C4: The Word Problem for Context-Free Languages

Task: Let G be the following context-free grammar:

$$
\begin{aligned}
S &\to AB \mid BC \\
A &\to BA \mid a \\
B &\to CC \mid b \\
C &\to AB \mid a
\end{aligned}
$$

$L(G) = \{ w \in \Sigma^* \mid S \Rightarrow^* w \}$

and let $w := baaba$. Employ the CYK-Algorithm to show that $w \in L(G)$. Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \qquad (1 \le i \le j \le 5)$$

where $w[i,j] := a_i \ldots a_j$ for $w = a_1 a_2 a_3 a_4 a_5$.

$i \backslash j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	

Solution:

$i \backslash j$	1	2	3	4	5
1	B	S, A	$\emptyset$	$\emptyset$	S, A, C
2	X	A, C	B	B	S, A, C
3	X	X	A, C	S, C	B
4	X	X	X	B	S, A
5	X	X	X	X	A, C

$S \in N_{1,5} \Rightarrow w \in L(G)$

$w[1,2] = w[1,1] \cdot w[2,2]$

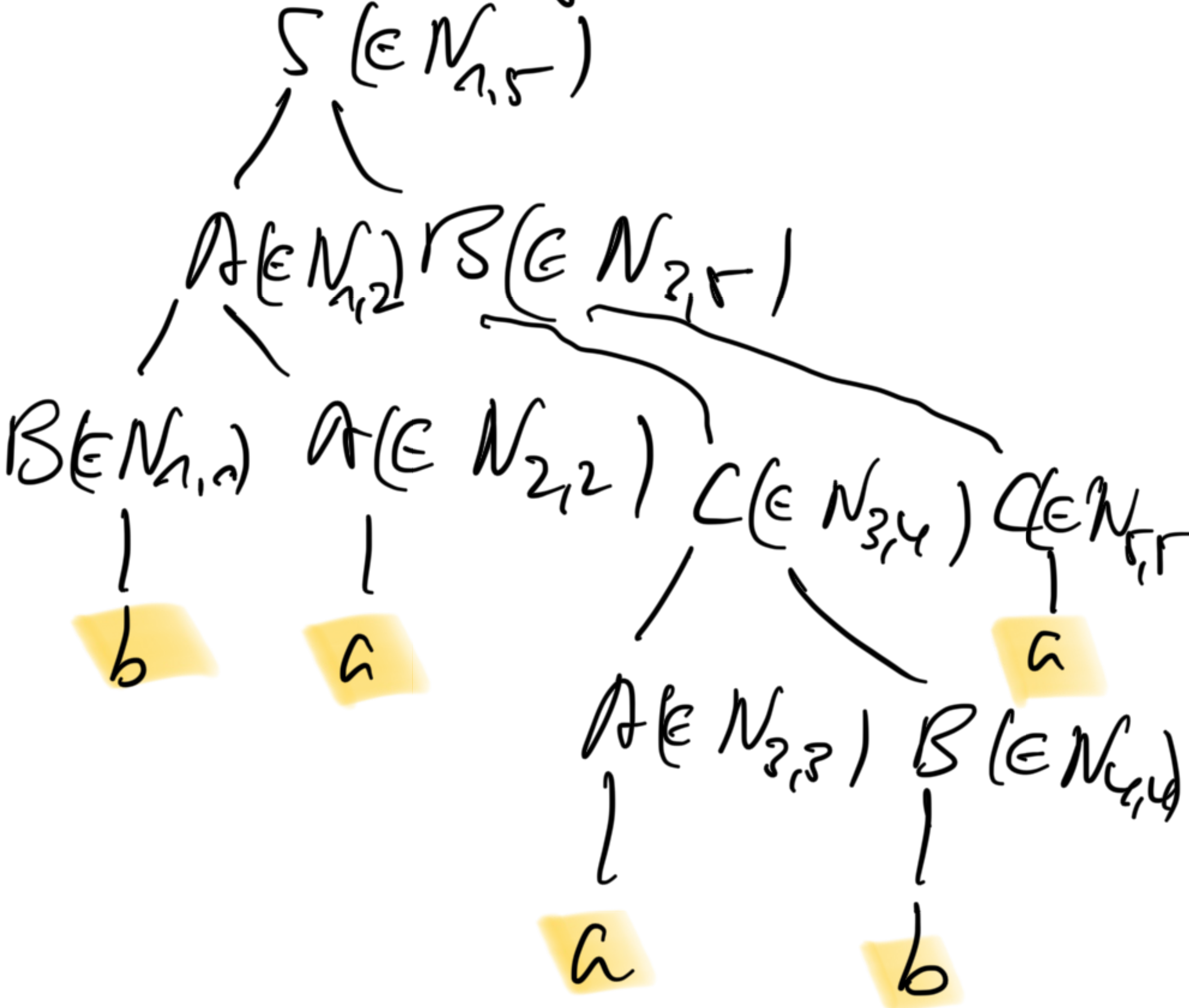
$w[1,5] = w[1,1] \cdot w[2,5]$

$B \in N_{1,1},\ A \in N_{2,5},\ A \to BA \in P \Rightarrow A \in N_{1,5}$

$B \in N_{1,1},\ C \in N_{2,5},\ S \to BC \in P \Rightarrow S \in N_{1,5}$

$A \in N_{1,2}$, $B \in N_{3,5}$, $S \to AB \in P$, $C \to AB$ $\Rightarrow S, C \in N_{1,5}$

Reconstruction of syntax tree:



$\Rightarrow baaba \in L(G)$

C5: The Emptiness Problem for Context-Free Languages

Task: Using the marking algorithm, check whether the following grammars generate empty languages or not:

(a)
$$\begin{aligned} S &\to XU \mid UW \\ U &\to SaW \mid VbU \mid SbX \\ V &\to U \mid ab \mid XW \\ W &\to bXaUb \mid VbX \mid SX \\ X &\to bV \mid WX \end{aligned}$$

S unmarked
$\Rightarrow S$ non-productive
$\Rightarrow L(G) = \emptyset$

(b)
$$\begin{aligned} S &\to AC \mid DA \\ A &\to SbC \mid BaB \mid AaD \\ B &\to A \mid ba \mid DC \\ C &\to aDbAa \mid BaD \mid SD \\ D &\to aB \mid CD \end{aligned}$$

S marked
$\Rightarrow S$ productive
$\Rightarrow L(G) \neq \emptyset$

Solution: Compute productive nonterminals

$A \in N$ is productive iff there exists $w \in \Sigma^*$ such that $A \Rightarrow^* w$

(Thus: $L(G) \neq \emptyset$ iff S is productive)

0. Initialisation
1. 1st phase
2. 2nd phase
3. 3rd phase

Generated:

$S \Rightarrow DA \Rightarrow aBA \Rightarrow abaA \Rightarrow abaBaB$

ababaaba

C7: From Context-Free Grammars to Pushdown Automata

Task: Construct a PDA that accepts the language generated by the grammar

$$S \to aSa \mid bSb \mid \varepsilon$$

(which is $L = \{ww^R \mid w \in \{a,b\}^*\}$, i.e., palindromes of even length).

Example: $S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$

Solution: In general:

$$\mathcal{A}_G = (Q, \Sigma, \Gamma, \Delta, q_0, Z_0, F)$$

where: $Q = \{q_0\}$ $\quad \Gamma = \Sigma \cup N$ $\quad Z_0 = S$

for $A \to \alpha \in P$:

$((q_0, A, \varepsilon), (q_0, \alpha)) \in \Delta$ ("expansion")

for $a \in \Sigma$: $((q_0, a, a), (q_0, \varepsilon))$ ("matching")

$F = Q$

Here: $Q = \{q_0\}$ $\quad \Gamma = \Sigma \cup N$ $\quad Z_0 = S$

Δ: $((q_0, S, \varepsilon), (q_0, aSa))$

$((q_0, S, \varepsilon), (q_0, bSb))$

$((q_0, S, \varepsilon), (q_0, \varepsilon))$

$((q_0, a, a), (q_0, \varepsilon))$

$((q_0, b, b), (q_0, \varepsilon))$

~~**A12: “Toolchain”**~~

~~**Task:** Construct a DFA accepting the language described by $\alpha = (ab^*)^*$.~~

$$
\begin{aligned}
(q_0, S, abba) &\vdash (q_0, aSa, abba) \\
&\vdash (q_0, Sa, bba) \\
&\vdash (q_0, bSba, bba) \\
&\vdash (q_0, Sba, ba) \\
&\vdash (q_0, ba, ba) \\
&\vdash (q_0, a, a) \\
&\vdash (q_0, \varepsilon, \varepsilon) \quad \text{(final conf.)}
\end{aligned}
$$

$$\Longrightarrow abba \in L(\mathfrak{A}_0)$$

$$(q_0, S, abba) \vdash (q_0, bSb, abba)$$

$$\not\vdash \ \lightning$$