Concurrency Theory

Winter Semester 2019/20

Lecture 15: Hennessy-Milner Logic and Bisimilarity

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https://moves.rwth-aachen.de/teaching/ws-19-20/ct/

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Outline of Lecture 15

Recap: Weak Bisimilarity

Weak Bisimilarity

1. Weak bisimilarity is based on mutually mimicking processes.
2. But: τ-actions do not need to be mimicked, as they are internal.
3. Weak bisimilarity is (non-τ) deadlock sensitive.
4. Divergence is weakly bisimilar to a deadlock process.
5. Weak bisimilarity is not a congruence for choice ($+$).
6. Observation congruence remedies this by forcing initial τ-actions to be mimicked.
7. Checking weak (non-)bisimilarity can be done using a two-player game.

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- Intuitively, saturation amounts to:
  1. pre-computing the weak transition relation $\stackrel{\alpha}{\Longrightarrow}$ (for $\alpha \neq \tau$) based on given relation $\longrightarrow$
  2. constructing a new finite-state process by replacing original transitions with weak transitions

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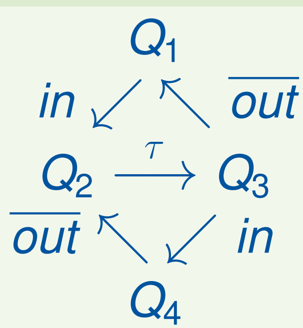
Example 15.1 (Parallel two-place buffer; cf. Example 14.10)



(on the board)

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Bisimilarity and HML

- Weak and strong bisimilarity are based on mutually mimicking of processes.
- They possess the required properties of behavioural equivalences.[1]
- In particular, $\sim$ and $\approx^c$ are deadlock-sensitive CCS congruences.
- Hennessy-Milner Logic (HML) is a logic for expressing properties of processes.

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Aim of this lecture

Study the connection between strong bisimilarity and satisfaction of HML formulae

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Verifying Correctness of Reactive Systems

Equivalence-checking approach

$$\mathit{Impl} \equiv \mathit{Spec}$$

- $\equiv$ is some equivalence, e.g., $\sim$ or $\approx^c$.
- *Spec* is often expressed in the same language as *Impl*, e.g., CCS.
- *Spec* provides the full specification of the intended behaviour.

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- *Spec* provides the full specification of the intended behaviour.

Model-checking approach

$$\mathit{Impl} \models \mathit{Prop}$$

- $\models$ is the satisfaction relation.
- *Prop* is a particular feature, often expressed via a logic, e.g., HML.
- *Prop* is a partial specification of the intended behaviour.

Outline of Lecture 15

Recap: HML

Definition (Syntax of HML; Definition 3.4) (Hennessy & Milner 1985)

The set *HMF* of Hennessy-Milner formulae over a set of actions *Act* is defined by the following syntax:

$$F ::= \mathrm{tt} \mid \mathrm{ff} \mid F_1 \wedge F_2 \mid F_1 \vee F_2 \mid \langle\alpha\rangle F \mid [\alpha]F$$

where $\alpha \in Act$.

Intuitive interpretation

- tt: all processes satisfy this property
- ff: no process satisfies this property
- $\wedge$, $\vee$: logical conjunction and disjunction
- $\langle\alpha\rangle F$: there is at least one α-successor that satisfies F
- $[\alpha]$: all α-successors have to satisfy F

Hennessy-Milner Logic

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Lemma (HML and process traces; Lemma 3.9)

Let $(Prc, Act, \longrightarrow)$ be an LTS, and let $P, Q \in Prc$ satisfy the same HMF (i.e., $\forall F \in HMF : P \models F \iff Q \models F$). Then $Tr(P) = Tr(Q)$.

Outline of Lecture 15

Image-Finite Transition Systems

Definition 15.1 (Image-finiteness)

- A process $P \in \mathit{Prc}$ is image-finite iff the set $\{P' \in \mathit{Prc} \mid P \stackrel{\alpha}{\longrightarrow} P'\}$ is finite for every $\alpha \in \mathit{Act}$.
- A labelled transition system is image-finite if so is each of its states.

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Example 15.2

1. The process $A_{rep} = a.\mathrm{nil} \parallel A_{rep}$ is not image-finite.
   By induction on n, one can prove that for each $n \in \mathbb{N}$:

$$A_{rep} \xrightarrow{a} \underbrace{a.\mathrm{nil} \parallel \cdots \parallel a.\mathrm{nil}}_{n \text{ times}} \parallel \mathrm{nil} \parallel A_{rep}$$

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2. Also the process $A^{<\omega} = \sum_{i \geq 0} a^i$ with $a^0 = \mathrm{nil}$ and $a^{i+1} = a.a^i$ is not image-finite:
   for each $i \geq 0$, $A^{<\omega} \xrightarrow{a} a^i$.

Relationship Between HML and Strong Bisimilarity

Theorem 15.3 (Hennessy-Milner Theorem)

Let $(Prc, Act, \{\xrightarrow{a} \mid a \in Act\})$ *be an image-finite LTS and* $P, Q \in Prc$. *Then:*

$$P \sim Q$$

if and only if

for every $F \in HMF : (P \models F \text{ iff } Q \models F)$.

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on the board □

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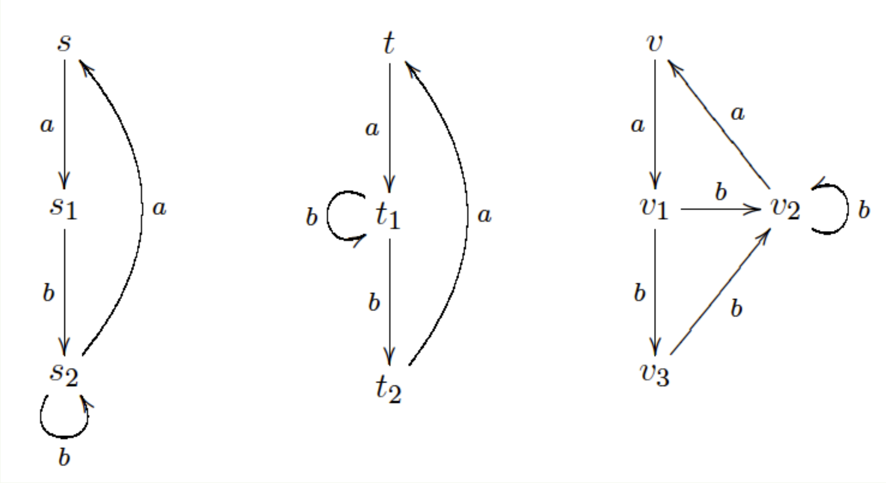
Proving non-bisimilarity

Showing $P \not\sim Q$ thus amounts to finding a single HML-formula F with $P \models F$ and $Q \not\models F$.

Example

Example 15.4



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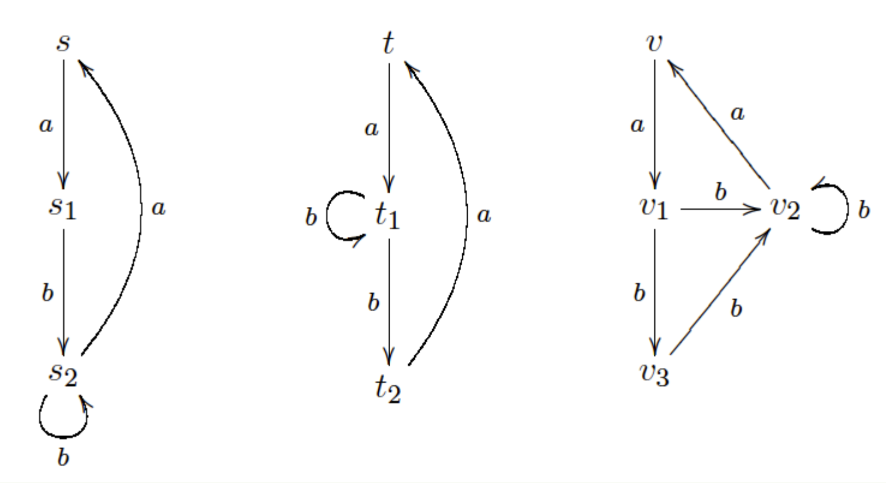

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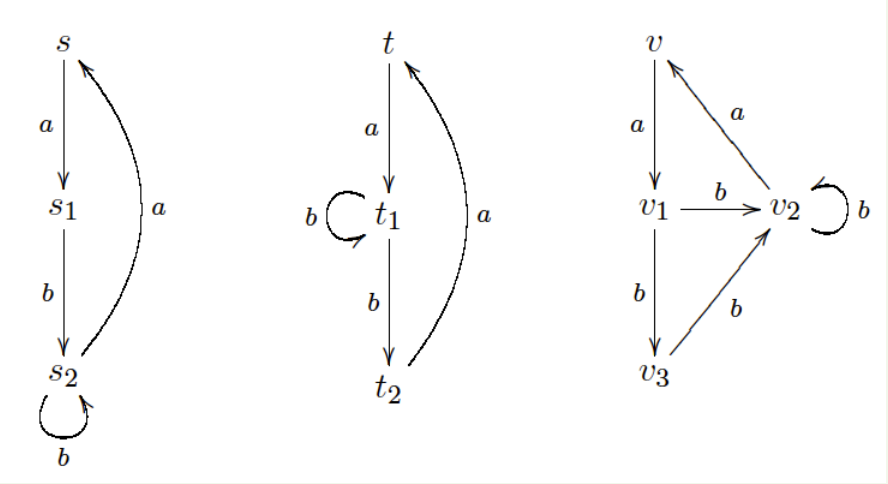

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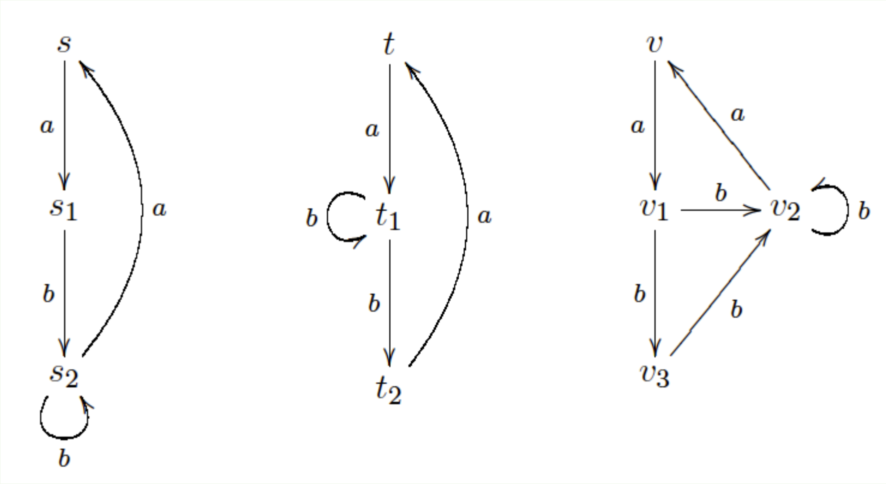

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Correspondence of HML and Strong Bisimilarity

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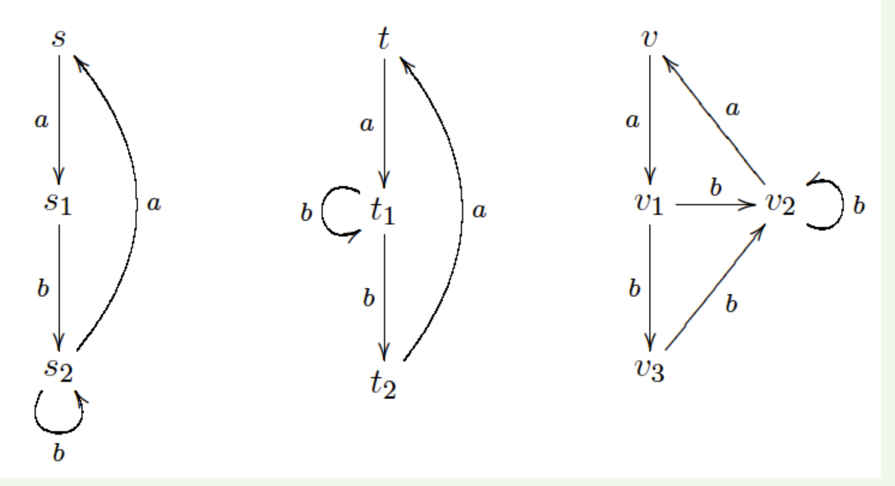

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- $t \not\sim v$: $F = \langle a \rangle \langle b \rangle (\langle a \rangle \mathrm{tt} \wedge \langle b \rangle \mathrm{tt})$ (as $v \models F$ and $t \not\models F$)

Counterexample for Non Image-Finite Processes

Lemma 15.5

Let $A^{<\omega} = \sum_{i\geq 0} a^i$ *and* $A^\omega = a.A^\omega$. *Then* $A^{<\omega}$ *and* $A^{<\omega} + A^\omega$

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For every $F \in HMF$, $A^\omega \models F$ iff $a^k \models F$, where k is the modal depth[2] of F.

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- As a next step, we show that for finite transition systems, the equivalence classes under $\sim$ can be characterised by a system of formulae in HML extended with recursion – one for each state.
- For a finite process P, this HML-formula is called P's characteristic formula.

The Need for Recursion

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- Obviously $a^i \not\sim A^\omega$ for every $i \geq 0$.
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- Obviously $a^i \not\sim A^\omega$ for every $i \geq 0$.
- On the other hand, $A^\omega \models F$ implies (by Lemma 15.6) that $a^k \models F$ where k is the modal depth of F.
- Thus, $a^k \sim A^\omega$, which contradicts $a^i \not\sim A^\omega$. □

Recap: HML with Recursion

Definition (Syntax of mutually recursive equational systems; Def. 6.4)

Let $\mathcal{X} = \{X_1, \ldots, X_n\}$ be a set of variables. The set $HMF_{\mathcal{X}}$ of Hennessy-Milner formulae over $\mathcal{X}$ is defined by the following syntax:

$$
\begin{array}{rll}
F ::= & X_i & \text{(variable)} \\
| & \mathrm{tt} & \text{(true)} \\
| & \mathrm{ff} & \text{(false)} \\
| & F_1 \wedge F_2 & \text{(conjunction)} \\
| & F_1 \vee F_2 & \text{(disjunction)} \\
| & \langle \alpha \rangle F & \text{(diamond)} \\
| & [\alpha] F & \text{(box)}
\end{array}
$$

where $1 \leq i \leq n$ and $\alpha \in Act$. A mutually recursive equational system has the form

$$(X_i = F_{X_i} \mid 1 \leq i \leq n)$$

where $F_{X_i} \in HMF_{\mathcal{X}}$ for every $1 \leq i \leq n$.

Characteristic Formulae by Example

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Therefore:

$$X_M = \langle m \rangle X_{M'} \wedge [m] X_{M'} \wedge [\{\overline{c}, \overline{t}\}]\mathrm{ff}$$

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$$X_M = \langle m \rangle X_{M'} \wedge [m] X_{M'} \wedge [\{\overline{c}, \overline{t}\}]\mathrm{ff}$$

Similarly:

$$X_{M'} = \langle \overline{c} \rangle X_M \wedge \langle \overline{t} \rangle X_M \wedge [\{\overline{c}, \overline{t}\}] X_M \wedge [m]\mathrm{ff}$$

Characteristic Formulae

The General Case

Enabled actions: $P \models \bigwedge_{\{\alpha, P' \mid P \xrightarrow{\alpha} P'\}} \langle\alpha\rangle X_{P'}$

Resulting states: $P \models \bigwedge_{\{\alpha \mid P \xrightarrow{\alpha}\}} [\alpha] \left(\bigvee_{\{P' \mid P \xrightarrow{\alpha} P'\}} X_{P'}\right)$

Disabled actions: $P \models \bigwedge_{\{\alpha \mid P \not\xrightarrow{\alpha}\}} [\alpha]\mathrm{ff}$

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Theorem 15.9 (Characteristic Formula) (Ingolfsdottir et al. 1987)

For a finite-state process $P \in Prc$, let the characteristic formula $X_P \in HMF_{\mathcal{X}}$ be defined by:

$$X_P \overset{max}{=} \bigwedge_{\{\alpha,P' \mid P \xrightarrow{\alpha} P'\}} \langle\alpha\rangle X_{P'} \wedge \bigwedge_{\alpha \in Act} [\alpha] \left(\bigvee_{\{P' \mid P \xrightarrow{\alpha} P'\}} X_{P'} \right)$$

Then, for every $Q \in Prc$: $Q \models X_P$ iff $P \sim Q$.

The General Case

Enabled actions: $P \models \bigwedge_{\{\alpha, P' \mid P \xrightarrow{\alpha} P'\}} \langle\alpha\rangle X_{P'}$

Resulting states: $P \models \bigwedge_{\{\alpha \mid P \xrightarrow{\alpha}\}} [\alpha] \left(\bigvee_{\{P' \mid P \xrightarrow{\alpha} P'\}} X_{P'} \right)$

Disabled actions: $P \models \bigwedge_{\{\alpha \mid P \not\xrightarrow{\alpha}\}} [\alpha]\mathrm{ff}$

can be combined!

Theorem 15.9 (Characteristic Formula) (Ingolfsdottir et al. 1987)

For a finite-state process $P \in Prc$, let the characteristic formula $X_P \in HMF_{\mathcal{X}}$ be defined by:

$$X_P \stackrel{max}{=} \bigwedge_{\{\alpha, P' \mid P \xrightarrow{\alpha} P'\}} \langle\alpha\rangle X_{P'} \wedge \bigwedge_{\alpha \in Act} [\alpha] \left(\bigvee_{\{P' \mid P \xrightarrow{\alpha} P'\}} X_{P'} \right)$$

Then, for every $Q \in Prc$: $Q \models X_P$ iff $P \sim Q$.

Proof.

omitted □

Outline of Lecture 15

Recap: Weak Bisimilarity

Deciding Weak Bisimilarity

Bisimilarity and HML

Hennessy-Milner Logic

Correspondence of HML and Strong Bisimilarity

Characteristic Formulae

Summary

Summary

- Strong bisimilarity and HML-equivalence coincide for image-finite processes.
- This result does not hold for processes that are not image-finite.
- For each finite-state process, a recursive HML-formula does exist that precisely characterises the strongly bisimilar processes.