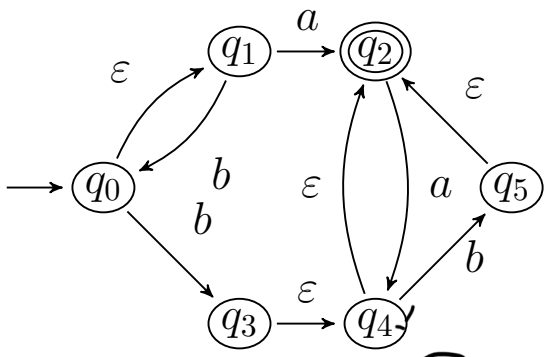
A6: Elimination of ε-Transitions

Task: Eliminate all ε-transitions of the following ε-NFA $\mathfrak{A}$ over $\Sigma := \{a, b\}$ to obtain an equivalent NFA.



DFA: $\mathfrak{A} = (Q, \Sigma, \delta, q_0, F)$

$\delta : Q \times \Sigma \to Q$

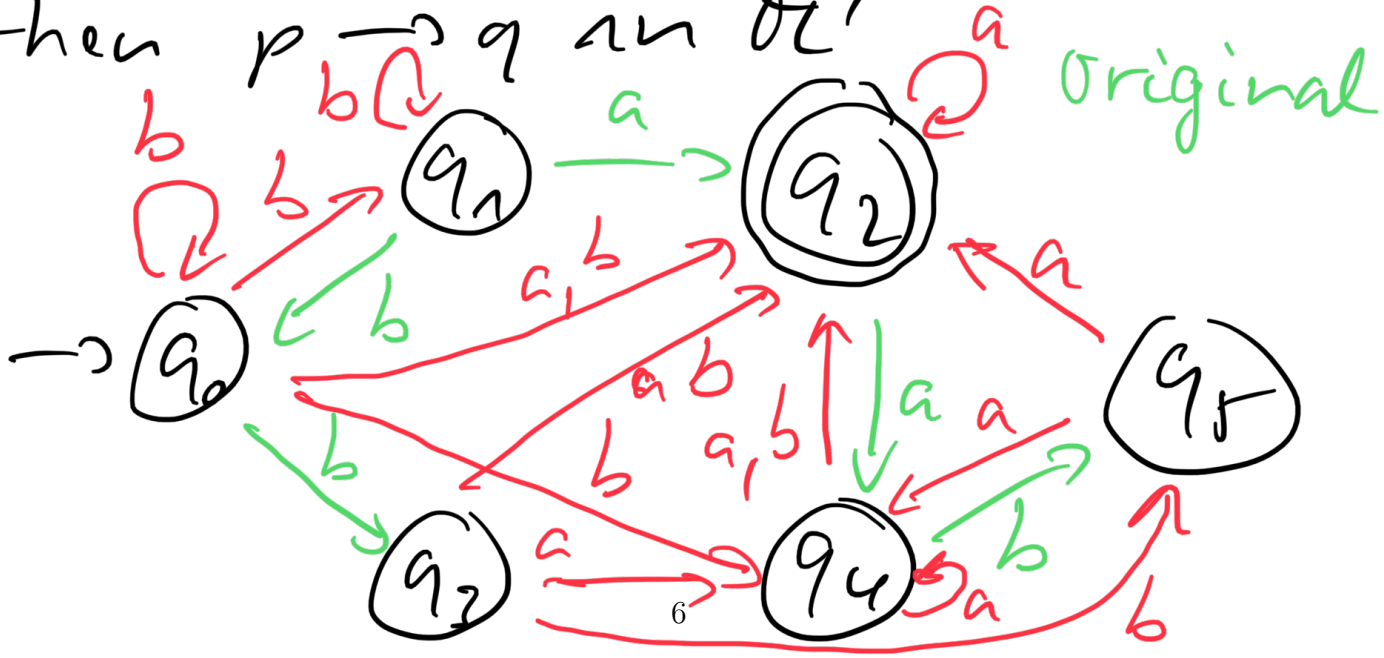
NFA: $\Delta : Q \times \Sigma \times Q$

ε-NFA: $\Delta : Q \times \Sigma_\varepsilon \times Q$, $\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$

ε-NFA $\mathfrak{A} \rightsquigarrow$ NFA $\mathfrak{A}'$:

if $p \xrightarrow{\varepsilon}{}^{*} \cdot \xrightarrow{a} \cdot \xrightarrow{\varepsilon}{}^{*} q$ in $\mathfrak{A}$, (*)

then $p \xrightarrow{a} q$ in $\mathfrak{A}'$



A8: Construction of Regular Expressions

Task: Give regular expressions that describe the following languages.

(a) $L := \{w \in \{a,b\}^* \mid |w| \text{ divisible by } 3\}$

(b) $L := \{w \in \{a,b,c\}^* \mid w \text{ does not contain } a, b, \text{ or } c\}$

(c) $L := \{w \in \{a,b\}^* \mid \text{substring } ab \text{ occurs exactly twice in } w, \text{ but not at the end}\}$

Reg. expr: $\emptyset, \Sigma, a\ (\in\Sigma)$,
$\alpha|\beta,\ \alpha\cdot\beta,\ \alpha^*$

(a) $((a|b)\cdot(a|b)\cdot(a|b))^*$

(for all reg. expr. α, $\Sigma \in L(\alpha^*)$)

(b) $(b|c)^* \mid (a|c)^* \mid (a|b)^*$

(c) $b^*ab^+ \quad +\ ab\,(b^+a^* \mid b^*a^+)$

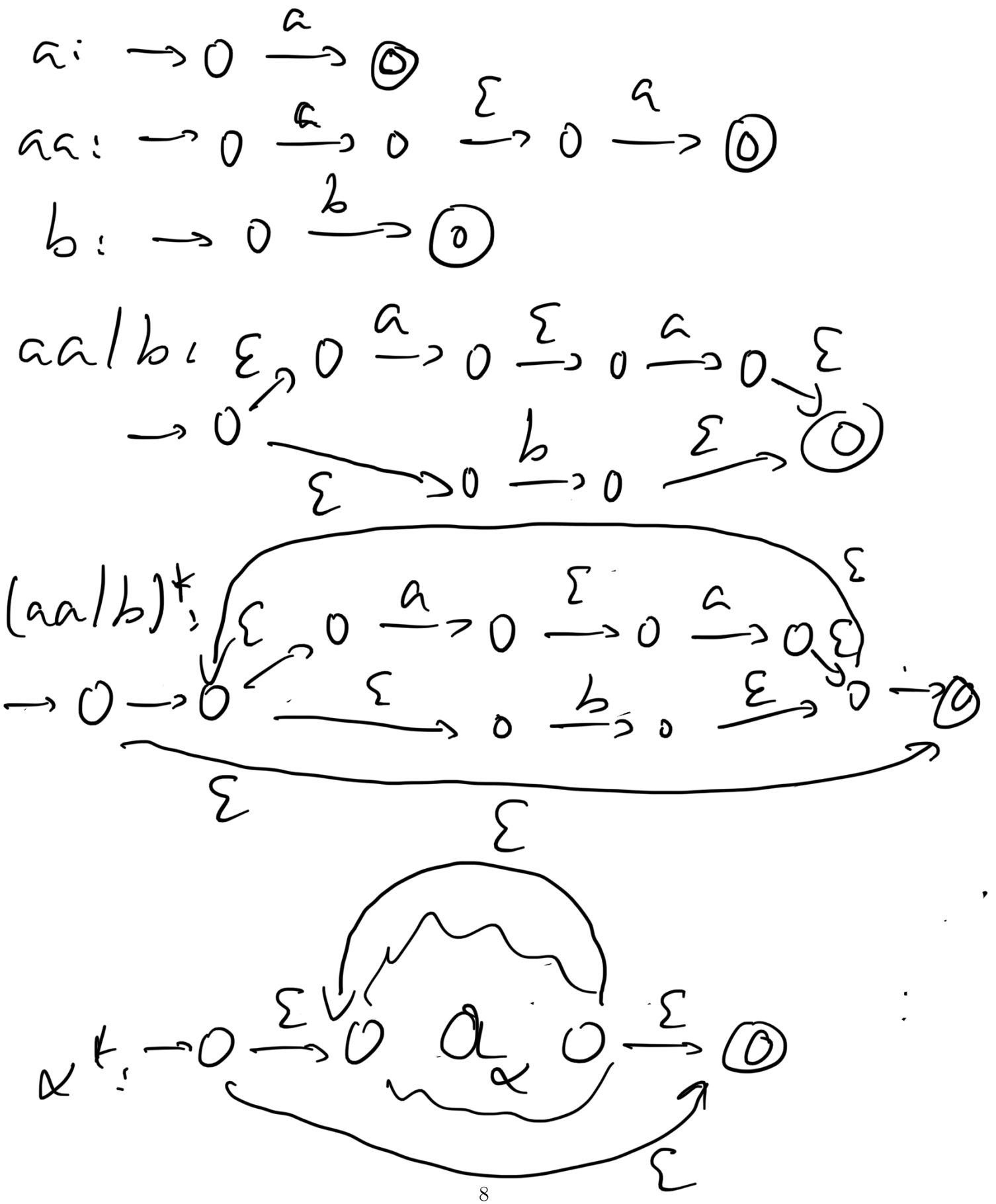
1st 2nd

baacbabab

for reg. expr. α,
$\alpha^+ := \alpha\alpha^*$

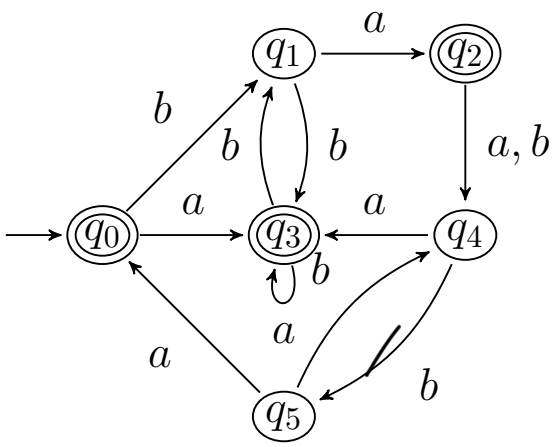
A9: From Regular Expressions to Finite Automata

Task: Using Kleene's construction, give the ε-NFA for the regular expression $(aa \mid b)^*$.



A10: Minimisation of Deterministic Finite Automata

Task: Minimise the following DFA.



$\delta : Q \times \Sigma \to Q$ $\quad$ $\delta^* : Q \times \Sigma^* \to Q$

Minimisation of DFA:

$p, q \in Q$ equivalent iff

$\forall w \in \Sigma^* : \delta^*(p, w) \in F$

$\iff \delta^*(q, w) \in F$

	q_0	q_1	q_2	q_3	q_4	q_5
q_0		Σ	a	-	Σ	Σ
q_1			Σ	Σ	b	a
q_2				a	Σ	Σ
q_3					Σ	Σ
q_4						-
q_5						

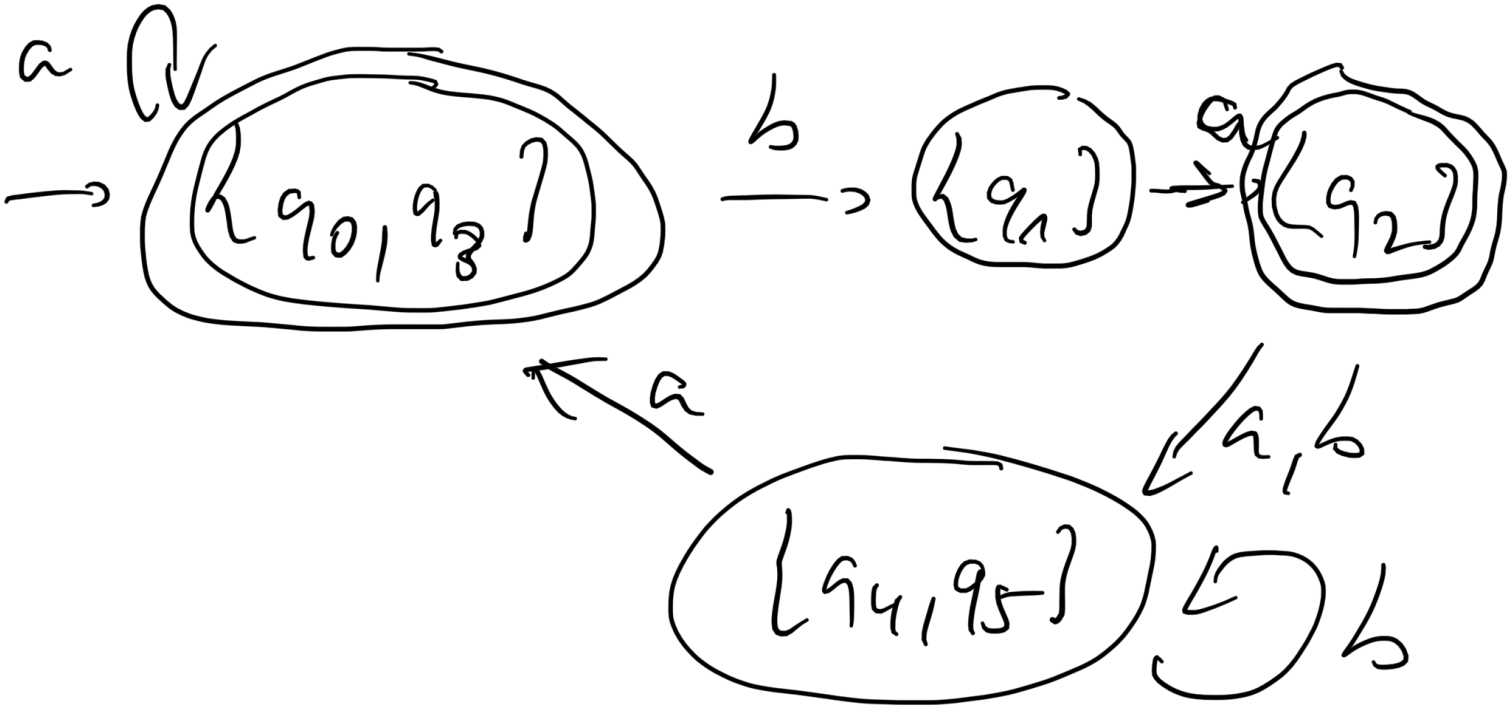
$\Rightarrow q_0 \sim q_3$

$q_4 \sim q_5$

A: "Toolchain"

Task: Construct a DFA accepting the language

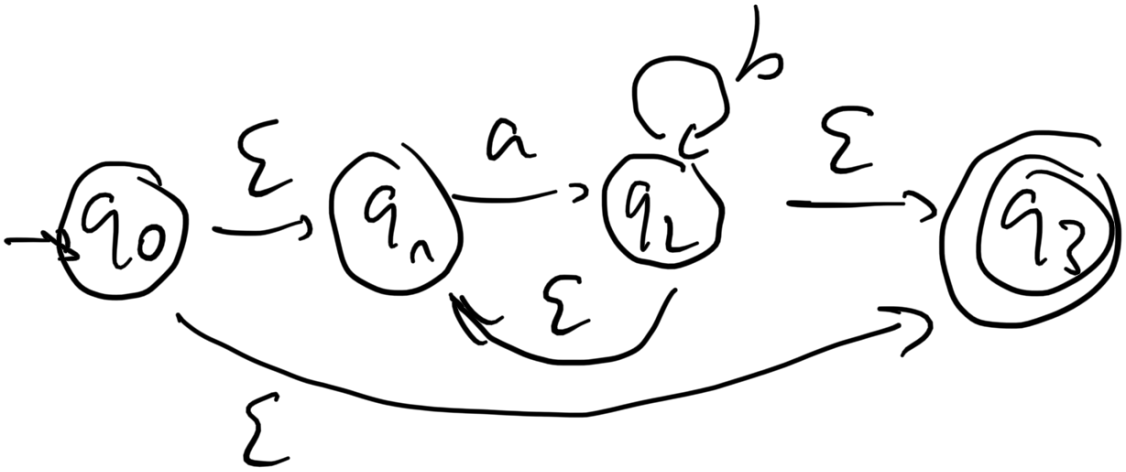
$$L = \{w \in \{a, b\}^* \mid w \text{ contains at least one } a \text{ and at least one } b\}.$$



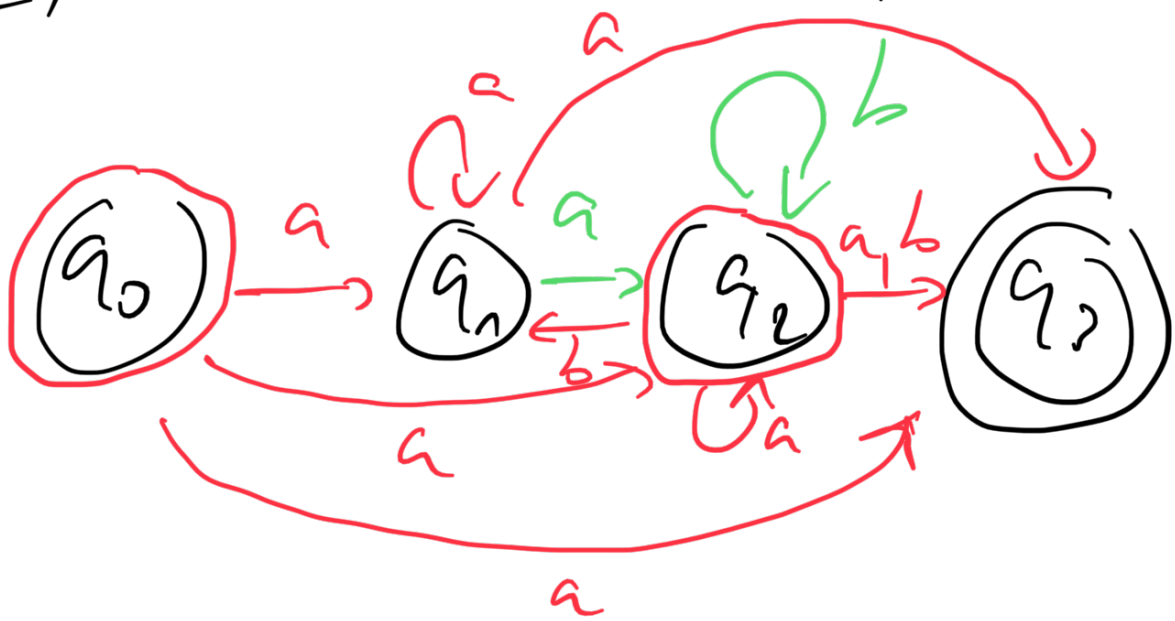
A: "Toolchain"

Task: Construct a DFA accepting the language described by $\alpha = (ab^*)^*$.

(1) ε-NFA:



(2) ε-elimination:



(3) Powerset construction: