Exercises (Regular Languages)

A1: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{a, b\}$ that accepts the following language:

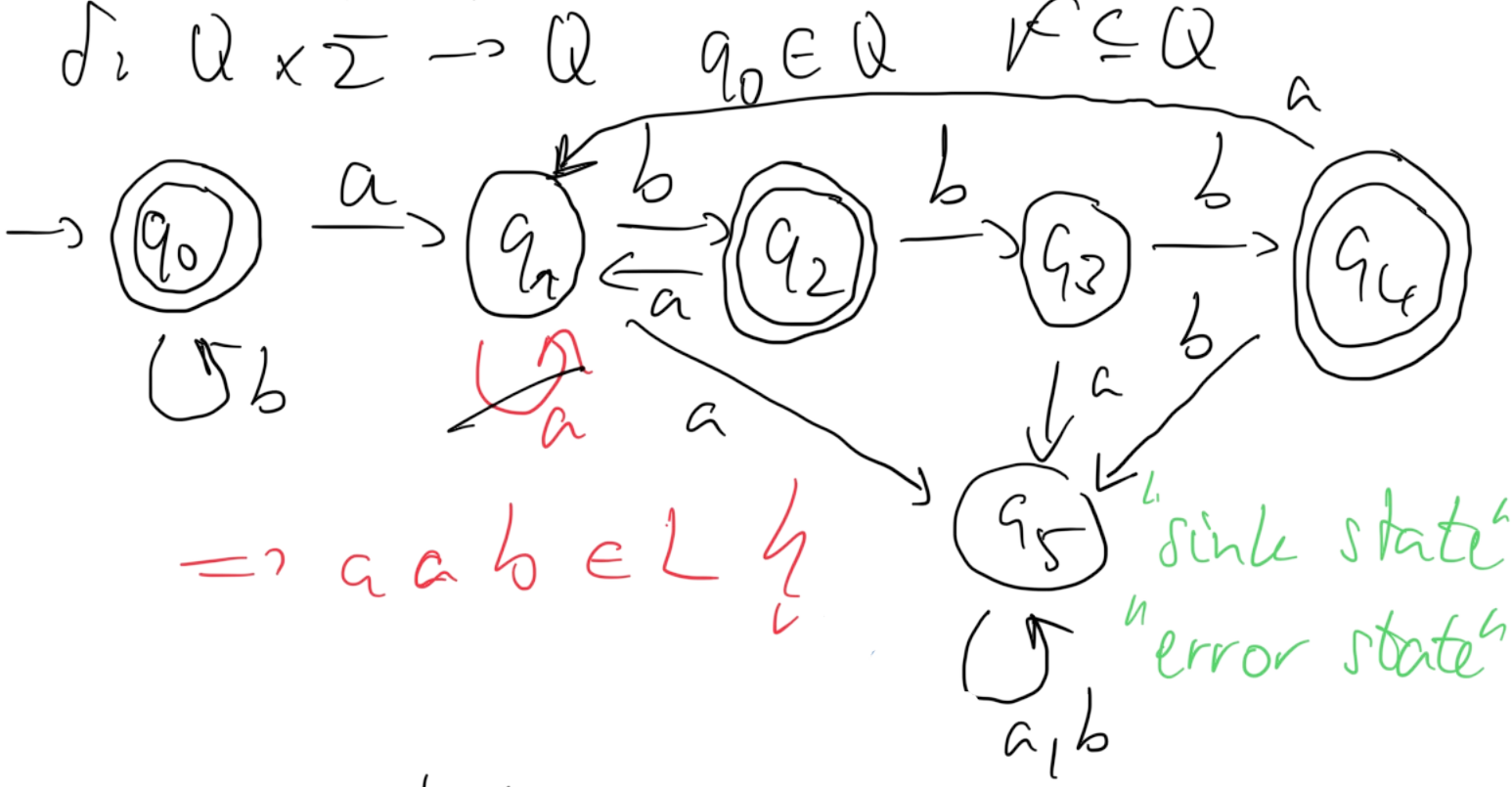
$$\{w \in \Sigma^* \mid \text{each } a \text{ followed by exactly 1 or 3 } b\text{'s}\}$$

bb ✓ baba.. ✓ babbba ✓

baa ↯ babba ↯

$A = (Q, \Sigma, \delta, q_0, F)$

$\delta: Q \times \Sigma \to Q \quad q_0 \in Q \quad F \subseteq Q$



$\Rightarrow aaab \in L$ ↯

"sink state"

"error state"

aa .... $\notin L$

$$L(A) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$$

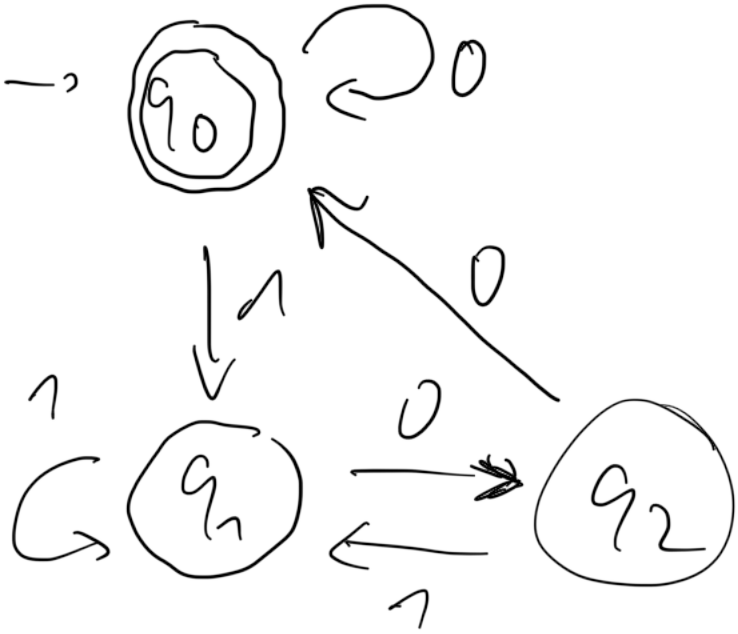
A2: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{0, 1\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{decimal value of } w \text{ divisible by } 4\}$$

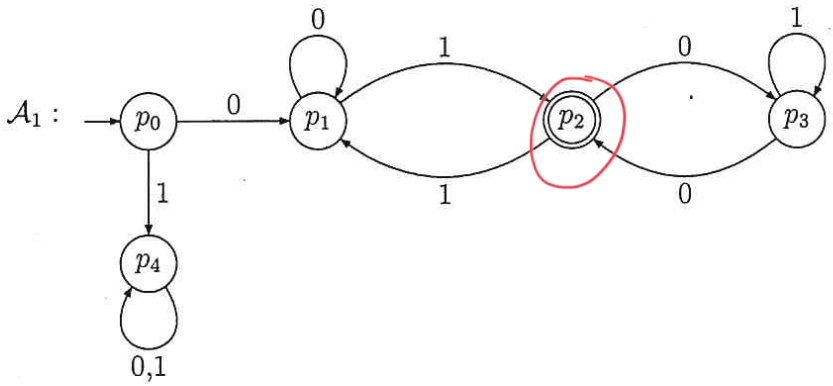
Not: 01, 11, 10 ↯ 1

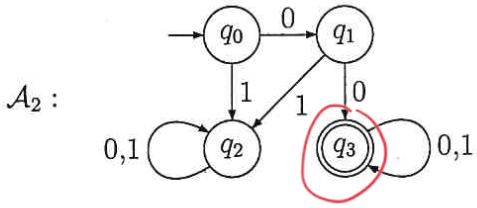
But: Σ, 0, 00, 100, 1100, $v00$ $(v \in \Sigma^*)$



A3: Operations on Automata

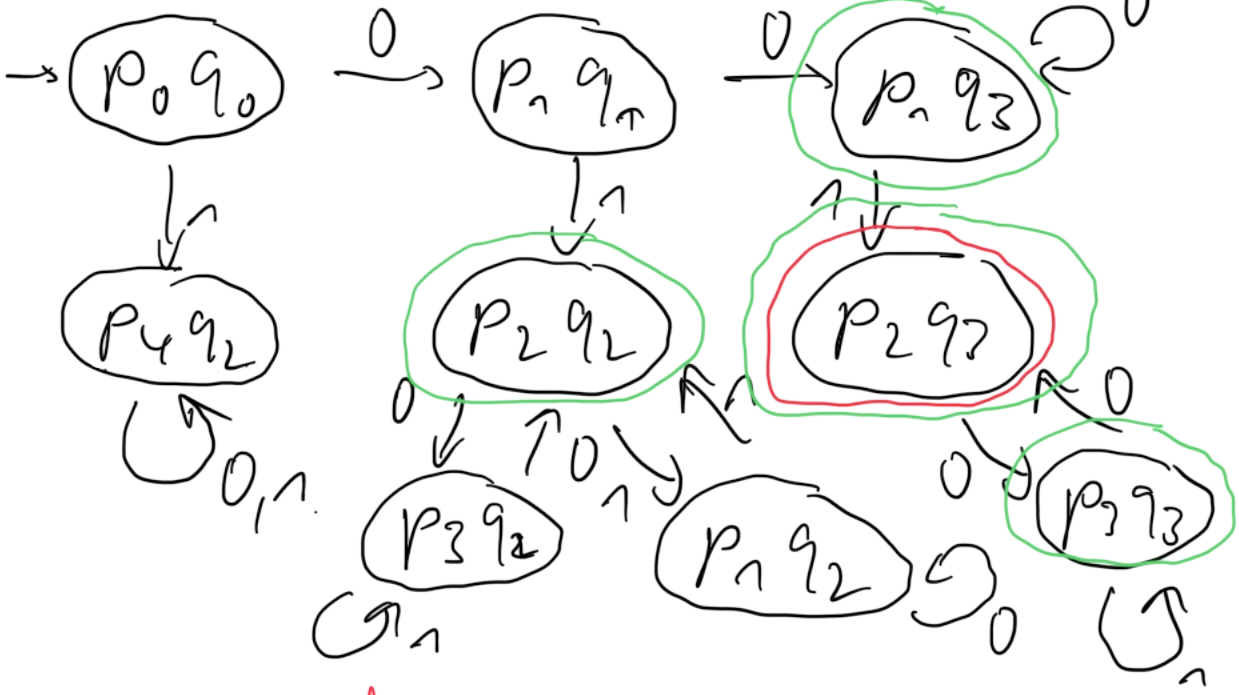
Task: Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be the following automata over $\Sigma = \{0, 1\}$:





Construct two automata that respectively recognise the intersection and the union of the languages accepted by $\mathfrak{A}_1$ and $\mathfrak{A}_2$.

"Product construction"

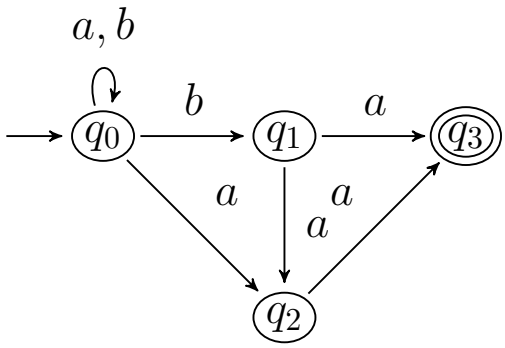


final for $\cap$

final for $\cup$

A4: Nondeterministic Finite Automata

Task: Let $\mathfrak{A}$ be the following NFA over $\Sigma := \{a, b\}$.



Determine the reachability sets $R_{\mathfrak{A}}(\varepsilon)$, $R_{\mathfrak{A}}(b)$, $R_{\mathfrak{A}}(ba)$, and $R_{\mathfrak{A}}(baa)$.

$\delta: Q \times \Sigma \to Q$ deterministic (DFA)

$\Delta: Q \times \Sigma \times Q$ NFA

$L(\mathfrak{A}) = \{w \in \Sigma^* \mid q_0 \xrightarrow{w} q, q \in F\}$

$R_{\mathfrak{A}}(\varepsilon) = \{q_0\}$

$R_{\mathfrak{A}}(b) = \{q_0, q_1\}$

$R_{\mathfrak{A}}(ba) = \{q_0, q_2, q_3\}$ ✓

$R_{\mathfrak{A}}(baa) = \{q_0, q_2, q_3\}$ ✓

A5: Powerset Construction

Task: Apply the powerset construction to transform the following NFA $\mathfrak{A}$ over $\Sigma := \{a, b, c\}$ into an equivalent DFA.