

# Exercises (Context-Free Languages)

## C1: Construction of Context-Free Grammars

**Task:** Give context-free grammars that generate the following languages.

(a)  $L := \{a^k b^l c^{k+l} \mid k, l \in \mathbb{N}\}$

(b)  $L := \{a^k b^k c^l d^l \mid k, l \in \mathbb{N}\}$

(c)  $L := \{a^k b^l \mid k \geq 1, l > k\}$

(d)  $L := \{w \in \{a, b\}^* \mid |w| \text{ odd, } a \text{ in middle position}\} (= \{uav \in \{a, b\}^* \mid |u| = |v|\})$

(e)  $L = \{a^k b^l c^m \mid k, l, m \in \mathbb{N}, k = l \text{ or } k = m\}$



## C2: From Regular to Context-Free Languages

**Task:** Show that every regular expression can directly be translated into an equivalent context-free grammar.



### C3: Chomsky Normal Form

**Task:** Transform the following grammar into Chomsky Normal Form:

$$S \rightarrow xAx \mid CyBA \mid BB \mid z \mid xxx$$

$$A \rightarrow C \mid xy$$

$$B \rightarrow A$$

$$C \rightarrow yyy \mid B$$

## C4: The Word Problem for Context-Free Languages

**Task:** Let  $G$  be the following context-free grammar:

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

and let  $w := baaba$ . Employ the CYK-Algorithm to show that  $w \in L(G)$ . Use the following table to compute the sets

$$N_{i,j} := \{A \in N \mid A \Rightarrow^* w[i,j]\} \quad (1 \leq i \leq j \leq 5)$$

where  $w[i,j] := a_i \dots a_j$  for  $w = a_1 a_2 a_3 a_4 a_5$ .

$i \setminus j$	1	2	3	4	5
1					
2	X				
3	X	X			
4	X	X	X		
5	X	X	X	X	