Prof. Dr. Ir. Dr. h. c. Joost-Pieter Katoen
apl. Prof. Dr. Thomas Noll
Christoph Matheja, Kevin Batz

Concurrency Theory WS 2019/2020

— Exercise 5 —

Hand in until November 14th before the exercise class.

Exercise 1 (50 Points)

In this exercise, we use the tool CAAL (`http://caal.cs.aau.dk/`.), which you already know from the lecture, to model the *dining philosphers problem* in CCS. You can find examples on the syntax of CCS expressions and HML formulae, respectively, on the webiste.

The philosophical society employs two philosophers Phil_1 and Phil_2. Both spend their time either thinking or eating at a table with a large spaghetti bowl, one spoon, and one fork. Each philosopher usually keeps thinking, but he may decide to eat at any point in time. When philosopher Phil_i decides to eat, he *either* picks up the fork or he picks up the spoon, then he picks up the remaining piece of cutlery, then eats, and then releases the fork and the spoon.

(a) Complete the following CCS process definition such that it describes the behavior of the philosophical society. Use the set of actions names $\text{A} = \{\text{eat}_1, \text{eat}_2, \text{pickUpFork}, \text{releaseFork}, \text{pickUpSpoon}, \text{releaseSpoon}\}$.

$$\begin{aligned}
\text{Society} &= (\text{Phil}_1 \parallel \text{Phil}_2 \parallel \text{Spoon} \parallel \text{Fork}) \ \setminus \ \ldots \\
\text{Phil}_1 &= \ ? \\
\text{Phil}_2 &= \ ? \\
\text{Spoon} &= \ ? \\
\text{Fork} &= \ ?
\end{aligned}$$

(b) Use CAAL to draw the corresponding LTS and argue by observation of the LTS that the system exhibits a deadlock. Explain why we encounter a deadlock.

(c) Give an HML formula with one variable D which is satisified by some LTS L iff L contains a deadlock. Use your formula to verify in CAAL that the LTS from (b) indeed contains a deadlock.

(d) Dijkstra's "resource hierarchy solution" ranks each piece of cutlery. A philosopher can only pick a piece of cutlery of lower rank first and then the higher rank, if available. Extend your process definition from (a) to incorporate the ranking of the cutlery. Verify that the resulting LTS is deadlock-free using CAAL and your formula from (c).

Exercise 2 (20 Points)

Let

$$\begin{aligned}
S \ = \ & \mathsf{new}\, x(\\
& (x(u)\,.\,u(y)\,.\,u(z)\,.\,\bar{y}\langle z\rangle\,.\,\mathsf{nil} \\
& \ \|\ x(t)\,.\,t(w)\,.\,t(v)\,.\,\bar{v}\langle w\rangle\,.\,\mathsf{nil}) \\
& \ \|\ !\mathsf{new}\, s(\bar{x}\langle s\rangle\,.\,\bar{s}\langle a\rangle\,.\,\bar{s}\langle b\rangle\,.\,\mathsf{nil}) \\
&).
\end{aligned}$$

Show that

$$S \longrightarrow^{\leq 12} (\bar{a}\langle b\rangle\,.\,\mathsf{nil} \ \|\ \bar{b}\langle a\rangle\,.\,\mathsf{nil}) \ \|\ \mathsf{new}\, x(!\mathsf{new}\, s(\bar{x}\langle s\rangle\,.\,\bar{s}\langle a\rangle\,.\,\bar{s}\langle b\rangle\,.\,\mathsf{nil}))$$

where $\longrightarrow^{\leq 12}$ denotes at most 12 applications of the reaction relation.

Prof. Dr. Ir. Dr. h. c. Joost-Pieter Katoen
apl. Prof. Dr. Thomas Noll
Christoph Matheja, Kevin Batz

Exercise 3 (30 Points)

Consider the following process definition in polyadic π-calculus:

$$x(y_1, y_2)\,.\,P \parallel \bar{x}\langle z_1, z_2\rangle\,.\,Q \parallel \bar{x}\langle z_1', z_2'\rangle\,.\,Q'.$$

Provide the corresponding encoding in monadic π-calculus (See Lecture 10, Slide 9). Furthermore, do at least two reduction sequences to the resulting process definition in order to convince yourself of the correctness of your translation.