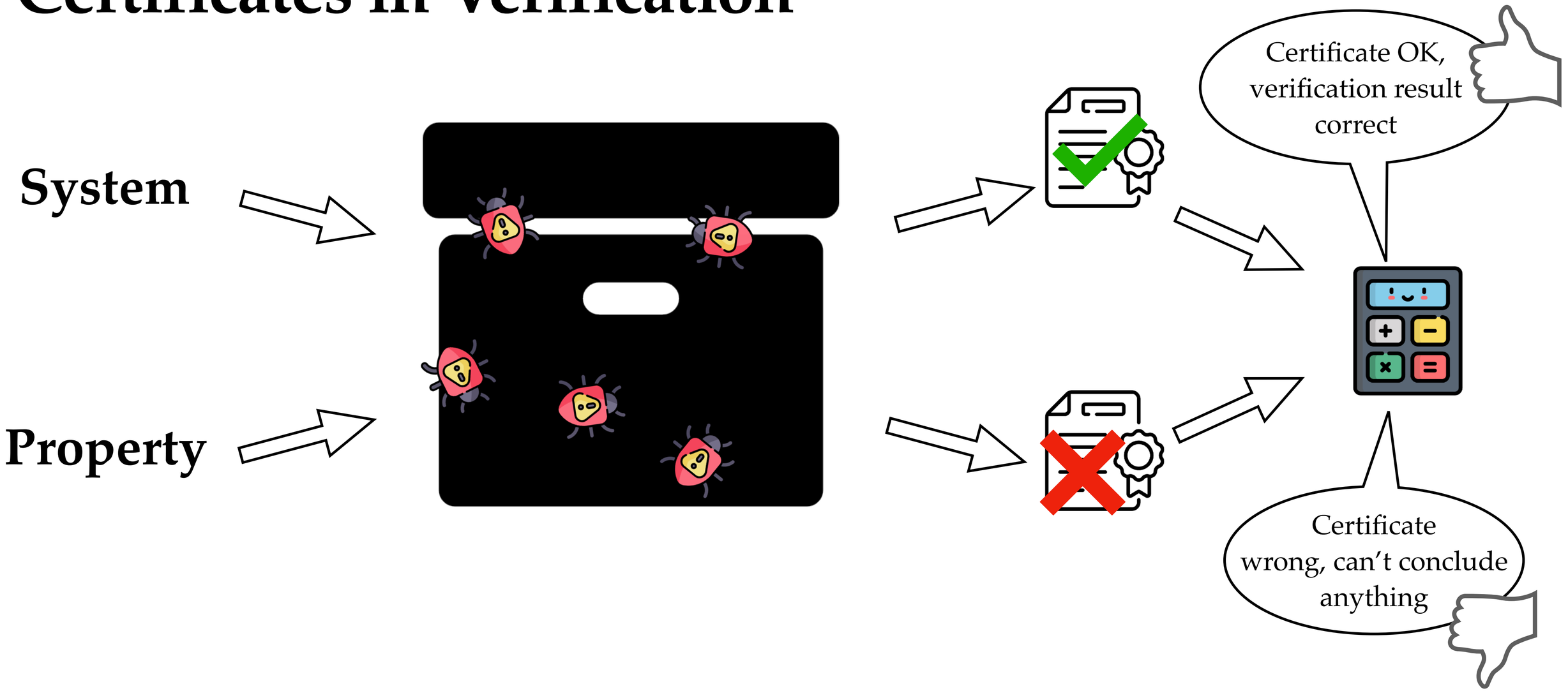

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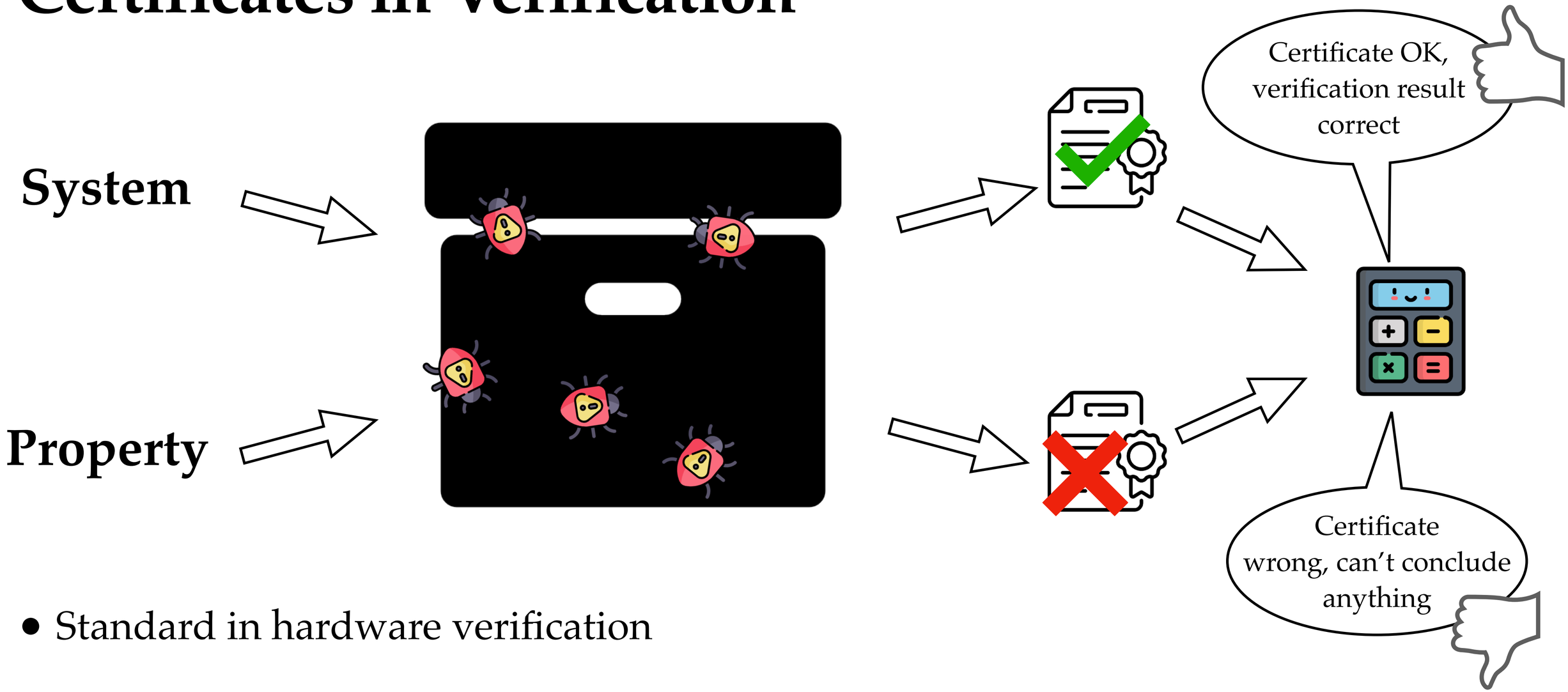
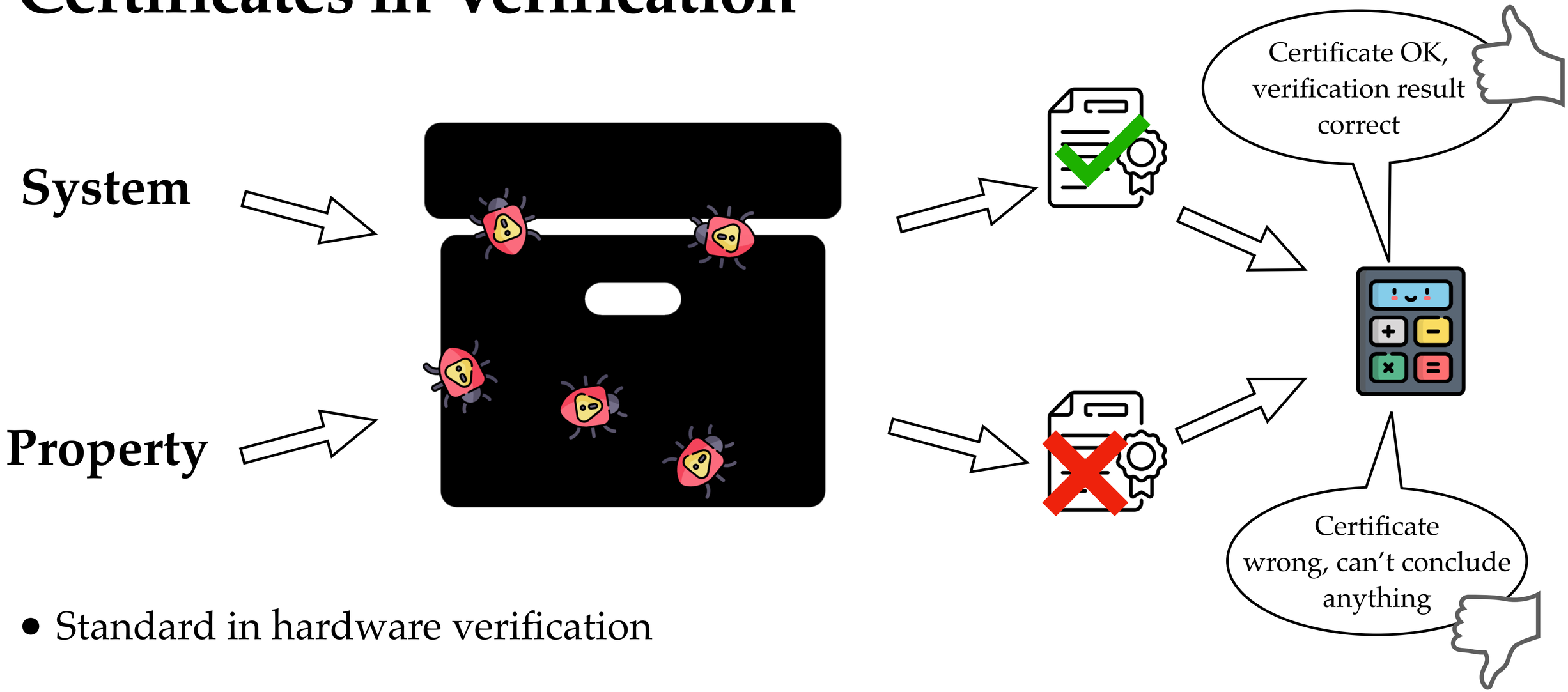


- Standard in hardware verification

Certificates in Verification



- Standard in hardware verification
- Our goal: Certified verification of **probabilistic** systems

Probabilistic Pushdown Automata

[Esparza, Kucera, Mayr, LICS 2004 + 2005]

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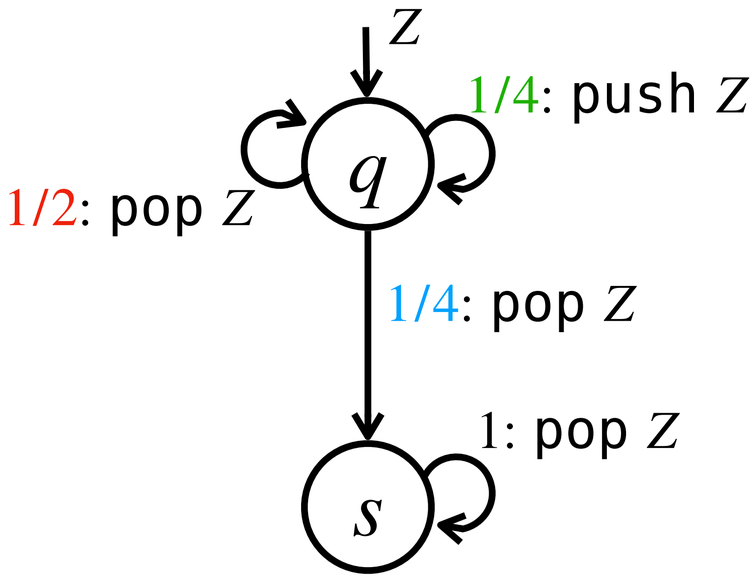

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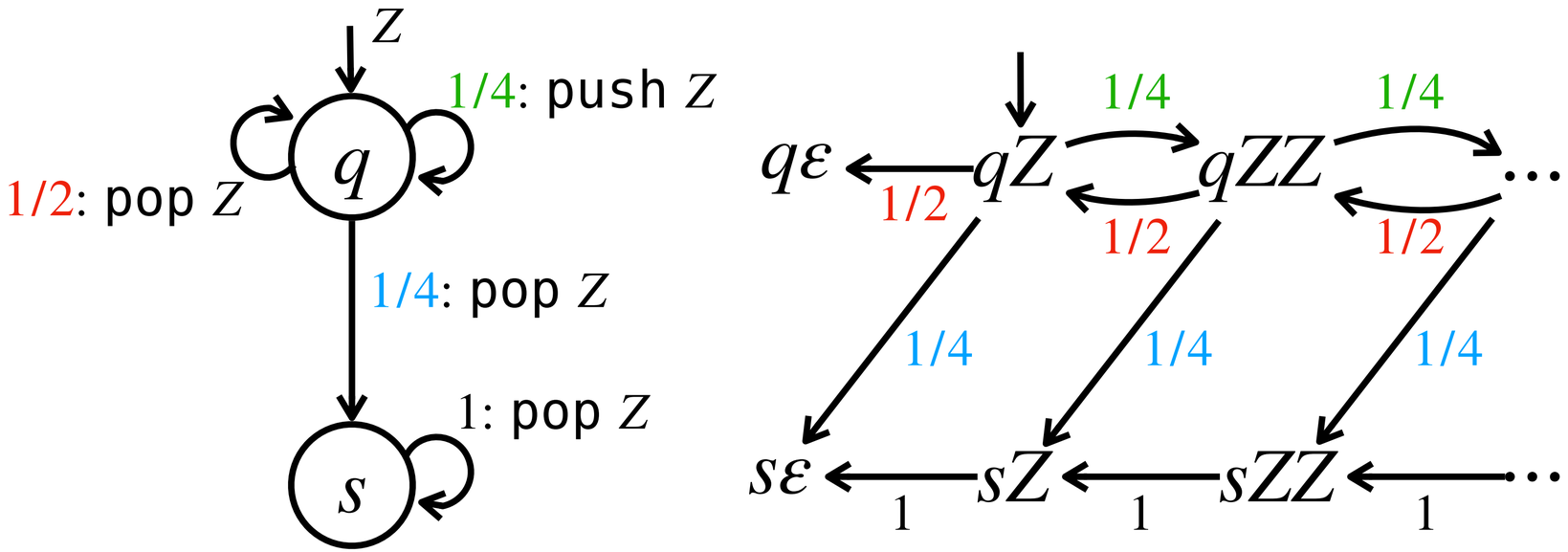

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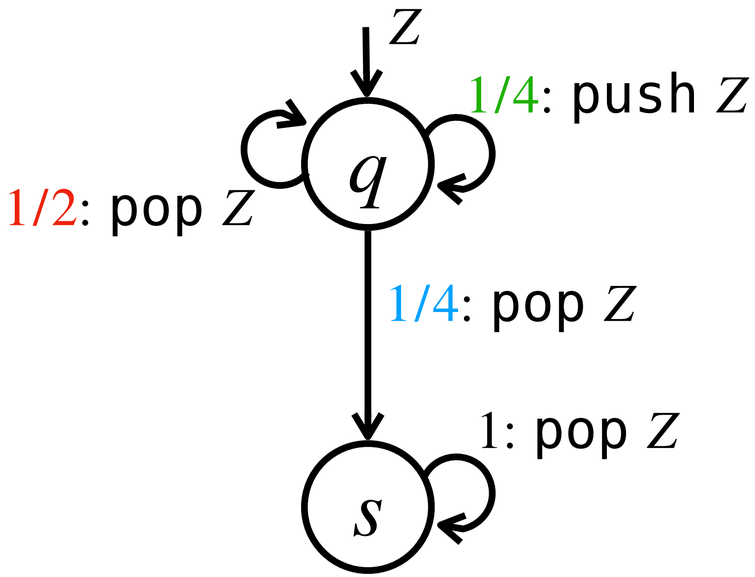

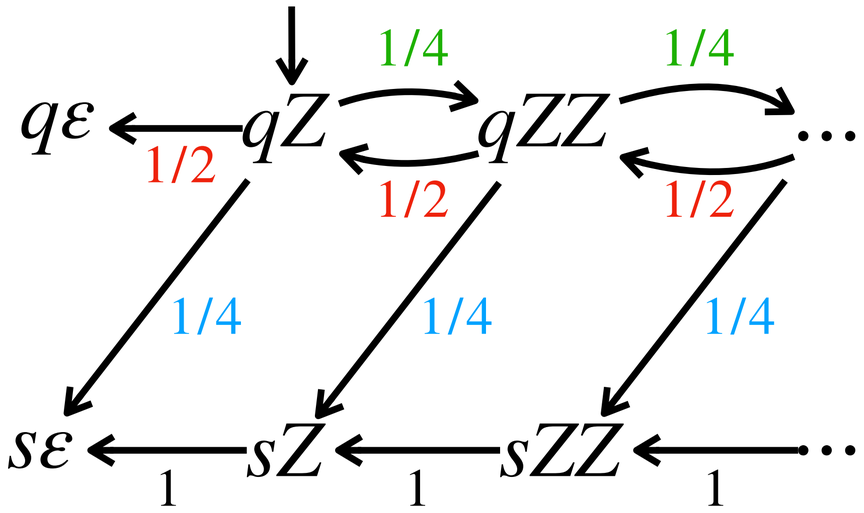


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bool f(bool b) {
  if(b) {
    choice {
      0.25: return f(f(b));
      0.50: return true;
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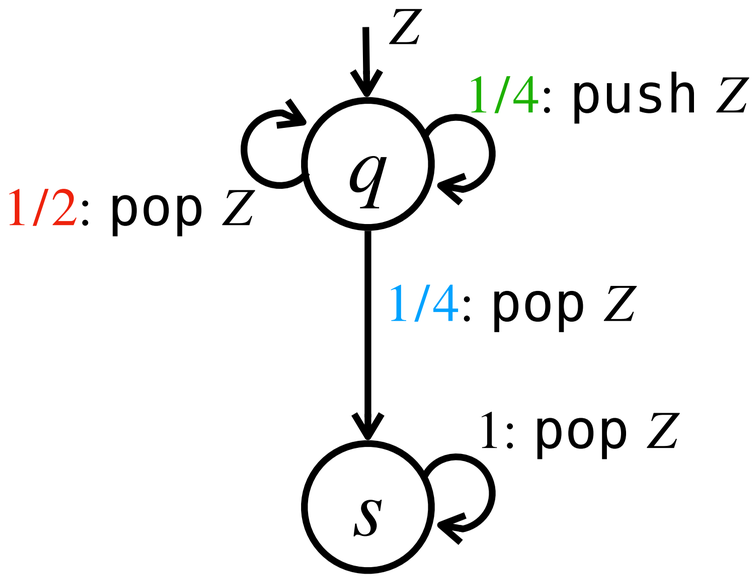

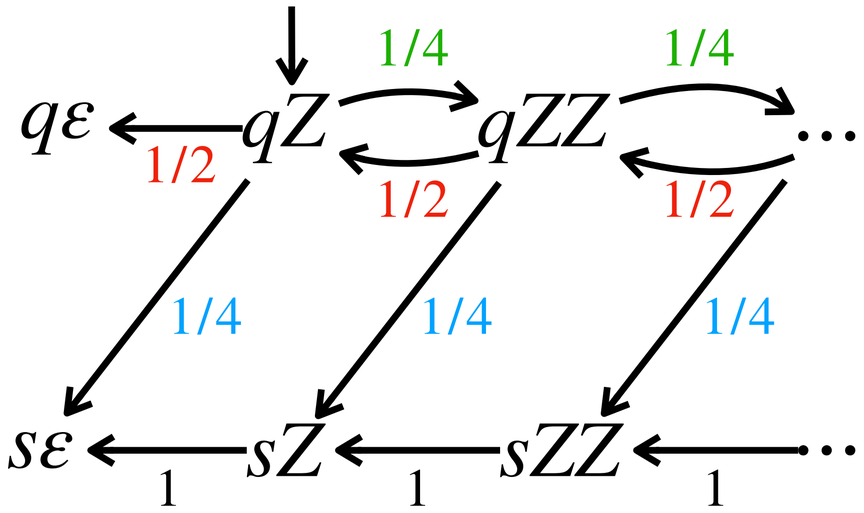

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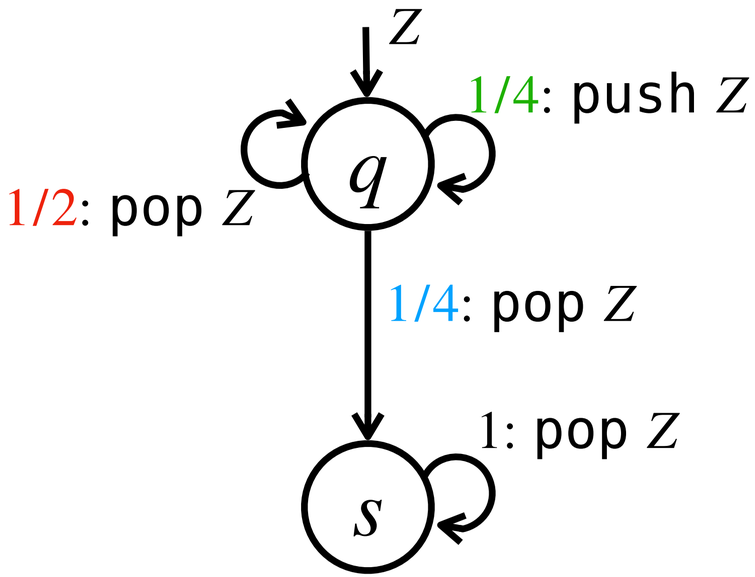

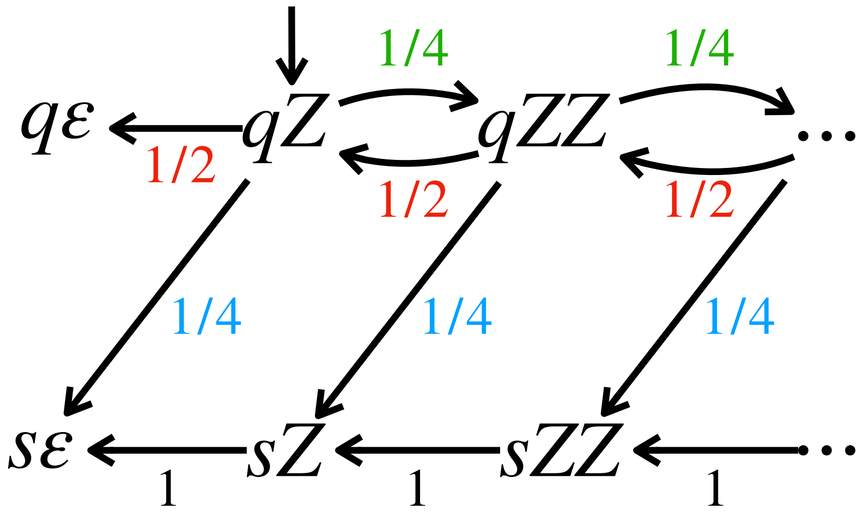

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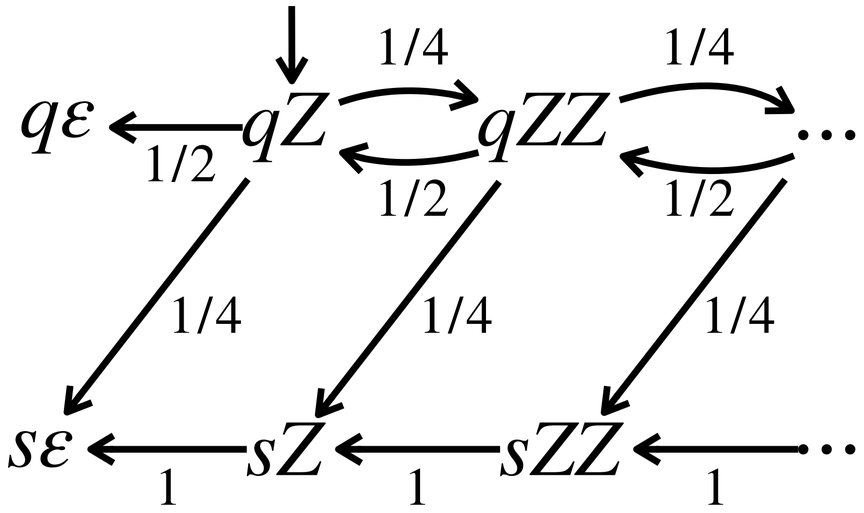
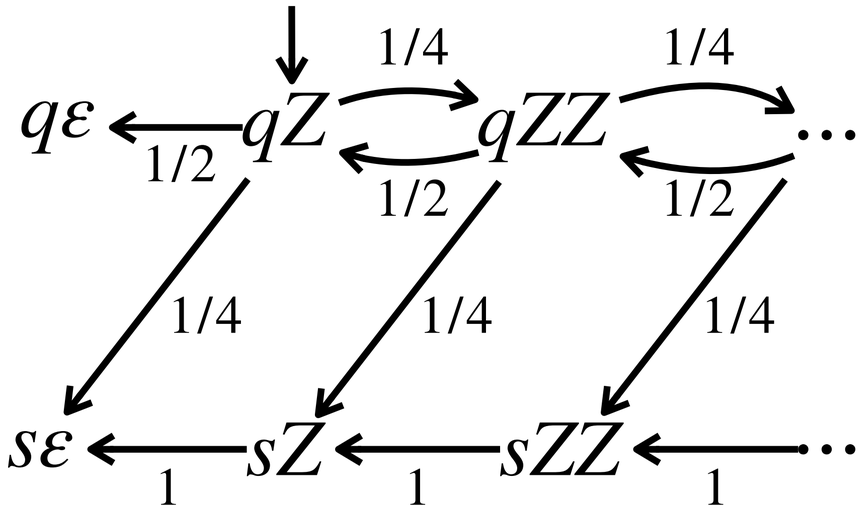
Certificates for **positive almost sure termination** (PAST)
= empty stack is reached after *finitely* many steps *on average*

Certifying Empty Stack Reachability



Certifying Empty Stack Reachability

Assign a rank to each configuration:



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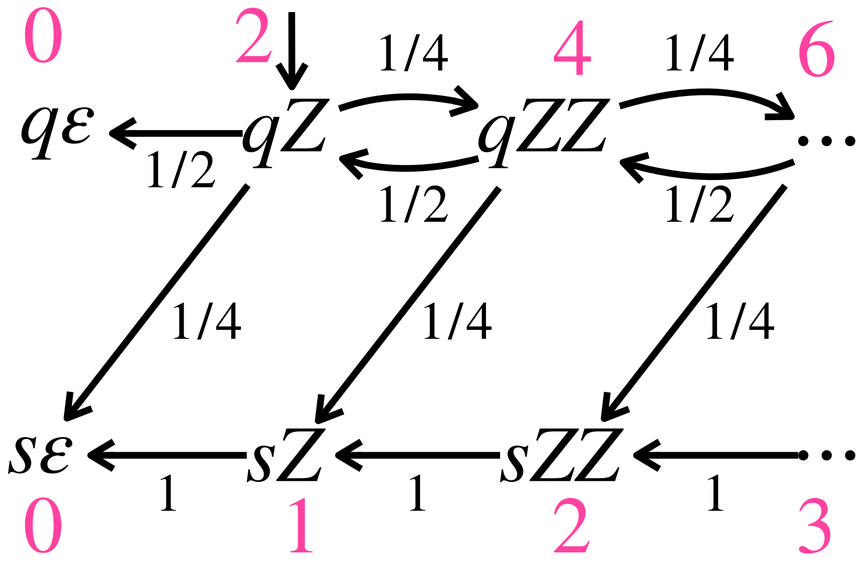

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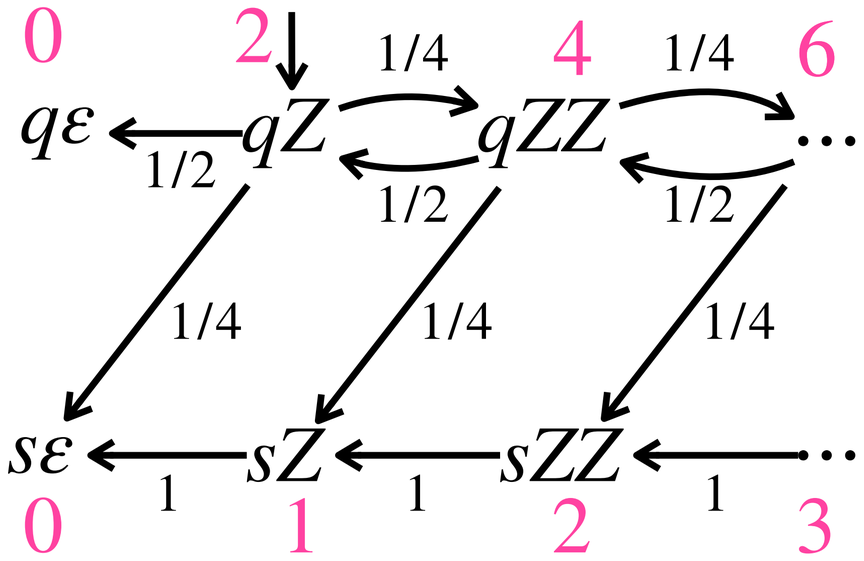
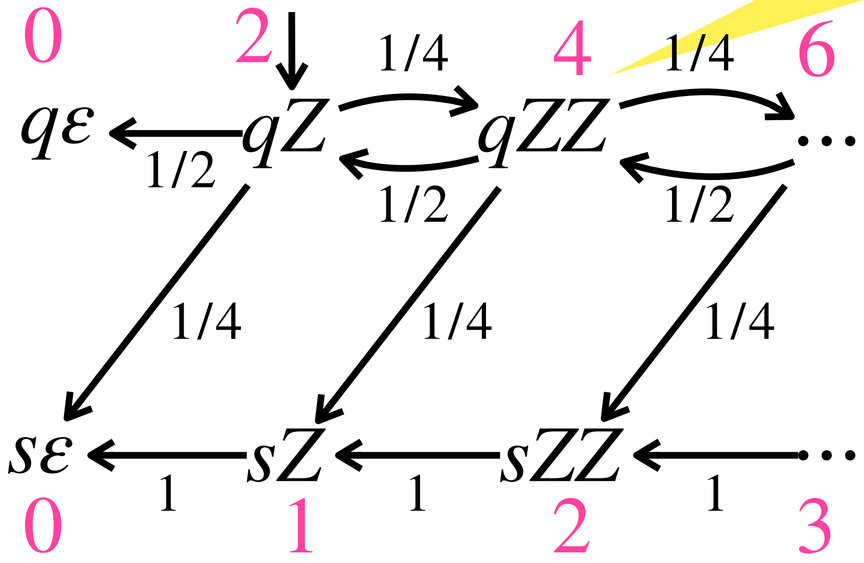


Ranks decrease by ≥ 1 *on average* in one step $\implies$ PAST

Certifying Empty Stack Reachability

Assign a rank to each configuration:

$$\frac{1}{2}\cdot 2+\frac{1}{4}\cdot 6+\frac{1}{4}\cdot 1 = 2.75 < 4$$



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Finite Representation of ∞-many Ranks?

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We only need

- ranks for each configuration qZ — quadratically many
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solution of polynomial equation system

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Theorem: Certificates for PAST

[W & Katoen, LICS 2023]

The probabilistic PDA Δ is **PAST**

$$\Longleftrightarrow$$

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there exist non-negative ranks $\vec{r}$ and probabilities $\vec{p}$ satisfying a system of polynomial inequalities (that can be "read off" from Δ in polynomial time).

Theorem: Certificates for PAST

[W & Katoen, LICS 2023]

The probabilistic PDA Δ is **PAST**

$\Leftrightarrow$

there exist non-negative rational ranks $\vec{r}$ and probabilities $\vec{p}$ satisfying a system of polynomial inequalities (that can be "read off" from Δ in polynomial time).

Theorem: Certificates for PAST

[W & Katoen, LICS 2023]

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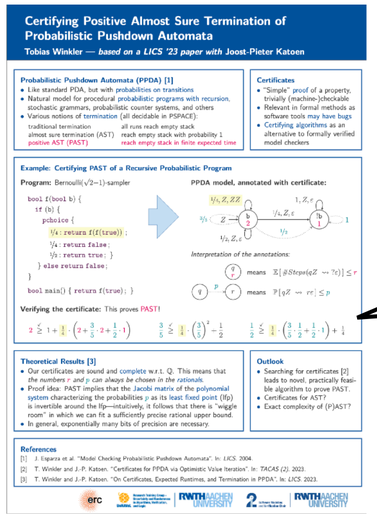
rational



Check poster for concrete example

Link to paper

images: Flaticon.com