

Exercises (Regular Languages)

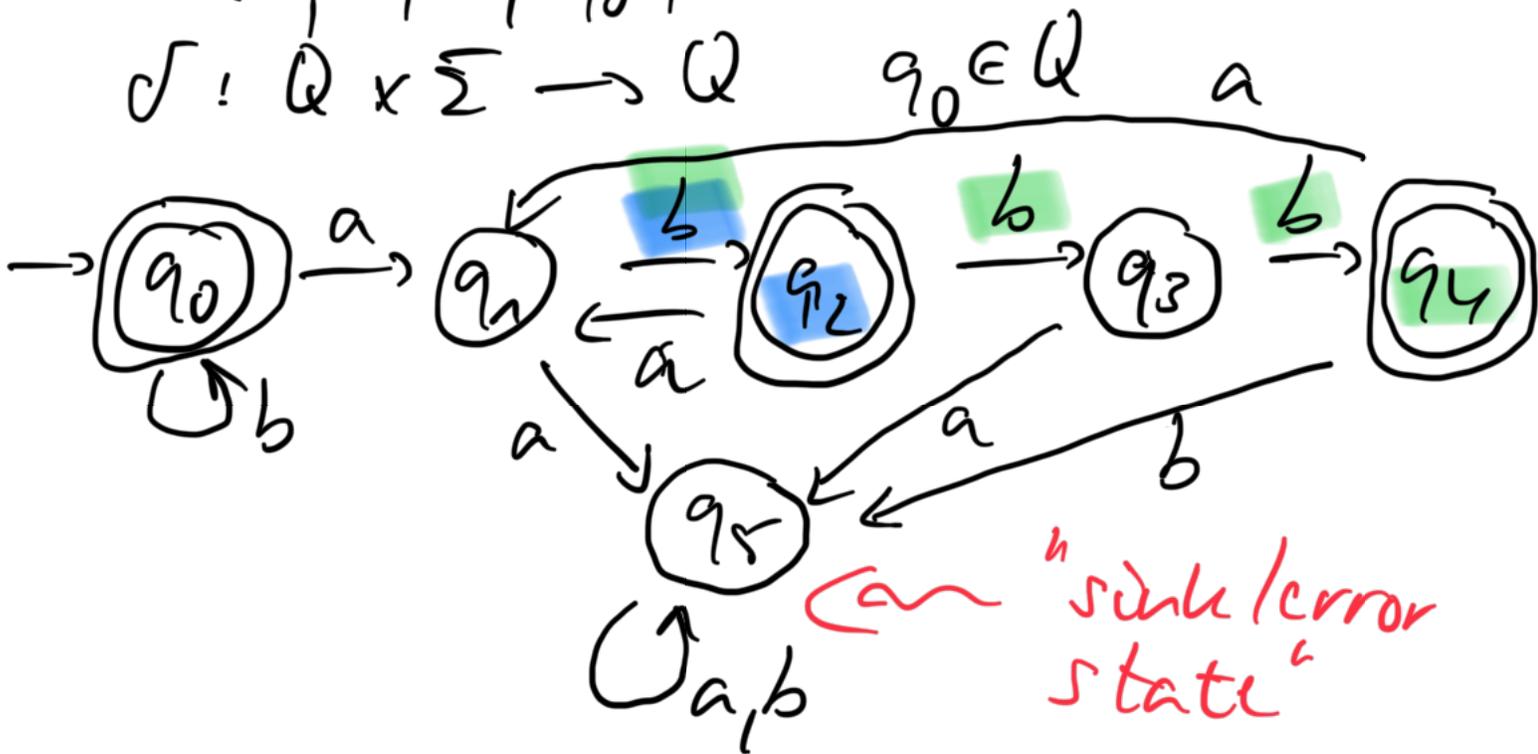
A1: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{a, b\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{each } a \text{ followed by exactly 1 or 3 } b\text{'s}\}$$

$$\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$$

$$\delta: Q \times \Sigma \rightarrow Q \quad q_0 \in Q$$



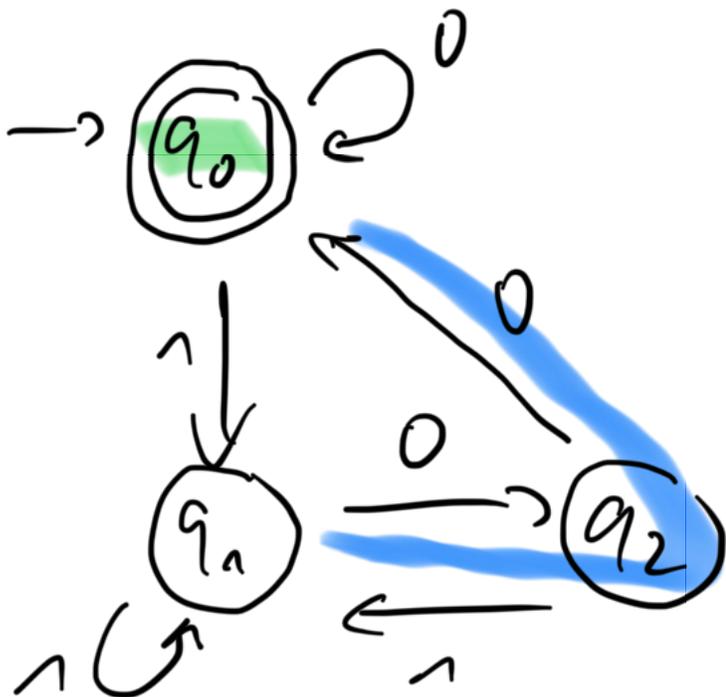
A2: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{0, 1\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{decimal value of } w \text{ divisible by } 4\}$$

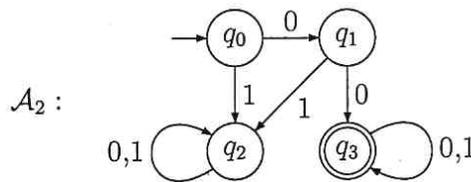
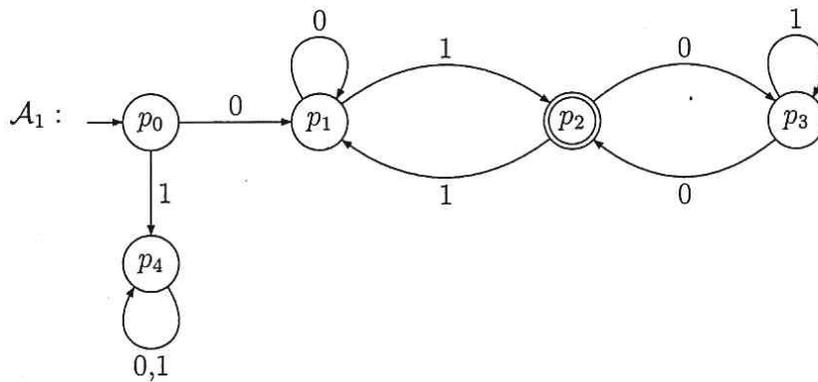
(where the empty word ε is assumed to have value 0).

Equivalent: w is of the form
 $v00$ ($v \in \Sigma^*$) or ε or 0

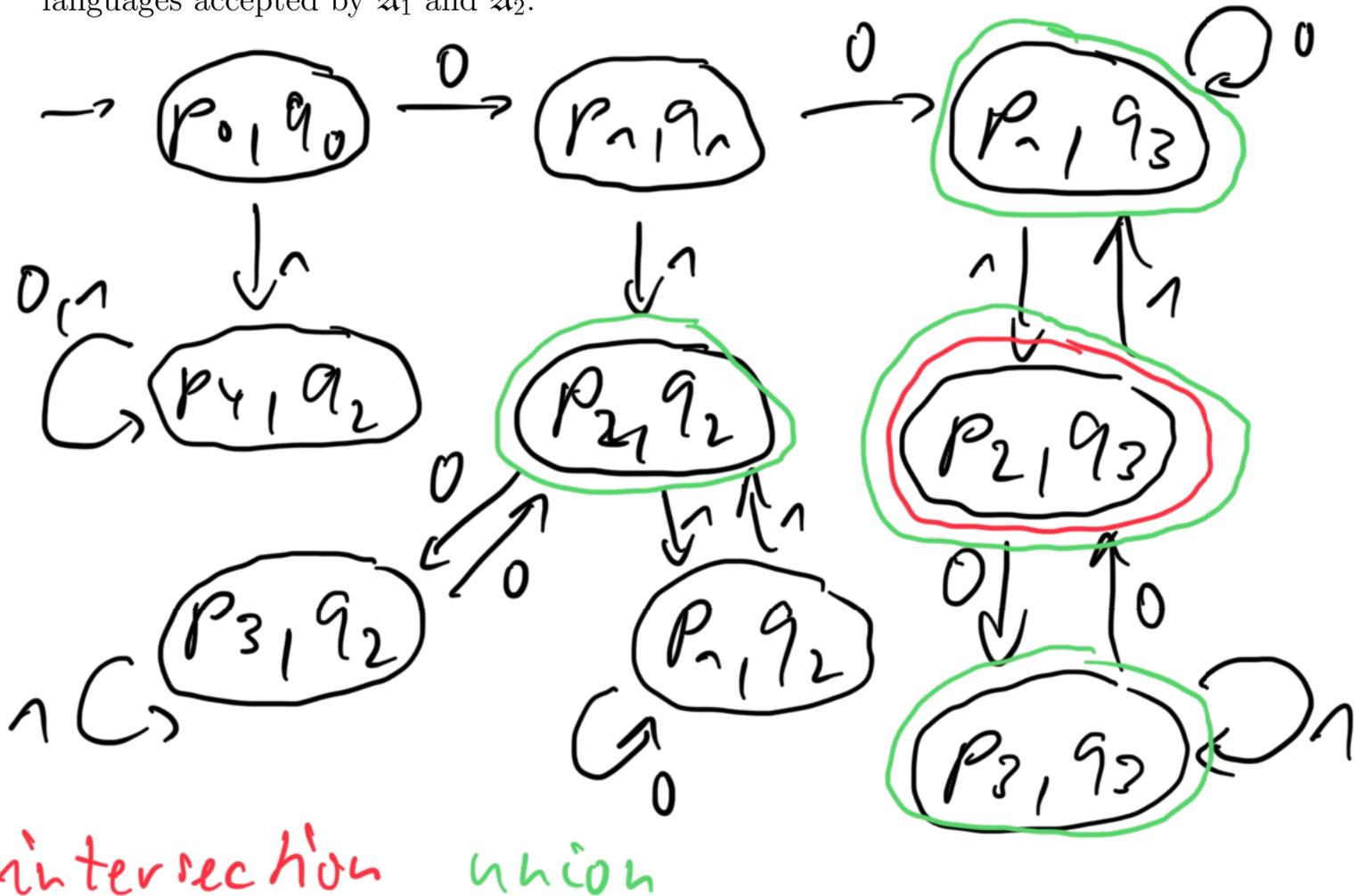


A3: Operations on Automata

Task: Let \mathcal{A}_1 and \mathcal{A}_2 be the following automata over $\Sigma = \{0, 1\}$:

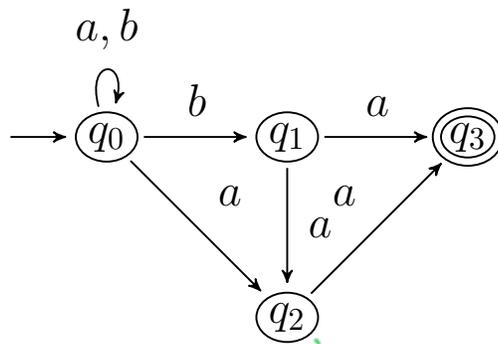


Construct two automata that respectively recognise the intersection and the union of the languages accepted by \mathcal{A}_1 and \mathcal{A}_2 .



A4: Nondeterministic Finite Automata

Task: Let \mathcal{A} be the following NFA over $\Sigma := \{a, b\}$.



Determine the reachability sets $\underline{R_{\mathcal{A}}(\varepsilon)}$, $\underline{R_{\mathcal{A}}(b)}$, $\underline{R_{\mathcal{A}}(ba)}$, and $\underline{R_{\mathcal{A}}(baa)}$.

$$R_{\mathcal{A}}(\varepsilon) = \{q_0\} \Rightarrow \varepsilon \notin L(\mathcal{A})$$

$$R_{\mathcal{A}}(b) = \{q_0, q_1\} \Rightarrow b \notin L(\mathcal{A})$$

$$R_{\mathcal{A}}(ba) = \{q_0, q_2, q_3\} \Rightarrow ba \in L(\mathcal{A})$$

$$R_{\mathcal{A}}(baa) = \{q_0, q_2, q_3\} \Rightarrow baa \in L(\mathcal{A})$$

A5: Powerset Construction

Task: Apply the powerset construction to transform the following NFA \mathcal{A} over $\Sigma := \{a, b, c\}$ into an equivalent DFA.

