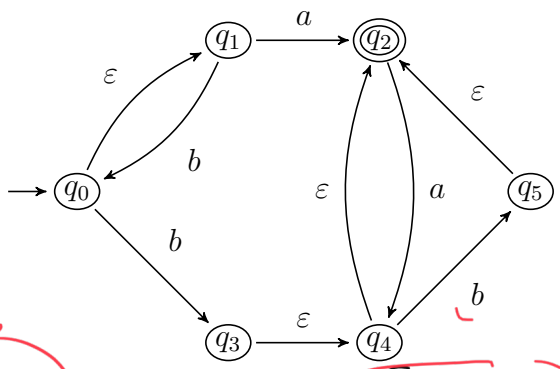
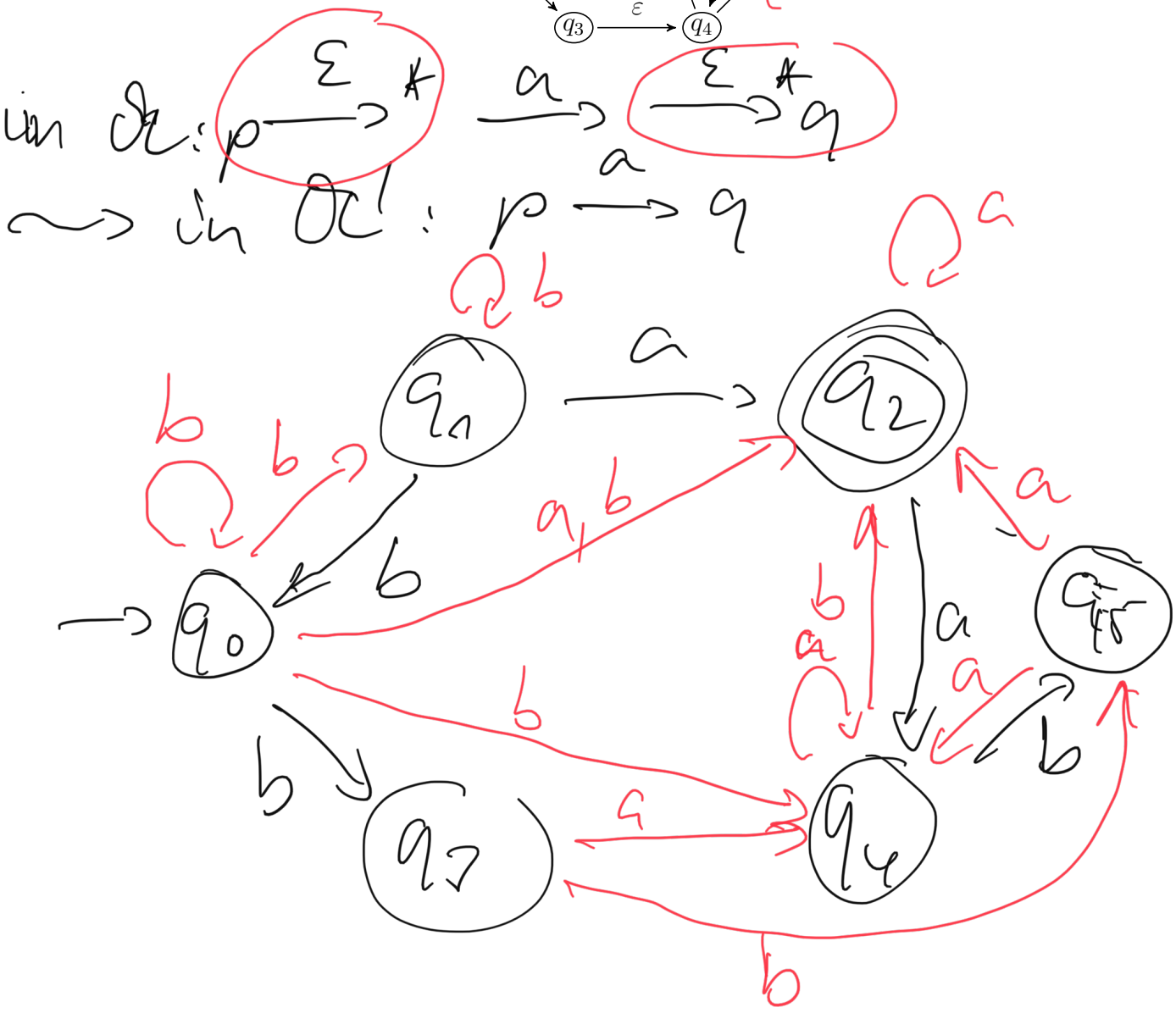
A6: Elimination of ε-Transitions

Task: Eliminate all ε-transitions of the following ε-NFA $\mathfrak{A}$ over $\Sigma := \{a, b\}$ to obtain an equivalent NFA.





A8: Construction of Regular Expressions

Task: Give regular expressions that describe the following languages.

(a) $L := \{w \in \{a,b\}^* \mid |w| \text{ divisible by } 3\}$

(b) $L := \{w \in \{a,b,c\}^* \mid w \text{ does not contain } a, b, \text{ or } c\}$

(c) $L := \{w \in \{a,b\}^* \mid \text{substring } ab \text{ occurs exactly twice in } w, \text{ but not at the end}\}$

Reg. expr.: $\emptyset$, Σ, $a \in \Sigma$

$\alpha \mid \beta$, $\alpha \cdot \beta$, α^*

(a) $((a \mid b)(a \mid b)(a \mid b))^*$ ∋ Σ, aaa, aba, ..., $aaabbb$, ...

(b) $(b \mid (b \mid c)^* \mid (a \mid c)^* \mid (a \mid b)^*$

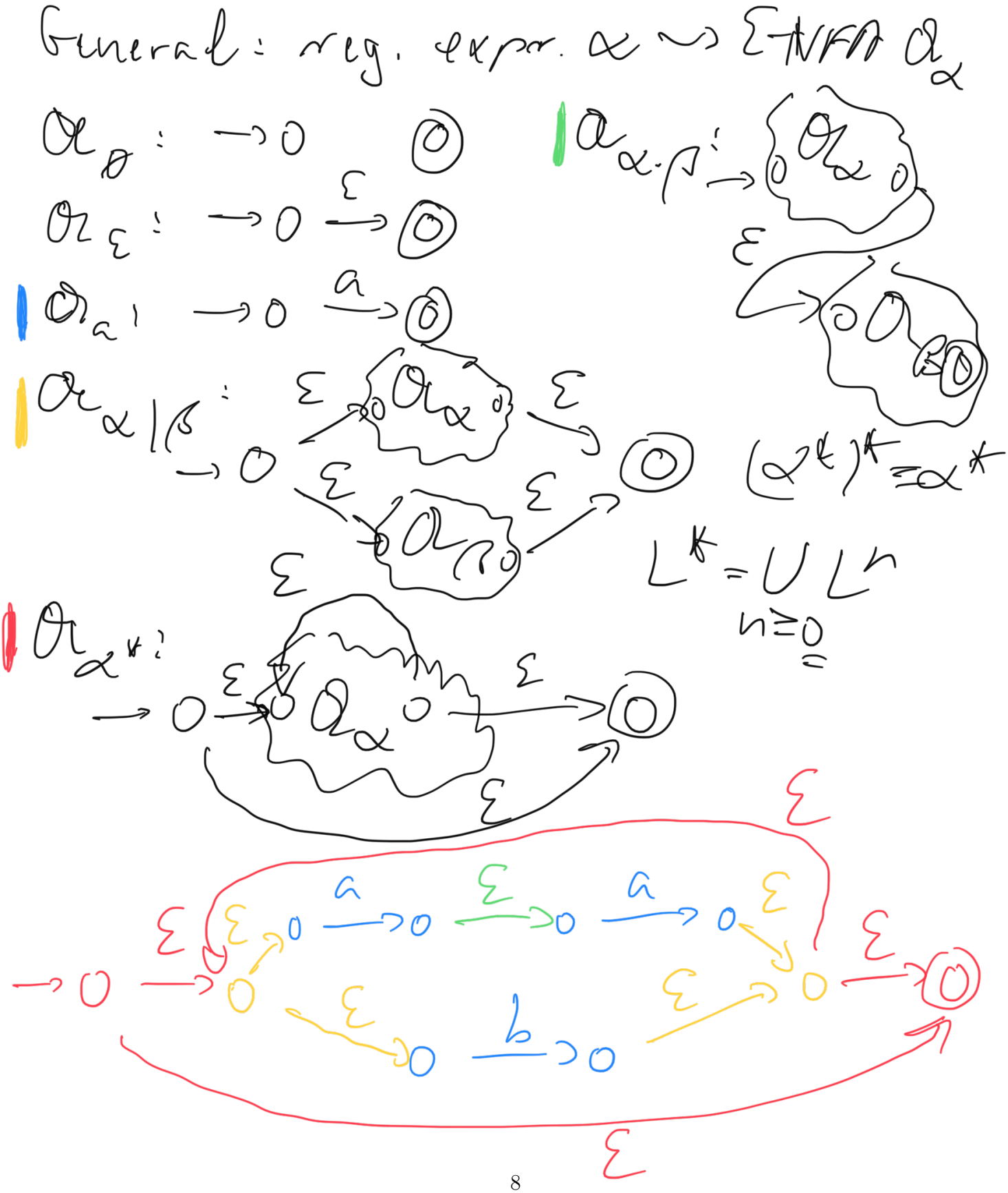
(c) $b^* \underbrace{a^+ b^+}_{1st} \underbrace{a^+ b}_{2nd} (b^+ a^* \mid b^* a^+)$

where $a^+ := aa^*$

$b^* \underbrace{a^+ b^+}_{ab} \underbrace{a^+ b}_{ab} \underbrace{b^* a^*}_{\varepsilon}$ ✓

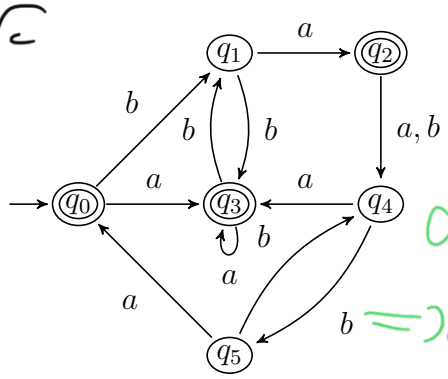
A9: From Regular Expressions to Finite Automata

Task: Using Kleene's construction, give the ε-NFA for the regular expression $(aa \mid b)^*$.



A10: Minimisation of Deterministic Finite Automata

Task: Minimise the following DFA.



(i) $p \in F,\ q \notin F$ → p, q distinguishable by Σ

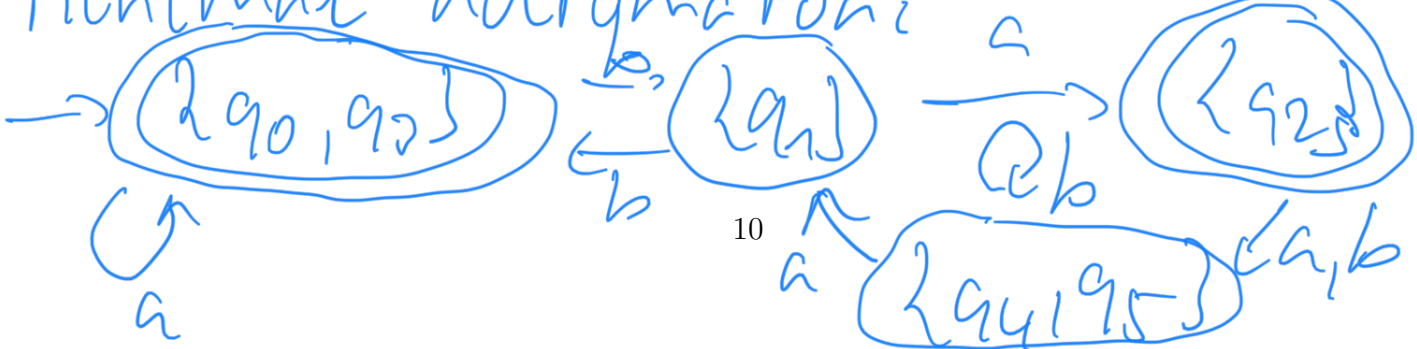
(ii) $\delta(q,a) = q'$, $\delta(p,a) = p'$, q', p' distinguishable by w ⇒ q, p distinguishable by aw

$p \sim q \triangleq \forall w \in \Sigma^*: \delta^*(p,w) \in F \iff \delta^*(q,w) \in F$

	q_0	q_1	q_2	q_3	q_4	q_5
q_0		ε	a	○	ε	ε
q_1			ε	ε	b	b
q_2				a	ε	ε
q_3					ε	ε
q_4						○
q_5						

$q_0 \sim q_3$, $q_4 \sim q_5$

Minimal automaton:

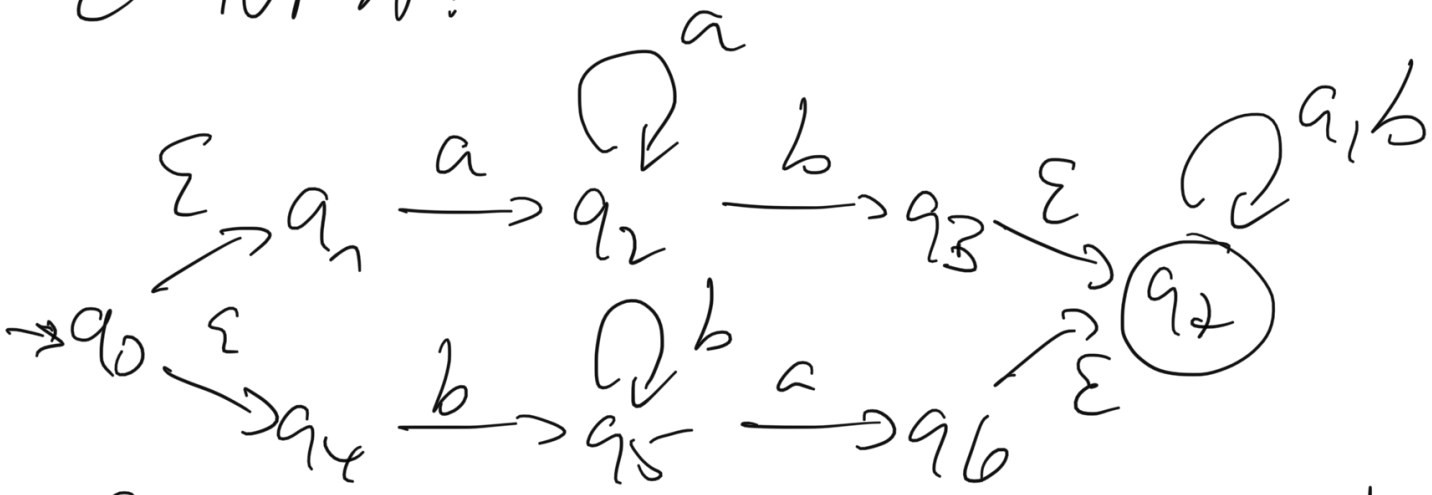


Lang. spec. $\longrightarrow$ min. DFA:

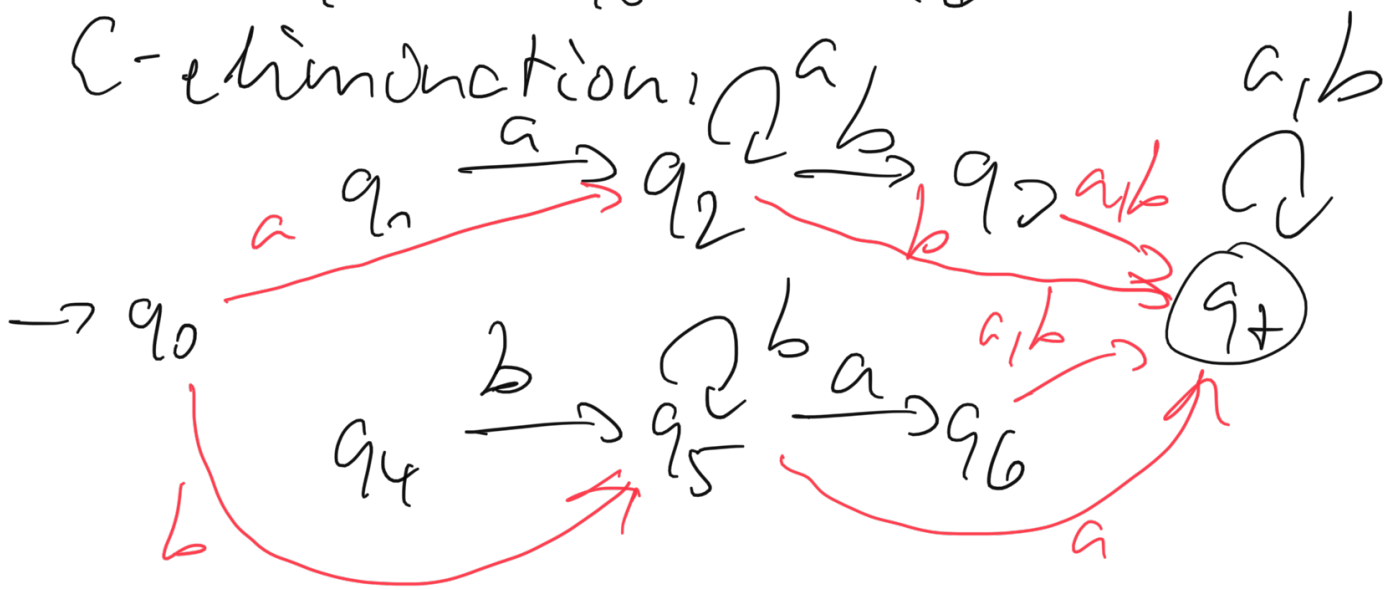
$L = \{ w \in \{a,b\}^* \mid$ at least one a and — n — one b in $w \}$

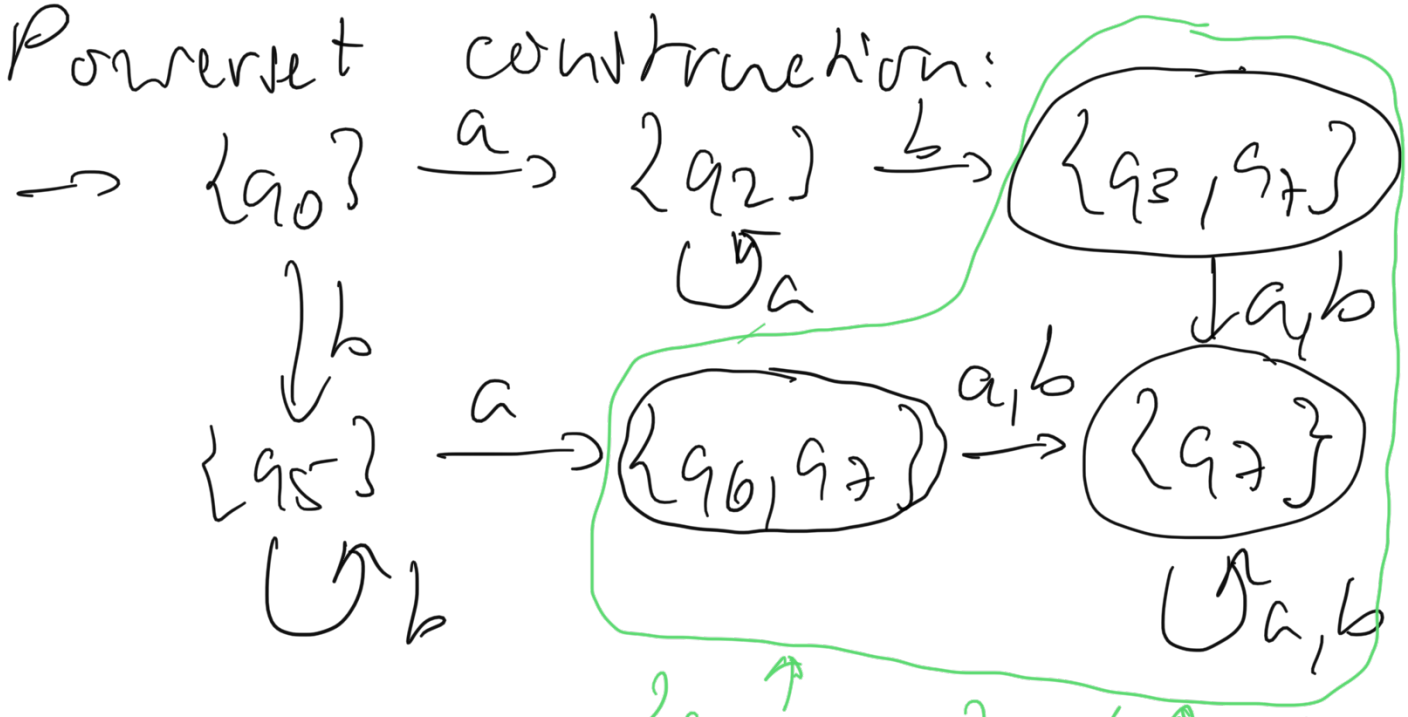
$\alpha = (a^+ b \mid b^+ a)(a|b)^*$

ε-NFA:



ε-elimination:





Minimization:

	q_0	q_2	q_5	q_{27}	q_{67}	q_7
q_0		b	a	Σ	Σ	Σ
q_2			a	Σ	Σ	Σ
q_5				Σ	Σ	Σ
q_{27}					×	×
q_{67}						×
q_7						

$\Rightarrow q_{27} \sim q_{67} \sim q_7$