Heterogeneous/homogeneous model checking GRM5.5-30

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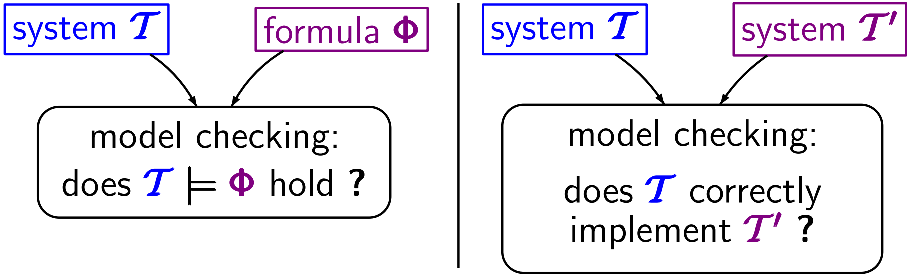


Heterogeneous/homogeneous model checking

system $\mathcal{T}$ | formula Φ

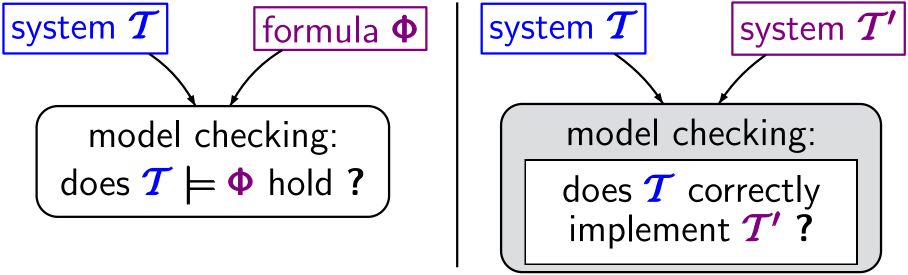
model checking:
does $\mathcal{T} \models \Phi$ hold ?

system $\mathcal{T}$ | system $\mathcal{T}'$

model checking:
does $\mathcal{T}$ correctly implement $\mathcal{T}'$?

trace inclusion checking
trace equivalence checking $\longleftarrow$ ***PSPACE*-complete**

Heterogeneous/homogeneous model checking



trace inclusion checking
trace equivalence checking ⟵ ***PSPACE*-complete**

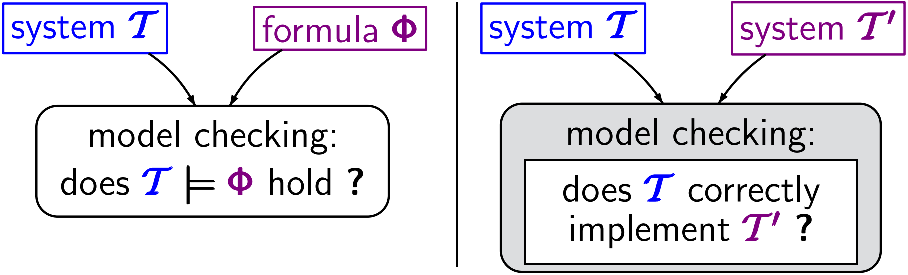
bisimulation equivalence checking ⟵ $\mathcal{O}(m \cdot \log n)$

"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

n = #states

m = #transitions

Heterogeneous/homogeneous model checking



trace inclusion checking
trace equivalence checking ⟵ ***PSPACE*-complete**

bisimulation equivalence checking ⟵ $\mathcal{O}(m \cdot \log n)$
"does $\mathcal{T} \sim \mathcal{T}'$ hold ?"

refinement checking via simulation ⟵ $\mathcal{O}(m \cdot n)$
"does $\mathcal{T} \preceq \mathcal{T}'$ hold ?"

Refinement checking via simulation

given: 2 finite transition system $\mathcal{T}_1$ and $\mathcal{T}_2$ over the same set of propositions AP

question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold ?

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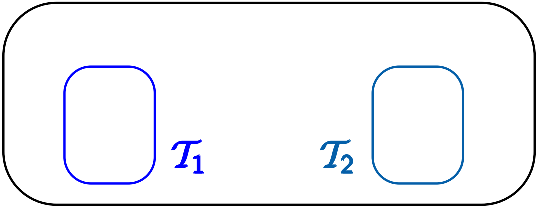

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

Refinement checking via simulation

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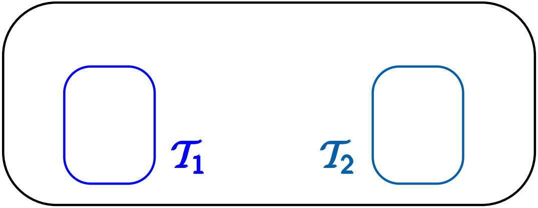

composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$

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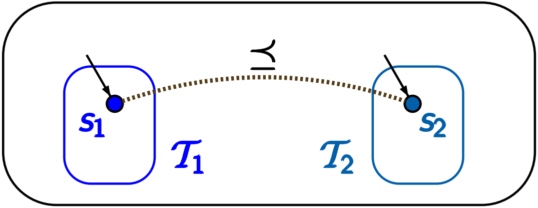
question: does $\mathcal{T}_1 \preceq \mathcal{T}_2$ hold ?



composite system $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$

- compute the simulation preorder $\preceq_{\mathcal{T}}$ on $\mathcal{T}$
- check whether for all initial states s_1 of $\mathcal{T}_1$ there is an initial state s_2 of $\mathcal{T}_2$ s.t. $s_1 \preceq_{\mathcal{T}} s_2$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

given: finite TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
possibly with terminal states

goal: compute the simulation preorder $\preceq_{\mathcal{T}}$

$\rightsquigarrow$ simulation equivalence classes

$\rightsquigarrow$ simulation quotient $\mathcal{T}/\simeq$

method: iterative refinement of relation $\mathcal{R} \subseteq S \times S$

$\mathcal{R} := \{ (s_1, s_2) \in S \times S : L(s_1) = L(s_2) \};$

WHILE $\mathcal{R}$ is no simulation DO

choose $(s_1, s_2) \in \mathcal{R}$ s.t. $s_1 \longrightarrow s_1'$, but there is no transition $s_2 \longrightarrow s_2'$ with $(s_1', s_2') \in \mathcal{R}$

$\mathcal{R} := \mathcal{R} \setminus \{(s_1, s_2)\}$

OD

return $\mathcal{R}$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

$\mathcal{R} := \{ (s_1, s_2) \in S \times S : L(s_1) = L(s_2) \}$;

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$\mathcal{R} := \mathcal{R} \setminus \{(s_1, s_2)\}$

OD

return $\mathcal{R}$ $\longleftarrow$ $\mathcal{R}$ is the coarsest simulation on $\mathcal{T}$ and therefore $\mathcal{R} = \preceq_{\mathcal{T}}$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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OD

return $\mathcal{R}$

#iterations: $\mathcal{O}(|S|^2)$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

$\mathcal{R} \; := \; \{ (s_1, s_2) \in S \times S \, : \, L(s_1) = L(s_2) \};$

WHILE $\mathcal{R}$ is no simulation DO

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$\mathcal{R} \; := \; \mathcal{R} \setminus \{(s_1, s_2)\}$

OD

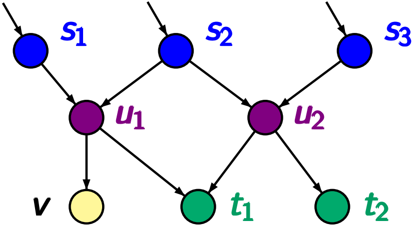
return $\mathcal{R}$

#iterations: $\mathcal{O}(|S|^2)$

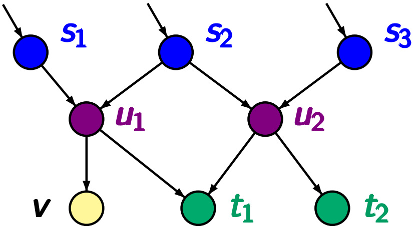
representation of $\mathcal{R}$ by simulator sets

$$Sim_{\mathcal{R}}(s_1) \; = \; \{ s_2 \in S \, : \, (s_1, s_2) \in \mathcal{R} \}$$

Example: computation of $\preceq$

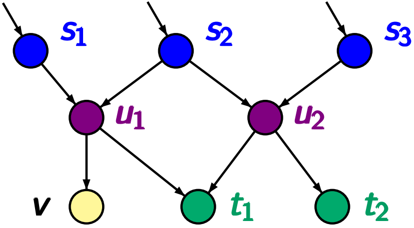


Example: computation of $\preceq$



initially:

$$
\begin{aligned}
Sim(s_i) &= \{s_1, s_2, s_3\} \\
Sim(u_1) &= Sim(u_2) = \{u_1, u_2\} \\
Sim(v) &= \{v\} \\
Sim(t_1) &= Sim(t_2) = \{t_1, t_2\}
\end{aligned}
$$



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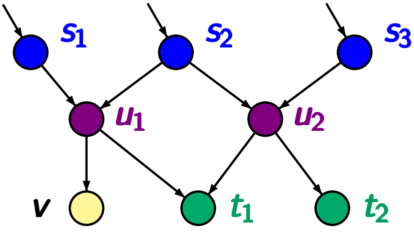
$u_1 \not\preceq u_2$, as $u_1 \rightarrow v$, $u_2 \not\rightarrow Sim(v)$

Example: computation of $\preceq$



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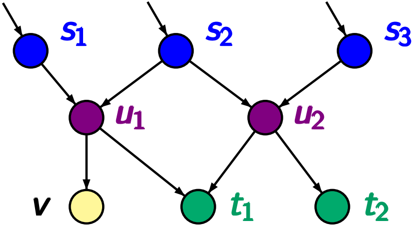

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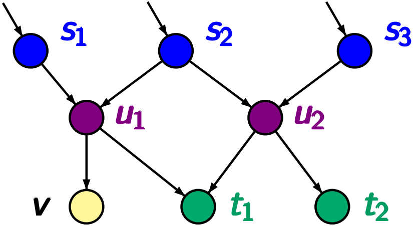
Example: computation of $\preceq$



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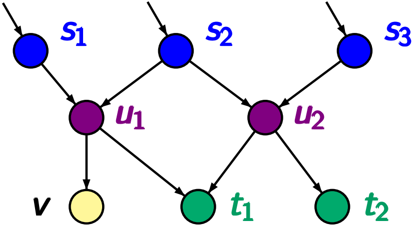

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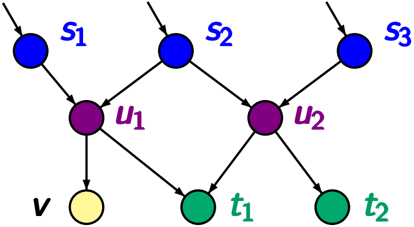
$s_2 \not\preceq s_3$, as $s_2 \to u_1$, $s_3 \not\to Sim(u_1)$

Example: computation of $\preceq$



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$s_2 \not\preceq s_3$, as $s_2 \rightarrow u_1$, $s_3 \not\rightarrow Sim(u_1)$ $\quad$ $Sim(s_2) = \{s_1, s_2\}$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

```
FOR ALL s1 ∈ S DO
    Sim(s1) := {s2 ∈ S : L(s1) = L(s2)}
OD
WHILE ∃s1 ∈ S ∃s2 ∈ Sim(s1) ∃s1' ∈ Post(s1)
            s.t. Post(s2) ∩ Sim(s1') = ∅      DO
    choose such states s1, s2
    Sim(s1) := Sim(s1) \ {s2}
OD
return {(s1, s2) : s2 ∈ Sim(s1)}
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FOR ALL $s_1 \in S$ DO

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OD

WHILE $\exists s_1 \in S\ \exists s_2 \in Sim(s_1)\ \exists s_1' \in Post(s_1)$

$\quad$ s.t. $Post(s_2) \cap Sim(s_1') = \varnothing$ DO

$\quad$ choose such states s_1, s_2

$\quad Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

OD

return $\{(s_1, s_2) : s_2 \in Sim(s_1)\}$

$s_1 \longrightarrow s_1'$

s_2

FOR ALL $s_1 \in S$ DO

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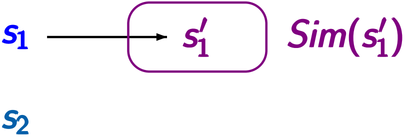

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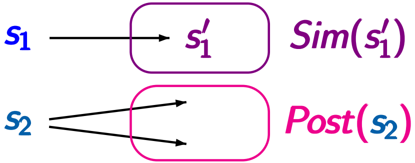

Computing the simulation preorder $\preceq_{\mathcal{T}}$

FOR ALL $s_1 \in S$ DO

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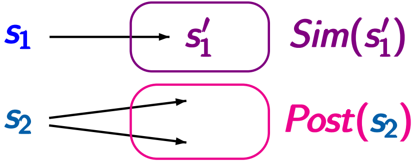
  s.t. $Post(s_2) \cap Sim(s_1') = \varnothing$ DO

  choose such states s_1, s_2

  $Sim(s_1) := Sim(s_1) \setminus \{s_2\} \longleftarrow$ $s_1 \not\preceq_{\mathcal{T}} s_2$

OD

return $\{(s_1, s_2) : s_2 \in Sim(s_1)\}$



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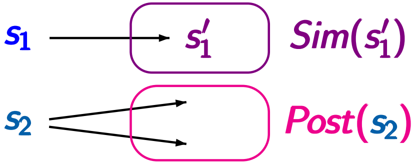
complexity:
$\mathcal{O}(m \cdot |S|^3)$



$m = \#\text{edges}$
$\geqslant |S|$

Computing the simulation preorder $\preceq_{\mathcal{T}}$

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complexity:
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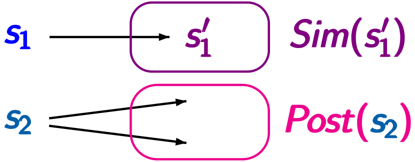

$m = \#\text{edges}$
$\geqslant |S|$

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_\mathcal{T}$

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reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$
by means of

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_{\mathcal{T}}$

reformulation of the algorithm to compute $\preceq_{\mathcal{T}}$ by means of

- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$

$\mathcal{O}(m \cdot |S|^2)$-algorithm for computing $\preceq_\mathcal{T}$

reformulation of the algorithm to compute $\preceq_\mathcal{T}$ by means of

- counters $\delta(s_1', s_2)$ for $|Post(s_2) \cap Sim(s_1')|$
- a set V that organizes all pairs (s_1', s_2) where $\delta(s_1', s_2) = 0$

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
```

$\texttt{FOR ALL}\ s_1 \in S\ \texttt{DO}\ \ Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}\ \texttt{OD}$

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left| Post(s_2) \cap Sim(s_1') \right|$ OD

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := \left|Post(s_2) \cap Sim(s_1')\right|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

WHILE $V \neq \varnothing$ DO

choose $(s_1', s_2) \in V$ and remove (s_1', s_2) from V

OD

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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FOR ALL $s_1 \in Pre(s_1')$ with $s_2 \in Sim(s_1)$ DO

$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

OD

OD

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
FOR ALL s1', s2 DO  δ(s1', s2) := |Post(s2) ∩ Sim(s1')| OD
V := {(s1', s2) : δ(s1', s2) = 0}
WHILE  V ≠ ∅ DO
   choose (s1', s2) ∈ V and remove (s1', s2) from V
   FOR ALL  s1 ∈ Pre(s1') with s2 ∈ Sim(s1) DO
      Sim(s1) := Sim(s1) \ {s2}
      FOR ALL  u2 ∈ Pre(s2) DO
         δ(s1, u2) := δ(s1, u2) − 1
         IF  δ(s1, u2) = 0 THEN  insert (s1, u2) in V FI
      OD
   OD
OD
```

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
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      OD
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OD
```

```
FOR ALL s1 ∈ S DO  Sim(s1) := {s2 : L(s1) = L(s2)} OD
FOR ALL s1', s2 DO  δ(s1', s2) := |Post(s2) ∩ Sim(s1')| OD
V := {(s1', s2) : δ(s1', s2) = 0}
WHILE V ≠ ∅ DO
   choose (s1', s2) ∈ V and remove (s1', s2) from V
   FOR ALL s1 ∈ Pre(s1') with s2 ∈ Sim(s1) DO
     Sim(s1) := Sim(s1) \ {s2}
     FOR ALL u2 ∈ Pre(s2) DO
       δ(s1, u2) := δ(s1, u2) − 1
       IF δ(s1, u2) = 0 THEN insert (s1, u2) in V FI
     OD
   OD
OD
```

in total: $\mathcal{O}(m \cdot |S|)$

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

FOR ALL s_1', s_2 DO $\delta(s_1', s_2) := |Post(s_2) \cap Sim(s_1')|$ OD

$V := \{(s_1', s_2) : \delta(s_1', s_2) = 0\}$

WHILE $V \neq \varnothing$ DO

  choose $(s_1', s_2) \in V$ and remove (s_1', s_2) from V

  FOR ALL $s_1 \in Pre(s_1')$ with $s_2 \in Sim(s_1)$ DO

    $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

    FOR ALL $u_2 \in Pre(s_2)$ DO

      $\delta(s_1, u_2) := \delta(s_1, u_2) - 1$

      IF $\delta(s_1, u_2) = 0$ THEN insert (s_1, u_2) in V FI

    OD

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```

cost per iteration
$\mathcal{O}(m)$

FOR ALL $s_1 \in S$ DO $Sim(s_1) := \{s_2 : L(s_1) = L(s_2)\}$ OD

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WHILE $V \neq \varnothing$ DO ⟵ #iterations $\leqslant |S|^2$

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FOR ALL $s_1 \in Pre(s_1')$ with $s_2 \in Sim(s_1)$ DO

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cost per iteration $\mathcal{O}(m)$

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OD

An $\mathcal{O}(m \cdot |S|)$-algorithm for computing $\preceq_{\mathcal{T}}$

... algorithm by Henzinger, Henzinger, and Kopke

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Then: if $Post(s_2) \cap Sim(s_1') = \{s_2'\}$ then $s_1 \not\preceq s_2$ and s_2 can be removed from $Sim(s_1)$.

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↑

idea: collect all such states s_2 in $Remove(s_1')$

Idea of the HHK-algorithm

If s'_2 is removed from $Sim(s'_1)$ then regard all direct predecessors s_1 of s'_1 and remove all states in

$$Remove(s'_1) \; = \; Pre(Sim_{old}(s'_1)) \setminus Pre(Sim(s'_1))$$

from $Sim(s_1)$.

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$$Sim_{old}(s_1') := Sim(s_1')$$

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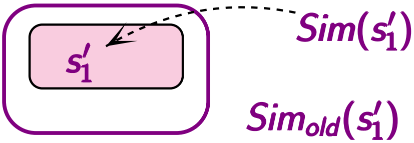

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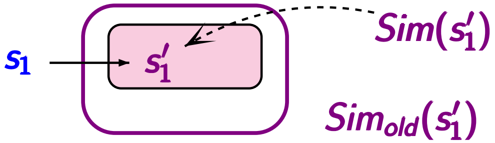

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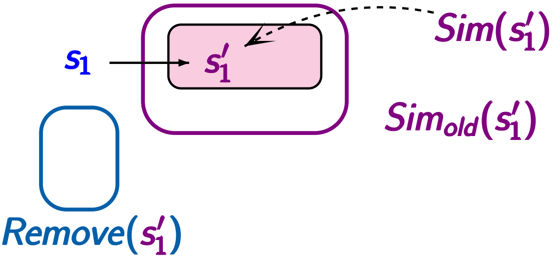

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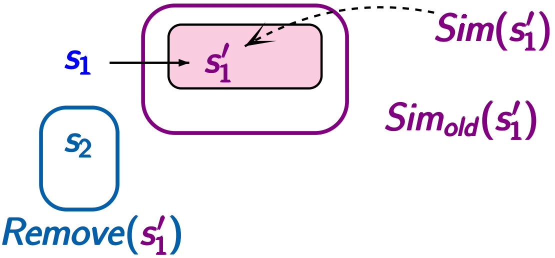

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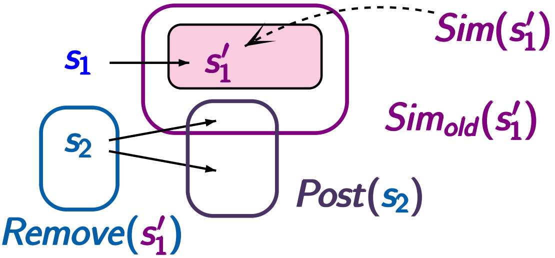

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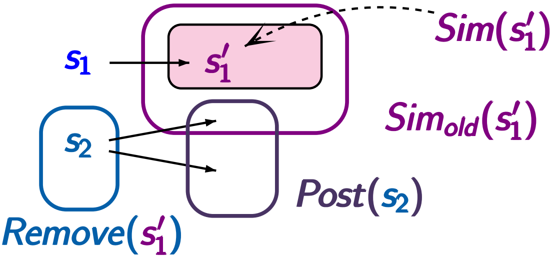


$s_1 \not\preceq_T s_2$

HHK-algorithm (first version)

FOR ALL states s_1' DO

$Sim_{old}(s_1') := S$

$Sim(s_1') := \{ s_2' \in S : L(s_1') = L(s_2') \}$

OD

HHK-algorithm (first version)


```
FOR ALL states s1' DO
    Sim_old(s1') := S
    Sim(s1') := { s2' ∈ S : L(s1') = L(s2') }
OD
WHILE ∃ state s1' with Sim(s1') ≠ Sim_old(s1') DO
    choose such a state s1';
    Remove(s1') := Pre(Sim_old(s1')) \ Pre(Sim(s1'));
    FOR ALL s1 ∈ Pre(s1') DO
        Sim(s1) := Sim(s1) \ Remove(s1')
    OD ;
    Sim_old(s1') := Sim(s1')
OD
return {(s1, s2') : s2' ∈ Sim(s1')}
```

HHK-algorithm (first version)

FOR ALL states s_1' DO

$Sim_{old}(s_1') := S$

$Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2')$ and ... $\}$

OD

> if s_2' is terminal then so is s_1'

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

choose such a state s_1';

$Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$;

FOR ALL $s_1 \in Pre(s_1')$ DO

$Sim(s_1) := Sim(s_1) \setminus Remove(s_1')$

OD ;

$Sim_{old}(s_1') := Sim(s_1')$

OD

return $\{(s_1, s_2') : s_2' \in Sim(s_1')\}$

HHK-algorithm (first version)

FOR ALL states s_1' DO

  $Sim_{old}(s_1') :=$ "undefined"

  $Sim(s_1') := \{ s_2' \in s : L(s_1') = L(s_2') \text{ and } \ldots \}$

OD

WHILE $\exists$ state s_1' with $Sim(s_1') \neq Sim_{old}(s_1')$ DO

  choose such a state s_1'

  IF $Sim_{old}(s_1') =$ "undefined"

    THEN $Remove(s_1') := S \setminus Pre(Sim(s_1'))$

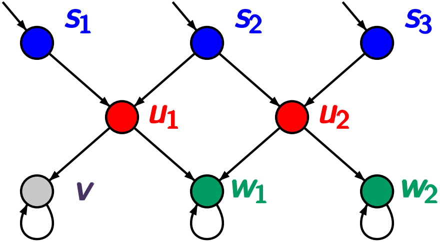
    ELSE $Remove(s_1') := Pre(Sim_{old}(s_1')) \setminus Pre(Sim(s_1'))$

  FI

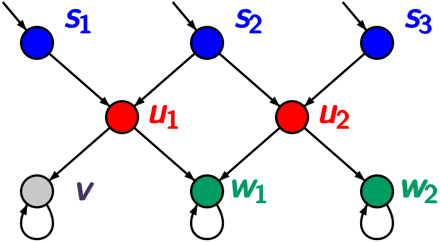
  FOR ALL $s_1 \in Pre(s_1')$ DO

  ...

Example: HHK-algorithm



Example: HHK-algorithm



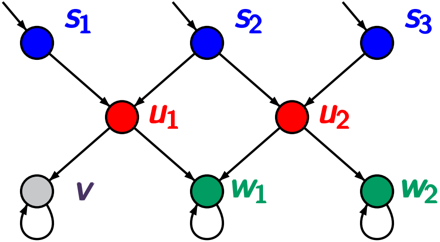
initially:

$Sim_{old}(t) = \perp$ for all states t

$Sim(s_1) = \{s_1, s_2, s_3\}$

⋮

Example: HHK-algorithm



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$\vdots$

choose state $s_1' = v$ with $Sim_{old}(v) \neq Sim(v)$:

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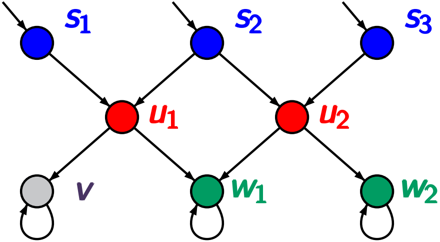

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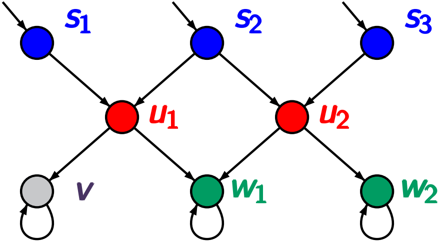
choose state $s'_1 = v$ with $Sim_{old}(v) \neq Sim(v)$:

$$Remove(v) = S \setminus Pre(Sim(v)) = S \setminus \{u_1\}$$

Example: HHK-algorithm



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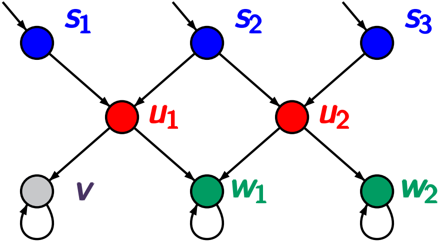

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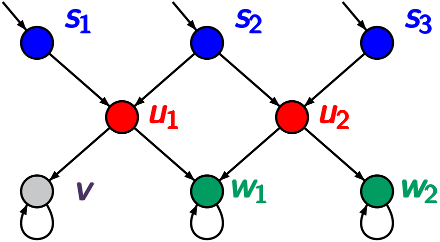
$$Remove(v) = S \setminus Pre(Sim(v)) = S \setminus \{u_1\}$$
$$Sim(u_1) := Sim(u_1) \setminus Remove(v) = \{u_1\}$$

$u_1 \longrightarrow v$ can't be simulated by any of the states in $Remove(v)$



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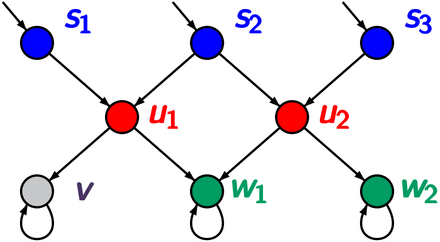

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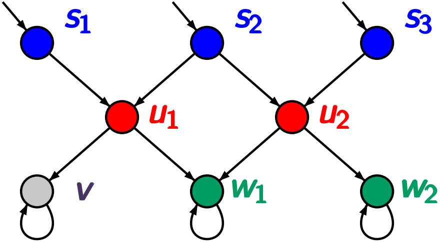
choose next state s_1'

Example: HHK-algorithm



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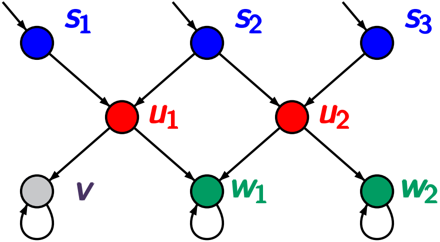
choose next state $s'_1 = s_1$ with $Sim_{old}(s_1) \neq Sim(s_1)$:

no change in $Sim(\ldots)$, as $Pre(s_1) = \varnothing$

Example: HHK-algorithm



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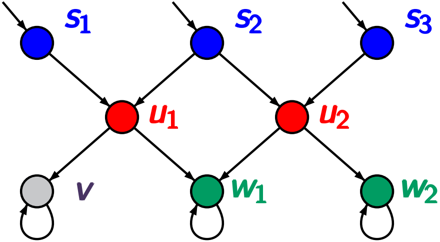
⋮

$s_1' = v$: $Sim(u_1) = \{u_1\}$, $Sim_{old}(v) = Sim(v) = \{v\}$

$s_1' = s_i$: $Sim_{old}(s_i) = Sim(s_i) = \{s_1, s_2, s_3\}$

Example: HHK-algorithm



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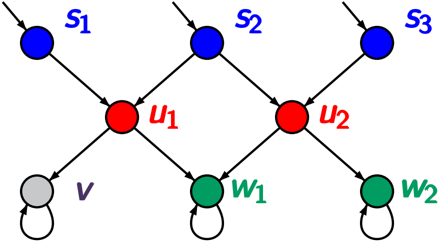

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choose next state $s_1' = u_1$ with $Sim_{old}(u_1) \neq Sim(u_1)$:

$Remove(u_1) = S \setminus Pre(Sim(u_1)) =$

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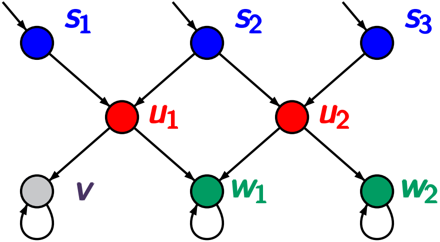

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choose next state $s_1' = u_1$ with $Sim_{old}(u_1) \neq Sim(u_1)$:

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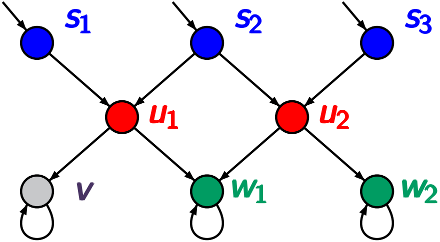

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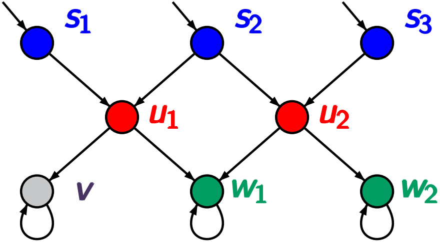


initially:

$Sim_{old}(t) = \bot$ for all states t

$Sim(s_1) = \{s_1, s_2, s_3\}$

$\vdots$

$s_1' = v$: $Sim(u_1) = \{u_1\}$, $Sim_{old}(v) = Sim(v) = \{v\}$

$s_1' = s_i$: $Sim_{old}(s_i) = Sim(s_i) = \{s_1, s_2, s_3\}$

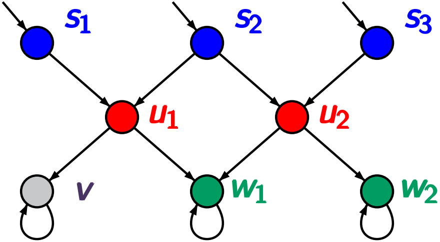
choose next state $s_1' = u_1$ with $Sim_{old}(u_1) \neq Sim(u_1)$:

$Remove(u_1) = S \setminus Pre(Sim(u_1)) = S \setminus \{s_1, s_2\}$

$Sim(s_1) := Sim(s_1) \setminus Remove(u_1) = \{s_1, s_2\}$

$s_1 \to u_1$ can't be simulated by any state $t \in Remove(u_1)$

Example: HHK-algorithm



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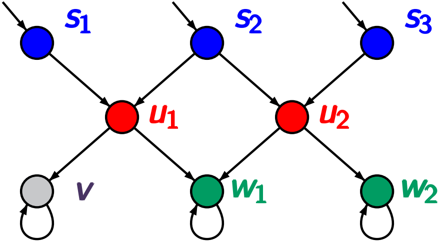

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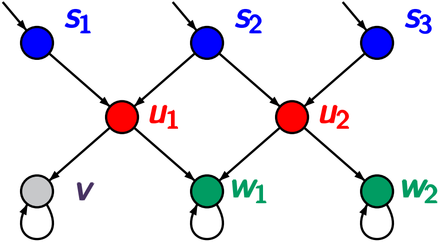
u_1: $Sim(s_i) = \{s_1, s_2\}$, i=1, 2, $Sim_{old}(u_1) = Sim(u_1) = \{u_1\}$

choose state $s'_1 = u_2$:

Example: HHK-algorithm



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$$Remove(u_2) = S \setminus Pre(Sim(u_2))$$

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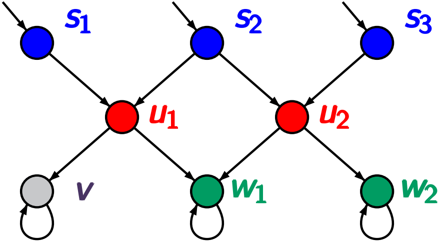

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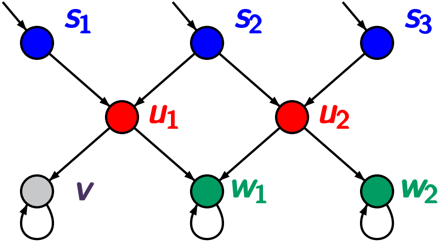

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choose state $s_1' = u_2$:

$Remove(u_2) = S \setminus Pre(Sim(u_2)) = S \setminus \{s_1, s_2, s_3\}$

$Sim(s_i) := Sim(s_2) \setminus Remove(u_2) = \{s_1, s_2\}$, $i = 1, 2$

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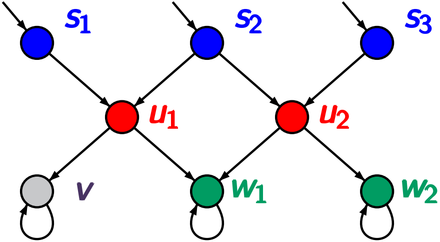

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choose state $s_1' = u_2$:

$Remove(u_2) = S \setminus Pre(Sim(u_2)) = S \setminus \{s_1, s_2, s_3\}$

$Sim(s_i) := Sim(s_2) \setminus Remove(u_2) = \{s_1, s_2\}$, $i = 1, 2$

$Sim(s_3) := Sim(s_3) \setminus Remove(u_2) = \{s_1, s_2, s_3\}$

HHK-algorithm (second version)

- an $\mathcal{O}(m \cdot |S|)$-algorithm for computing $\preceq_{\mathcal{T}}$
- relies on the techniques sketched so far
- but avoids the explicit use of the "old" simulator sets $Sim_{old}(s)$,
- adds dynamically elements to $Remove(s)$

Loop invariant of the HHK-algorithm

(1) $Sim(s_1') \supseteq \{s_2' \in S : s_1' \preceq_{\mathcal{T}} s_2'\}$

(2) $Remove(s_1') \subseteq S \setminus Pre(Sim(s_1'))$,

i.e., for all $s_2 \in Remove(s_1')$:

$$Post(s_2) \cap Sim(s_1') = \varnothing$$

hence: if $s_1 \to s_1'$ then $s_1 \not\preceq_{\mathcal{T}} s_2$

(3) if $s_2 \in Sim(s_1)$ and $s_1 \to s_1'$ then

- either $Post(s_2) \cap Sim(s_1') \neq \varnothing$
- or $s_2 \in Remove(s_1')$

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if $Remove(s_1') = \varnothing$ for all states s_1' then by (3):

$$s_2 \in Sim(s_1) \wedge s_1 \to s_1' \implies Post(s_2) \cap Sim(s_1') \neq \varnothing$$

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hence: $\{(s_1', s_2') : s_2' \in Sim(s_1')\}$ is a simulation

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hence: $\{(s_1', s_2') : s_2' \in Sim(s_1')\}$ is a simulation

since $\preceq_{\mathcal{T}}$ is the coarsest simulation, (1) yields:

$$s_2' \in Sim(s_1') \quad \text{iff} \quad s_1' \preceq_{\mathcal{T}} s_2'$$

HHK-algorithm (second version) GRM5.5-40B

HHK-algorithm (second version)


```
FOR ALL states s'_1 DO
```

$$Sim(s'_1) := \{s'_2 \in S : L(s'_1) = L(s'_2)\}$$

```
OD
```

HHK-algorithm (second version)


```
FOR ALL states s'_1 DO
```

$$Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2')\}$$

$$Remove(s_1') := S \setminus Pre(Sim(s_1'))$$

```
OD
```

HHK-algorithm (second version)


```
FOR ALL states s'_1 DO
        Sim(s'_1) := {s'_2 ∈ S : L(s'_1) = L(s'_2)}
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OD
WHILE there exists a state s'_1 with Remove(s'_1) ≠ ∅ DO
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     FOR ALL s_2 ∈ Remove(s'_1) DO
        FOR ALL s_1 ∈ Pre(s'_1) DO
           IF s_2 ∈ Sim(s_1) THEN
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        Sim(s1') := {s2' ∈ S : L(s1') = L(s2')}
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WHILE there exists a state s1' with Remove(s1') ≠ ∅ DO
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      FOR ALL s2 ∈ Remove(s1') DO
        FOR ALL s1 ∈ Pre(s1') DO
           IF s2 ∈ Sim(s1) THEN
              Sim(s1) := Sim(s1) \ {s2}
```

HHK-algorithm (second version)

FOR ALL states s_1' DO

$Sim(s_1') := \{s_2' \in S : L(s_1') = L(s_2')\}$

$Remove(s_1') := S \setminus Pre(Sim(s_1'))$

OD

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choose such a state s_1'

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$Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

$Remove(s_1) := Remove(s_1) \cup$

$\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$ FI

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OD

OD

$Remove(s_1') := \varnothing$

DO

HHK-algorithm (second version)

⋮

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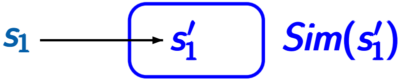
  FI

...



HHK-algorithm (second version)

⋮

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FOR ALL  s2 ∈ Remove(s1') DO
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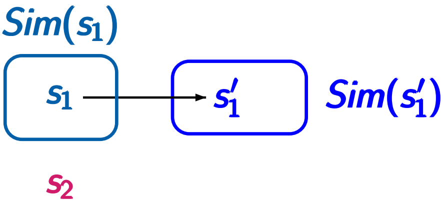
```



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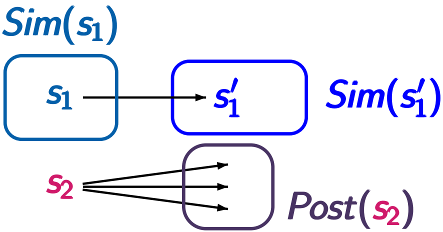

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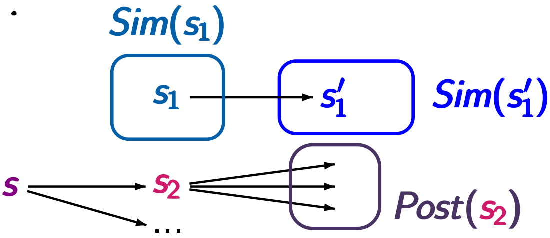

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FOR ALL $s_2 \in Remove(s_1')$ DO

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FI

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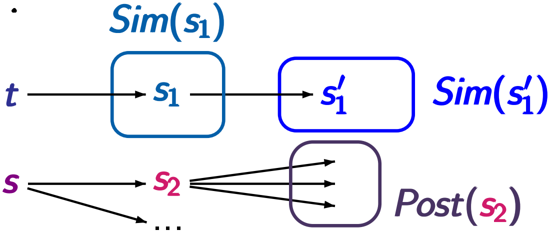

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    FI

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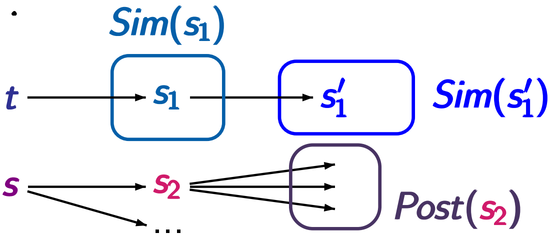

$t \not\lesssim_T s$

Termination of the HHK-algorithm

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for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $\textit{Remove}(s_1')$ at most once

Termination of the HHK-algorithm

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```
⋮
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FI
⋮
```

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if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. ...

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```

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. $s \longrightarrow s_2$ and ...

Termination of the HHK-algorithm

for each pair (s_2, s_1') of states: s_2 is inserted in (and removed from) $Remove(s_1')$ at most once

$$
\begin{array}{l}
\vdots \\
\texttt{IF}\ \ s_2 \in Sim(s_1)\ \texttt{THEN} \\
\quad Sim(s_1) := Sim(s_1) \setminus \{s_2\}; \\
\quad Remove(s_1) := Remove(s_1) \cup \\
\qquad \{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\} \\
\texttt{FI} \\
\vdots
\end{array}
$$

if s is inserted in $Remove(s_1)$ then there exists a state s_2 s.t. $s \rightarrow s_2$ and $Post(s) \cap Sim(s_1) = \{s_2\}$ immediately before s_2 has been removed from $Sim(s_1)$

Complexity of the HHK-algorithm

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show that the HHK-algorithm can be realized in time:

$$\mathcal{O}(m \cdot |S|)$$

where m = number of edges

S = state space

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$m \geqslant |S|$

Complexity of the HHK-algorithm

show that the HHK-algorithm can be realized in time:

$$\mathcal{O}(m \cdot |S|)$$

where m = number of edges

S = state space

$m \geqslant |S|$

and AP is fixed

Complexity of the initialization


```
FOR ALL states s'_1 DO
    Remove(s'_1) := S \ Pre(Sim(s'_1))
    Sim(s'_1) := { s'_2 ∈ S : L(s'_1) = L(s'_2) and
                   if s'_2 is terminal then so is s'_1 }
OD
```

FOR ALL states s'_1 DO

$$Remove(s'_1) := S \setminus Pre(Sim(s'_1))$$

$$Sim(s'_1) := \{ s'_2 \in S : L(s'_1) = L(s'_2) \text{ and if } s'_2 \text{ is terminal then so is } s'_1 \}$$

OD

↑

time complexity: $\mathcal{O}(|S| \cdot AP) = \mathcal{O}(|S|)$
(as in the bisimulation algorithms)

Complexity of the while-loop

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO

  choose such a state s_1'

  FOR ALL $s_2 \in Remove(s_1')$ DO

    FOR ALL $s_1 \in Pre(s_1')$ DO

      IF $s_2 \in Sim(s_1)$ THEN

        $Sim(s_1) := Sim(s_1) \setminus \{s_2\}$

        $Remove(s_1) := Remove(s_1) \cup$

          $\{s \in Pre(s_2) : Post(s) \cap Sim(s_1) = \varnothing\}$

      FI

    OD

  OD

  $Remove(s_1') := \varnothing$

DO

Complexity of the while-loop


```
WHILE there exists a state s1' with Remove(s1') ≠ ∅ DO
  choose such a state s1'
  FOR ALL s2 ∈ Remove(s1') DO
    FOR ALL s1 ∈ Pre(s1') DO
      IF s2 ∈ Sim(s1) THEN
        Sim(s1) := Sim(s1) \ {s2}
        Remove(s1) := Remove(s1) ∪
          {s ∈ Pre(s2) : Post(s) ∩ Sim(s1) = ∅}
      FI
    OD
  OD
  Remove(s1') := ∅
DO
```

Complexity of the while-loop


```
WHILE there exists a state s'_1 with Remove(s'_1) ≠ ∅ DO
  choose such a state s'_1
  FOR ALL s_2 ∈ Remove(s'_1) DO
    FOR ALL s_1 ∈ Pre(s'_1) DO
      IF s_2 ∈ Sim(s_1) THEN
        Sim(s_1) := Sim(s_1) \ {s_2}
        Remove(s_1) := Remove(s_1) ∪
          {s ∈ Pre(s_2) : Post(s) ∩ Sim(s_1) = ∅}
      FI
    OD
  OD
  Remove(s'_1) := ∅
OD
```

in total: $\mathcal{O}(m \cdot |S|)$

WHILE there exists a state s_1' with $Remove(s_1') \neq \varnothing$ DO

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in total: $\mathcal{O}(m \cdot |S|)$

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DO

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in total: $\mathcal{O}(m \cdot |S|)$

OD

OD

in total: $\mathcal{O}(m \cdot |S|)$

$Remove(s_1') := \varnothing$

DO

Summary: linear vs. branching time

	linear time	branching time
temporal logic	**LTL**	**CTL**
implementation relation	trace equivalence trace inclusion	bisimulation simulation

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	linear time	branching time
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bisimulation $\sim$: $\mathcal{O}(m \cdot \log |S|)$

stutter bisimulation $\approx$ or $\approx^{\mathsf{div}}$: $\mathcal{O}(m \cdot |S|)$

simulation $\preceq$: $\mathcal{O}(m \cdot |S|)$