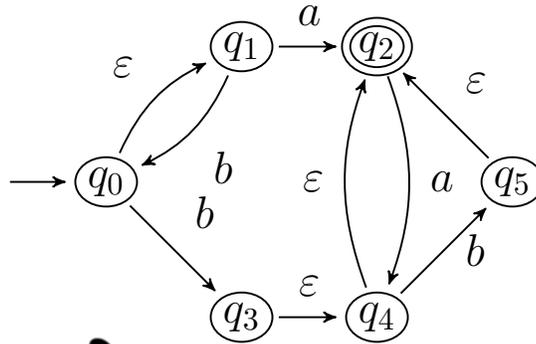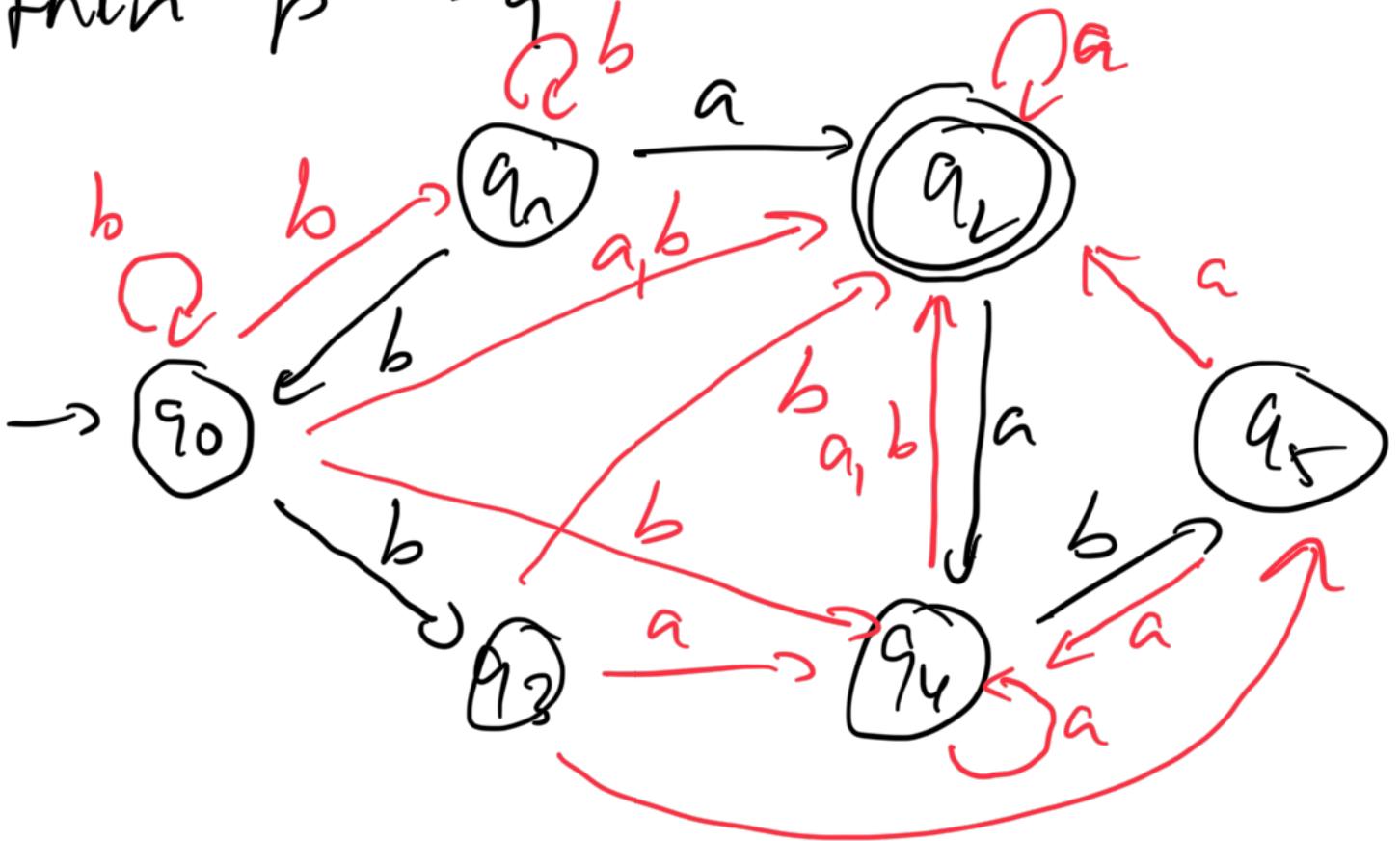# A6: Elimination of ε-Transitions

Task: Eliminate all ε-transitions of the following ε-NFA $\mathfrak{A}$ over $\Sigma := \{a, b\}$ to obtain an equivalent NFA.

$$\Sigma\text{-NFA}^{\mathfrak{A}} \rightsquigarrow \text{NFA } \mathfrak{A}':$$

$$\text{If } p \xrightarrow{\varepsilon, *} \cdot \xrightarrow{a} \cdot \xrightarrow{\varepsilon, *} q \text{ in } \mathfrak{A},$$

$$\text{then } p \xrightarrow{a} q$$

$$(\text{If } q_0 \xrightarrow{\varepsilon} q \text{ for some } q \in F \ (\text{in } \mathfrak{A}),$$
$$\text{then } q_0 \in F' \ (\text{in } \mathfrak{A}'))$$

A7: Construction of Regular Expressions

Task: Give regular expressions that describe the following languages.

(a) $L := \{w \in \{a,b\}^* \mid |w| \text{ divisible by } 3\}$

(b) $L := \{w \in \{a,b,c\}^* \mid w \text{ does not contain } a, b, \text{ or } c\}$

(c) $L := \{w \in \{a,b\}^* \mid \text{substring } ab \text{ occurs exactly twice in } w, \text{ but not at the end}\}$

Identifiers $(A|\ldots|Z|a|\ldots|z) \cdot$
$\qquad (A|..|Z|a|.-|z|0|\ldots|9|$
$\qquad \$|.\sim)^*$

(a) $w = a_1 \sim \cdot a_n \implies |w| = n$
$\quad ((a|b)(a|b)(a|b))^* \qquad \alpha^{t_i} = \alpha\alpha^*$

(b) $(a|b)^* \mid (a|c)^* \mid (b|c)^*$

(c) $(a|b)^* \, ab \, (a|b)^* \, ab \, (a|b|$
$\quad b^*a^*ab \, b^*a^*ab \, ((a|b)a^* \mid bb^*)$
$\quad b^* \, a^+ \, b^+ \, a^+ \, b \, (b^+a^* \mid b^*a^+)$

A8: From Regular Expressions to Finite Automata

Task: Using Kleene's construction, give the ε-NFA for the regular expression $(aa \mid b)^*$.

a: $\longrightarrow 0 \xrightarrow{a} \circledcirc$

$a \cdot a$: $\longrightarrow 0 \xrightarrow{a} 0 \xrightarrow{\varepsilon} 0 \xrightarrow{a} \circledcirc$

b: $\longrightarrow 0 \xrightarrow{b} \circledcirc$

$aa \mid b$: $\;\varepsilon \nearrow 0 \xrightarrow{a} 0 \xrightarrow{\varepsilon} 0 \xrightarrow{a} 0 \;\varepsilon$

$\longrightarrow 0 \searrow^{\varepsilon} 0 \xrightarrow{b} 0 \xrightarrow{\;\;\varepsilon\;\;} \circledcirc$

$(aa \mid b)^*$:

$\;\varepsilon \nearrow 0 \xrightarrow{a} 0 \xrightarrow{\varepsilon} 0 \xrightarrow{a} 0 \;\varepsilon$

$\longrightarrow 0 \xrightarrow{\varepsilon} 0 \;\varepsilon \qquad 0 \xrightarrow{\varepsilon} \circledcirc$

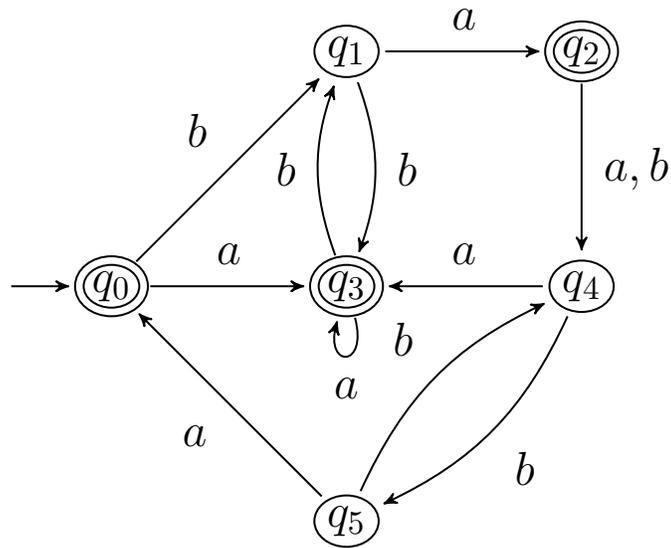$\varepsilon \searrow 0 \xrightarrow{b} 0 \xrightarrow{\varepsilon}$

ε
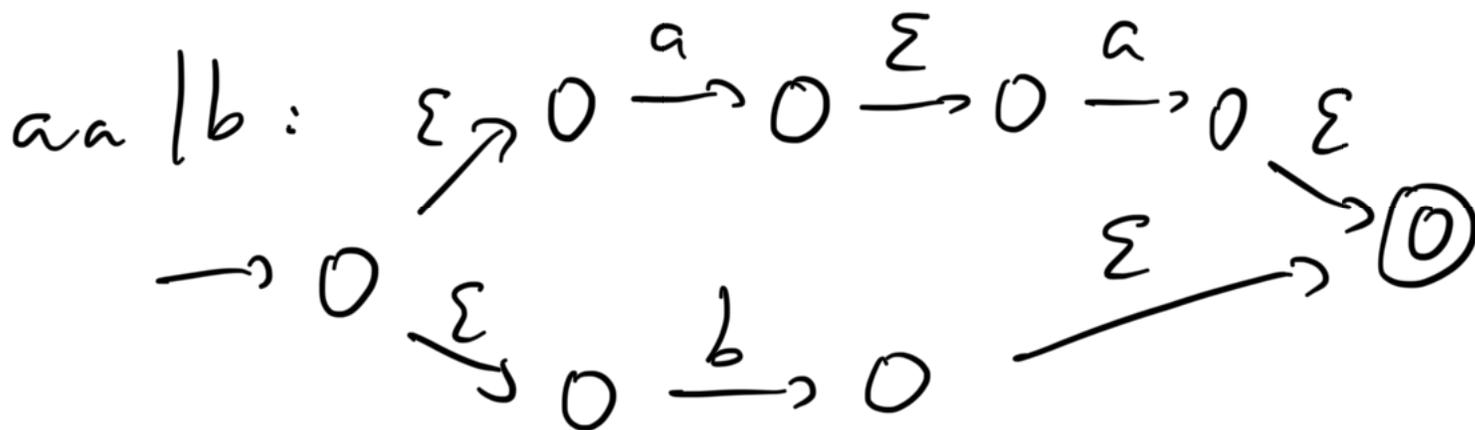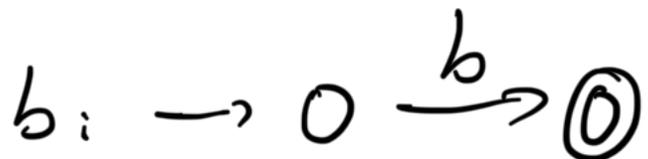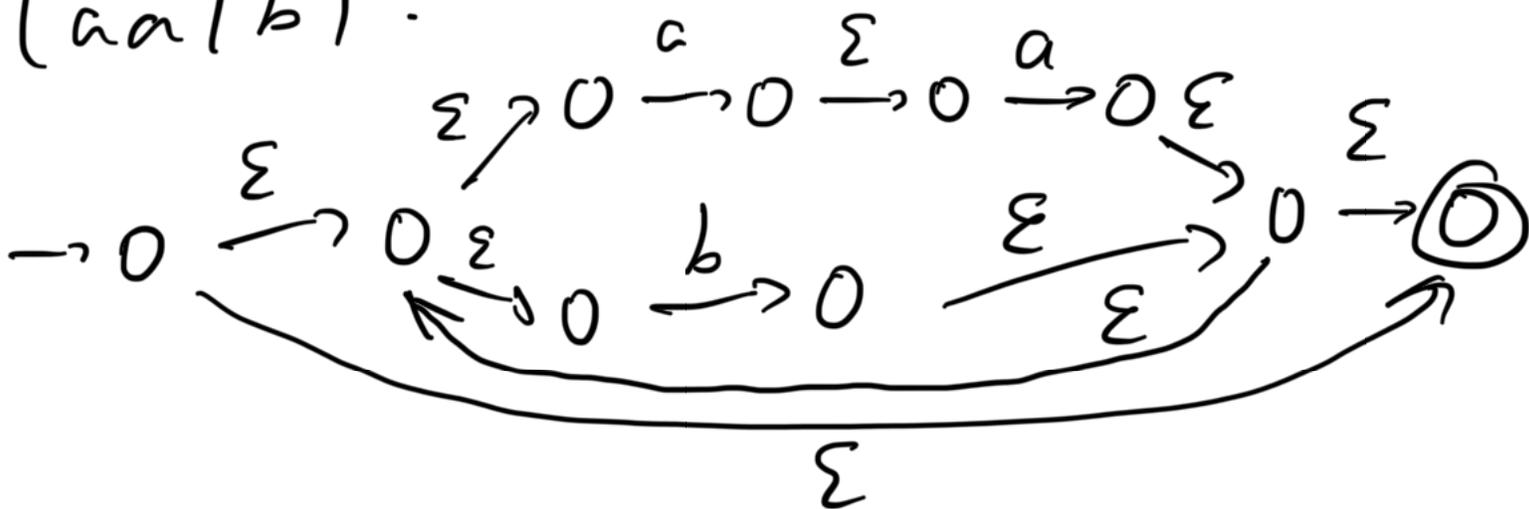
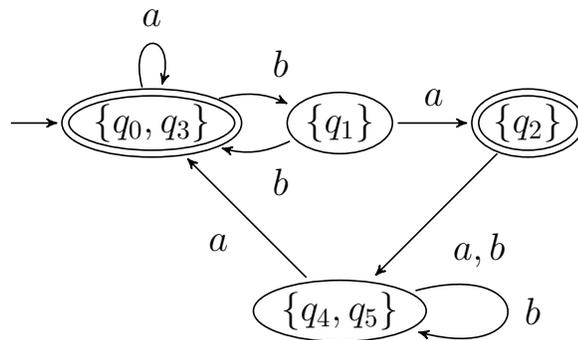# A10: Minimisation of Deterministic Finite Automata

Task: Minimise the following DFA.

Solution: For DFA $\mathfrak{A} = \langle Q, \Sigma, \delta, q_0, F \rangle$: $p, q \in Q$ equivalent if

$$\forall w \in \Sigma^* : \delta^*(p, w) \in F \iff \delta^*(q, w) \in F.$$

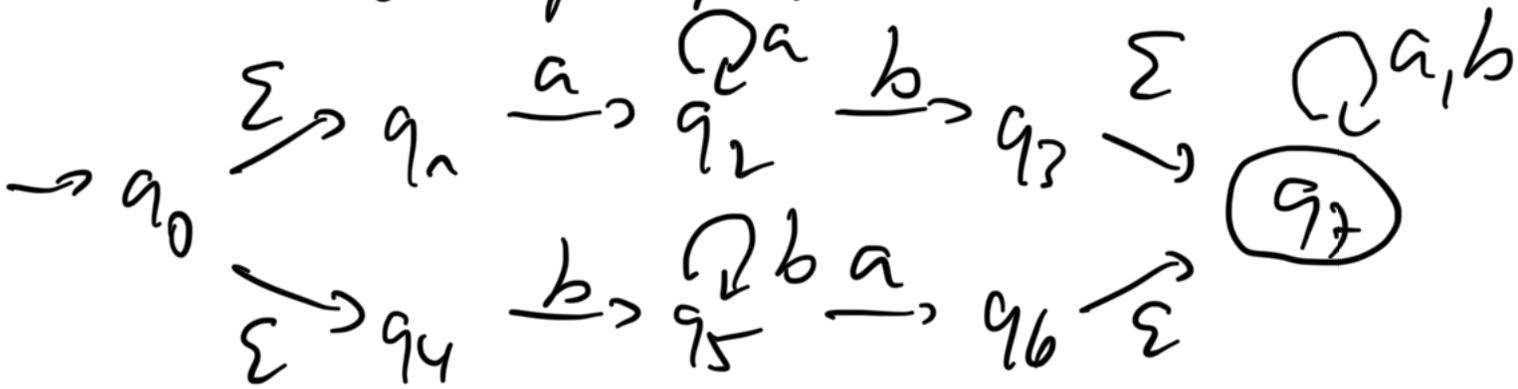|	q_0	q_1	q_2	q_3	q_4	q_5
q_0	—	ε	a		ε	ε
q_1	—	—	ε	ε	b	b
q_2	—	—	—	a	ε	ε
q_3	—	—	—	—	ε	ε
q_4	—	—	—	—	—	
q_5	—	—	—	—	—	—

10

A11: "Toolchain"

Task: Construct a DFA accepting the language
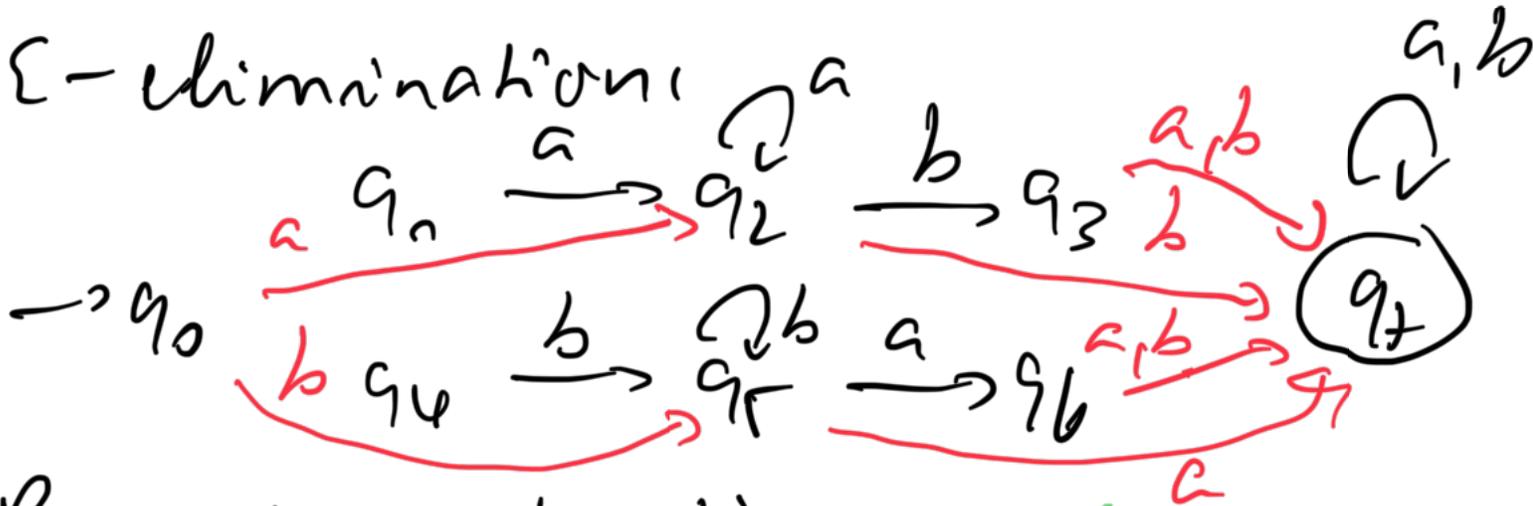
$$L = \{w \in \{a,b\}^* \mid w \text{ contains at least one } a \text{ and at least one } b\}.$$

Reg. expr: $\alpha = (a^+b \mid b^+a) \cdot (a \mid b)^*$

ε-NFA (simplified):

ε-elimination:

Powerset construction: