Theorem 1. *Finite $L \subseteq Act^*$ is realizable (by a weak CFM) if and only if L is closed under $\models$.*

$\Longrightarrow$. Assume L is realizable. Thus, there exists a weak CFM A (a CFM without synchronization messages) such that $L = Lin(A)$. As $Lin(A)$ only contains linearizations, and every linearization is well-formed, each word in L is well formed. Let $w \in \mathtt{Act}^*$, be well-formed, and assume $L \models w$. By definition of $\models$, this means that for every process p there exists a word $v^p \in L$ such that $v^p \restriction p = w \restriction p$. We show that $w \in L$. (Then it follows that L is closed under $\models$.) This goes as follows.

Let π be an accepting run of CFM A on v^p. (Such run does exist, otherwise, v^p would not belong to L.) Let $\pi \restriction p$ be the projection of run π of A by only considering the transitions along π that take place at process p. Thus, transitions along $\pi_p = \pi \restriction p$ correspond to the "local" transitions of process p. It follows from $v^p \in L$ that the local run π_p is an accepting run of local automaton (NFA) A_p (of process p) on $v^p \restriction p$ (which equals $w \restriction p$). Here, accepting means that the local run π_p ends in a local accept state of A_p. This applies to all processes $P = \{p_1, \ldots, p_n\}$ of the CFM. The local accepting runs $\pi_{p_1} = \pi \restriction p_1, \ldots, \pi_{p_n} = \pi \restriction p_n$ can be combined to obtain a run, π^w say, of CFM A on w in a straightforward manner. The run π^w is accepting, as all processes end in a local accepting state, and all channels are empty, as π was accepting and π^w is constituted from "bits" spanning π. Thus there cannot be "open" receipts. Thus $w \in L$.

$\Longleftarrow$. Assume L is closed under $\models$. As $\models$ is only defined for well-formed words, each word in L is well formed. Moreover, by definition of closure under $\models$, $L \models w$ implies $w \in L$ for each well-formed $w \in Act^*$. Proof obligation: L is realizable. This goes as follows:

Let A_p be an NFA over the alphabet $\mathtt{Act}_p$ accepting $L_p = \{w \restriction p \mid w \in L\}$. As L is finite, L_p is finite and regular, thus A_p is indeed an NFA. A_p thus accepts all projections to process p of words in L. Let weak CFM $A = ((A_p)_{p \in P}, s_{init}, F)$ with $F = \prod_{p \in P} F_p$. We now claim that A is a realization of L, i.e., $Lin(A) = L$. This claim can be proven as follows:

- $\supseteq$ Let $w \in L$. By construction of CFM A, $Lin(A_p) = L_p$. But then $w \in Lin(A)$.
- $\subseteq$ Let $w \in Lin(A)$. Then $w \restriction p \in Lin(A_p)$ for each $p \in P$. By definition of $\models$, it follows $L \models w$. Since L is closed under $\models$, it follows $w \in L$.