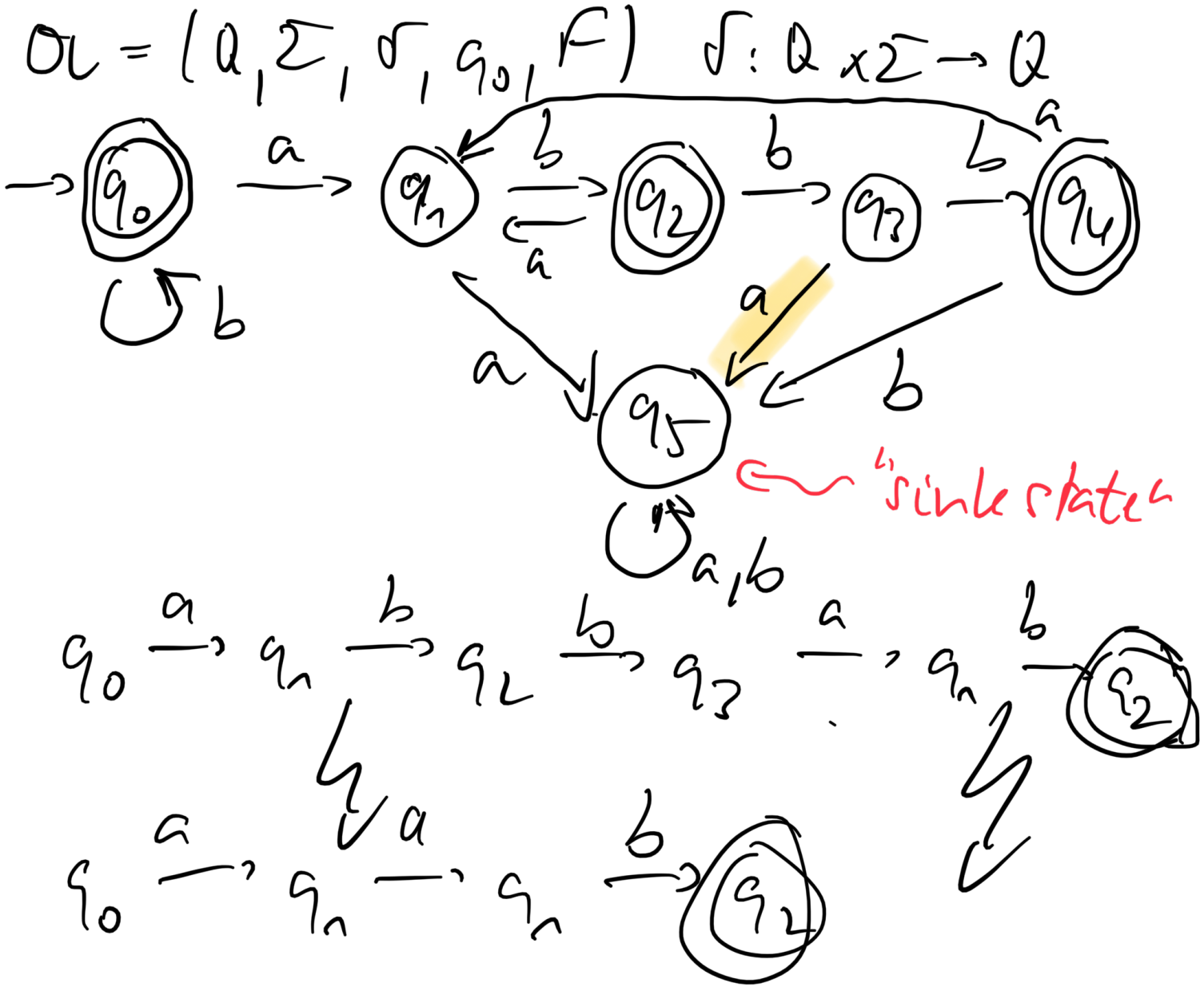
Exercises (Regular Languages)

A2: Construction of Deterministic Finite Automata

Task: Construct a DFA over $\Sigma := \{a, b\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{each } a \text{ followed by exactly 1 or 3 } b\text{'s}\}$$



A2: Construction of Deterministic Finite Automata

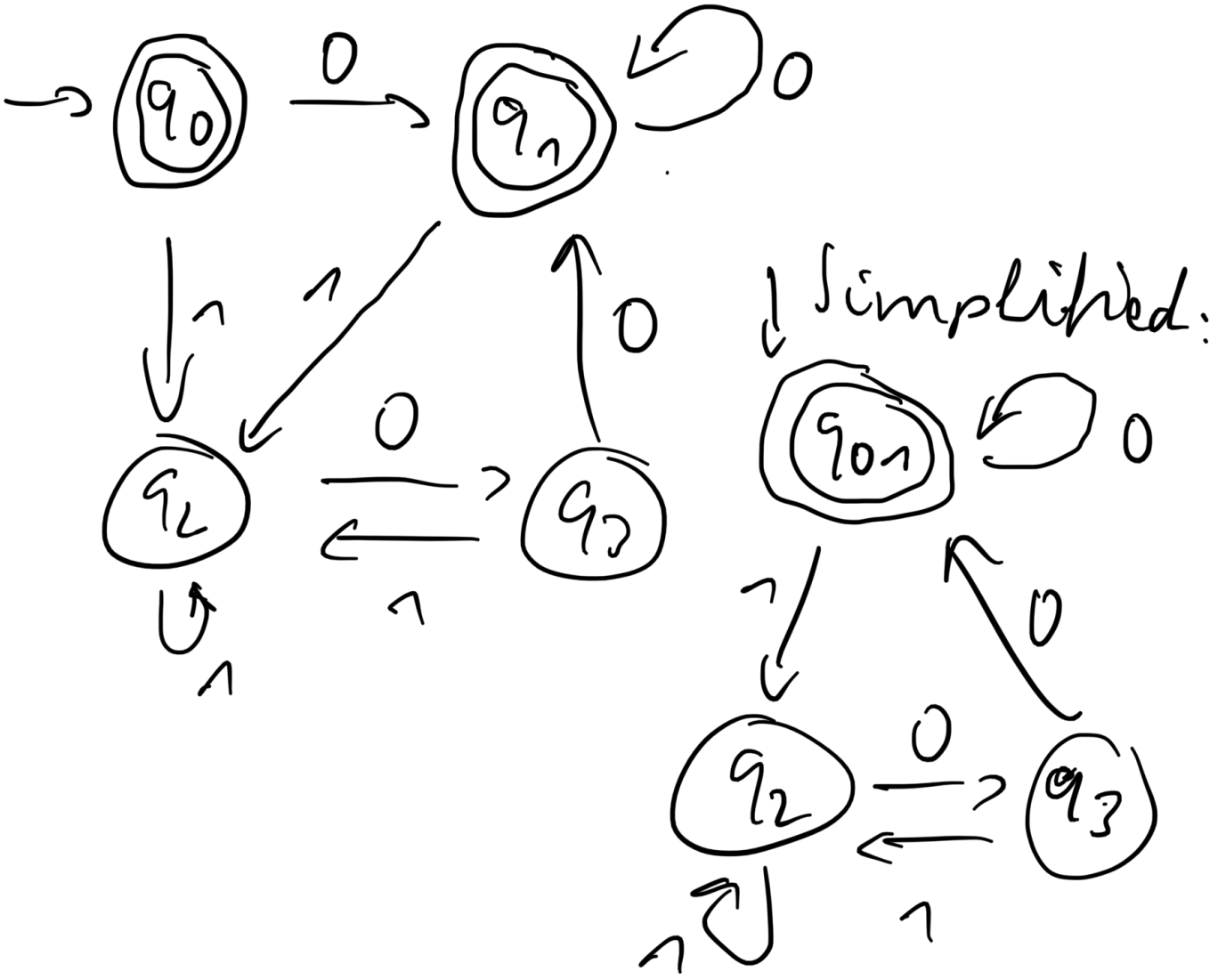
Task: Construct a DFA over $\Sigma := \{0, 1\}$ that accepts the following language:

$$\{w \in \Sigma^* \mid \text{decimal value of } w \text{ divisible by } 4\}$$

$101 \hat{=} 5$

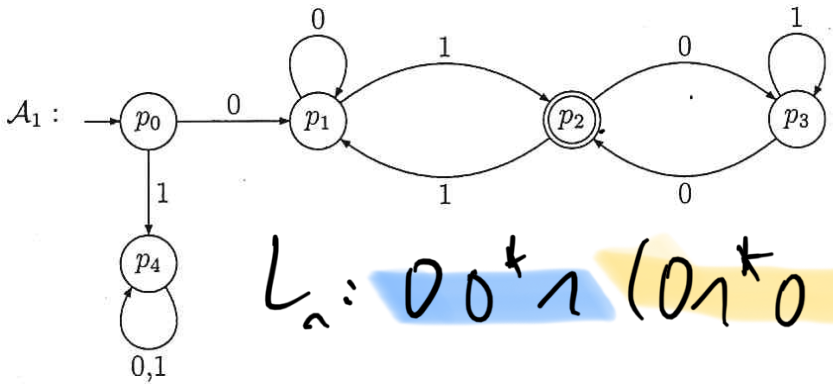
Equivalent: w of the form

$\varepsilon, 0, \cancel{1}, \cancel{10}, v00, \cancel{01}, \cancel{11}$ $v \in \{0,1\}^*$

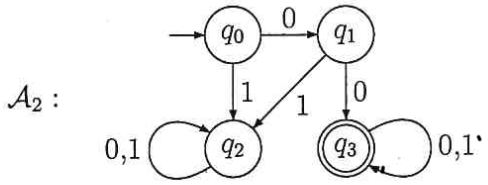


A3: Operations on Automata

Task: Let $\mathfrak{A}_1$ and $\mathfrak{A}_2$ be the following automata over $\Sigma = \{0, 1\}$:



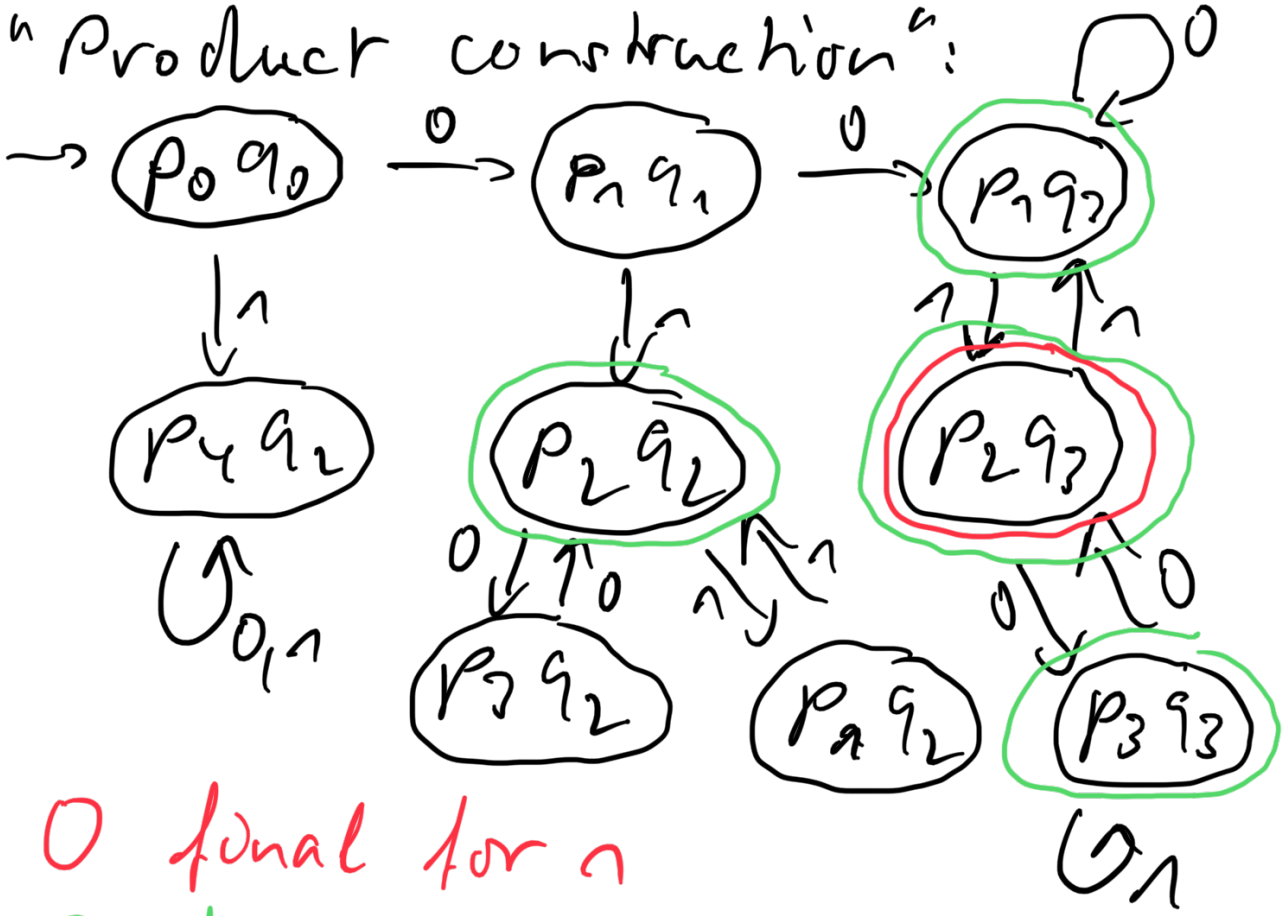
$L_1: 00^*1(01^*0 \mid 10^*1)^*$



$L_2: 00(0 \mid 1)^*$

Construct two automata that respectively recognise the intersection and the union of the languages accepted by $\mathfrak{A}_1$ and $\mathfrak{A}_2$.

"Product construction":



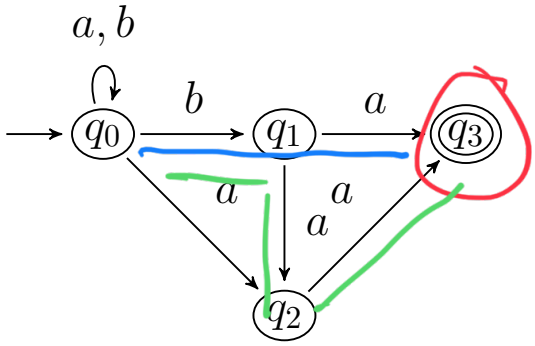
0 final for $\cap$

0 final for $\cup$

$L_\cap = 000^*1(01^*0 \mid 10^*1)^*$

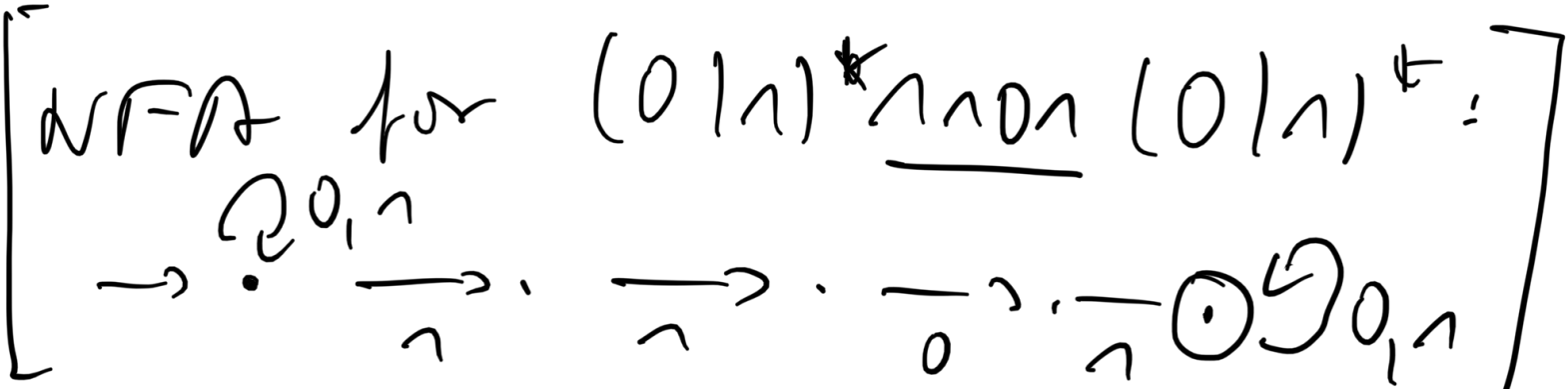
A4: Nondeterministic Finite Automata

Task: Let $\mathfrak{A}$ be the following NFA over $\Sigma := \{a, b\}$.



Determine the reachability sets $R_{\mathfrak{A}}(\varepsilon)$, $R_{\mathfrak{A}}(b)$, $R_{\mathfrak{A}}(ba)$, and $R_{\mathfrak{A}}(baa)$.

NFA for $(0 \mid 1)^* 1 1 0 1 (0 \mid 1)^*$:



$R_{\mathfrak{A}}(\varepsilon) = \{q_0\}$

$R_{\mathfrak{A}}(b) = \{q_0, q_1\}$

$R_{\mathfrak{A}}(ba) = \{q_0, q_2, q_3\}$

$R_{\mathfrak{A}}(baa) = \{q_0, q_2, q_3\}$

final

$\Rightarrow \varepsilon, b \notin L(\mathfrak{A})$, $ba, baa \in L(\mathfrak{A})$

A5: Powerset Construction

Task: Apply the powerset construction to transform the following NFA $\mathfrak{A}$ over $\Sigma := \{a, b, c\}$ into an equivalent DFA.