C5: The Emptiness Problem for Context-Free Languages

Task: Using the marking algorithm, check whether the following grammars generate empty languages or not:

(a)
$$
\begin{aligned}
S &\to XU \mid UW\\
U &\to SaW \mid VbU \mid SbX\\
V &\to U \mid ab \mid XW\\
W &\to bXaUb \mid VbX \mid SX\\
X &\to bV \mid WX
\end{aligned}
$$

Initialization
1st iteration
2nd iteration
3rd iteration

(b)
$$
\begin{aligned}
S &\to AC \mid DA\\
A &\to SbC \mid BaB \mid AaD\\
B &\to A \mid ba \mid DC\\
C &\to aDbAa \mid BaD \mid SD\\
D &\to aB \mid CD
\end{aligned}
$$

(a) S not prod.
$\Rightarrow L(G) = \emptyset$

(b) S productive
$\Rightarrow L(G) \neq \emptyset$

$G = (N, \Sigma, P, S)$

$A \in N$ productive

$\Leftrightarrow \exists w \in \Sigma^* : A \Rightarrow^* w$

$L(G) \neq \emptyset$

$\Leftrightarrow S$ productive

(b) $S \Rightarrow DA \Rightarrow aBA \Rightarrow abaA$
$\Rightarrow abaBab \Rightarrow^2 abababaaba$

C8: From Context-Free Grammars to Pushdown Automata

Task: Construct a PDA that accepts the language generated by the grammar

$$S \to aSa \mid bSb \mid \varepsilon$$

(which is $L = \{ww^R \mid w \in \{a,b\}^*\}$, i.e., palindromes of even length).

$L = \{a^n b^n \mid n \geq 0\} \subseteq a^* b^*$

Solution: $\mathfrak{A}_G := (Q, \Sigma, \Gamma, \Delta, q_0, Z_0, F)$

- $Q = \{q_0\}$, $\Gamma = N \cup \Sigma$,
  $Z_0 = S$
- for $A \to \alpha \in P$: ("expansion")
  $((q_0, A, \varepsilon), (q_0, \alpha)) \in \Delta$
- for each $a \in \Sigma$: ("matching")
  $((q_0, a, a), (q_0, \varepsilon)) \in \Delta$

Here:

- $Q = \{q_0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{S, a, b\}$,
  $Z_0 = S$
- Δ: $((q_0, S, \varepsilon), (q_0, aSa))$
  $((q_0, S, \varepsilon), (q_0, bSb))$
  $((q_0, S, \varepsilon), (q_0, \varepsilon))$
  $((q_0, a, a), (q_0, \varepsilon))$, $((q_0, b, b), (q_0, \varepsilon))$,

For example: Successful run:

$(q_0, S, abba) \vdash (q_0, aSa, abba)$
$\vdash (q_0, Sa, bba) \vdash (q_0, bSba, bba)$
$\vdash (q_0, Sba, ba) \vdash (q_0, ba, ba)$
$\vdash (q_0, a, a) \vdash (q_0, \varepsilon, \varepsilon)$

Failing run:

$(q_0, S, abba) \vdash (q_0, aSa, abba)$
$\vdash (q_0, Sa, bba) \vdash (q_0, a, bba) \lightning$