Model Checking

Lecture #4: Verifying Regular Properties

[Baier & Katoen, Chapter 4]

Joost-Pieter Katoen

Software Modeling and Verification Group

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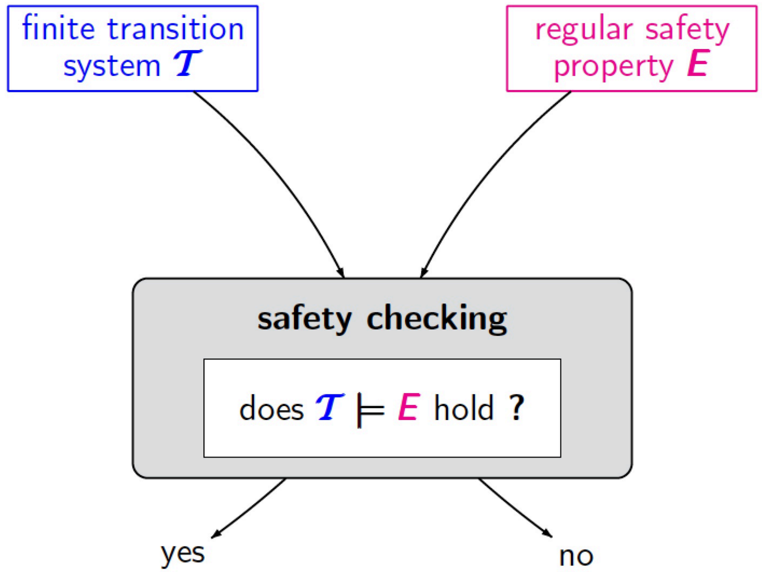
Overview

Overview

Topic



Safety Properties

Definition: Safety Property

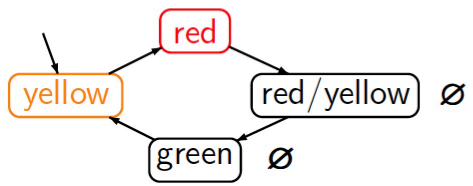
LT property E_{safe} over AP is a safety property if for all $\sigma \in \left(2^{AP}\right)^\omega \setminus E_{safe}$:

$$E_{safe} \cap \left\{ \sigma' \in \left(2^{AP}\right)^\omega \mid \hat{\sigma} \text{ is a prefix of } \sigma' \right\} \; = \; \varnothing.$$

for some prefix $\hat{\sigma}$ of σ.

- Path fragment $\hat{\sigma}$ is called a bad prefix of E_{safe}
- Let $BadPref(E_{safe})$ denote the set of bad prefixes of E_{safe}
- $\hat{\sigma} \in E_{safe}$ is minimal if no proper prefix of it is in $BadPref(E_{safe})$
- Let $MinBadPref(E_{safe})$ denote the set of minimal bad prefixes of E_{safe}

Examples



"every red phase is preceded by a yellow phase"

hence: $\mathcal{T} \models E$

E = set of all infinite words $A_0 A_1 A_2 \ldots$ over 2^{AP} such that for all $i \in \mathbb{N}$:
$$red \in A_i \implies i \geq 1 \text{ and } yellow \in A_{i-1}$$

is a safety property over $AP = \{red, yellow\}$ with

$BadPref$ = set of all finite words $A_0 A_1 \ldots A_n$ over 2^{AP} s.t. for some $i \in \{0, \ldots, n\}$:
$$red \in A_i \wedge (i{=}0 \vee yellow \notin A_{i-1})$$

Regular Safety Properties

Definition: regular safety property

Safety property E_{safe} is regular if $BadPref(E_{safe})$ is a regular language.

Or, equivalently:

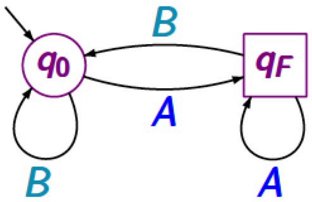
Safety property E_{safe} is regular if there exists a finite automaton over the alphabet 2^{AP} recognizing $BadPref(E_{safe})$

Overview

Finite Automata

A nondeterministic finite automaton (NFA) $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ with:

- Q is a finite set of states
- Σ is an alphabet
- $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function
- $Q_0 \subseteq Q$ a set of initial states
- $F \subseteq Q$ is a set of accept (or: final) states



Language of an Finite Automaton

- NFA $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ and finite word $w = A_1 \ldots A_n \in \Sigma^*$
- A run for w in $\mathfrak{A}$ is a finite sequence $q_0\, q_1 \, \ldots \, q_n \in Q^*$ such that:
  - $q_0 \in Q_0$ and $q_i \xrightarrow{A_{i+1}} q_{i+1}$ for all $0 \leq i < n$
- Run $q_0\, q_1 \, \ldots \, q_n$ is accepting if $q_n \in F$
- The accepted language of $\mathfrak{A}$:

$$\mathfrak{L}(\mathfrak{A}) = \{ \, w \in \Sigma^* \mid \mathfrak{A} \text{ has an accepting run for } w \, \}$$

  - $w \in \Sigma^*$ is accepted by $\mathfrak{A}$ if $\mathfrak{A}$ has an accepting run for w
- NFA $\mathfrak{A}$ and $\mathfrak{A}'$ are equivalent if $\mathfrak{L}(\mathfrak{A}) = \mathfrak{L}(\mathfrak{A}')$

Facts about Finite Automata

- They are as expressive as regular languages (Kleene's theorem)
- They are closed under $\cup$, $\cap$, and complementation
  - NFA $\mathfrak{A} \otimes B$ (= cross product) accepts $\mathfrak{L}(A) \cap \mathfrak{L}(B)$
  - Total DFA $\overline{\mathfrak{A}}$ (= swap all accept and normal states) accepts $\overline{\mathfrak{L}(A)} = \Sigma^* \setminus \mathfrak{L}(\mathfrak{A})$
- They are closed under determinization (= powerset construction)
  - although at an exponential cost ...
- $\mathfrak{L}(\mathfrak{A}) = \varnothing$? = check for a reachable accept state in NFA $\mathfrak{A}$
  - this can be done using a classical depth-first search
  - in linear-time complexity in the size of $\mathfrak{A}$
- For regular language $\mathfrak{L}$ there is a unique minimal DFA accepting $\mathfrak{L}$

Regular Safety Properties Revisited

Definition: regular safety property

Safety property E_{safe} is regular if $BadPref(E_{safe})$ is a regular language.

Or, equivalently:

if there exists a regular expression D over 2^{AP} with $\mathfrak{L}(\mathrm{D}) = BadPref(E_{safe})$

Or, equivalently:

Safety property E_{safe} is regular if there exists
an NFA $\mathfrak{A}$ over the alphabet 2^{AP} with $\mathfrak{L}(\mathfrak{A}) = BadPref(E_{safe})$

Or, equivalently:

... if there exists a DFA $\mathfrak{A}$ over 2^{AP} with $\mathfrak{L}(\mathfrak{A}) = BadPref(E_{safe})$

Sets as Formulas

Let NFA $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ over the alphaber $\Sigma = 2^{AP}$.

We adopt a shorthand notation for the transitions using propositional logic. If Φ is a propositional logic formula over AP then:

$p \xrightarrow{\Phi} q$ stands for the set of transitions $p \xrightarrow{A} q$ with $A \subseteq AP$ and $A \models \Phi$

where $A \subseteq AP$ such that $A \models \Phi$.

Examples. Let $A = \{ a, b, c \}$. Then:

- $p \xrightarrow{a \wedge \not b} q$ stands for $\{ p \xrightarrow{\{ a \}} q, p \xrightarrow{\{ a, c \}} q \}$
- $p \xrightarrow{\text{false}} q$ stands for $\{ p \xrightarrow{\varnothing} q \}$, i.e., $\{ p \xrightarrow{\neg(a \vee b \vee c)} q \}$
- $p \xrightarrow{\text{true}} q$ stands for $\{ p \xrightarrow{A} q \mid \forall A \subseteq AP \}$

Examples

- Every invariant (over AP) is a regular safety property
  - bad prefixes are of the form $\Phi^* (\neg\Phi)\, \text{true}^*$
  - where Φ is the invariant condition
- A regular safety property which is not an invariant:
  "a red light is immediately preceded by a yellow light"
- A non-regular safety property:
  "the # inserted coins is at least the # of dispensed drinks"

Details

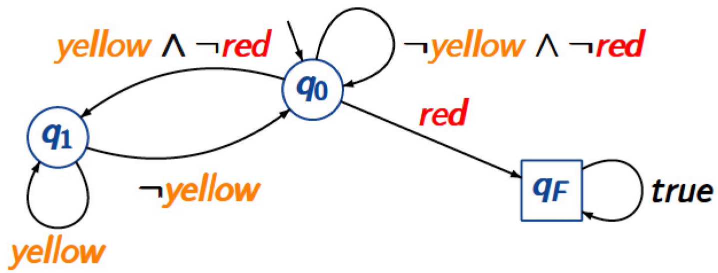
Example

"Every red phase is preceded by a yellow phase"

set of all infinite words $A_0 A_1 A_2 \ldots$ s.t. for all $i \geq 0$:

$$red \in A_i \implies i \geq 1 \text{ and } yellow \in A_{i-1}$$

DFA for all (possibly non-minimal) bad prefixes



Overview

Peterson's Algorithm

P_1	**loop forever**	
	$\vdots$	(* non-critical actions *)
	$\langle b_1 := \text{true};\ x := 2 \rangle$;	(* request *)
	wait until $(x = 1 \ \lor\ \neg b_2)$	
	do critical section **od**	
	$b_1 :=$ false	(* release *)
	$\vdots$	(* non-critical actions *)
	end loop	

b_i is true if and only if process P_i is waiting or in critical section
if both threads want to enter their critical section, x decides who gets access

Accessing a Bank Account

Thread Left behaves as follows:

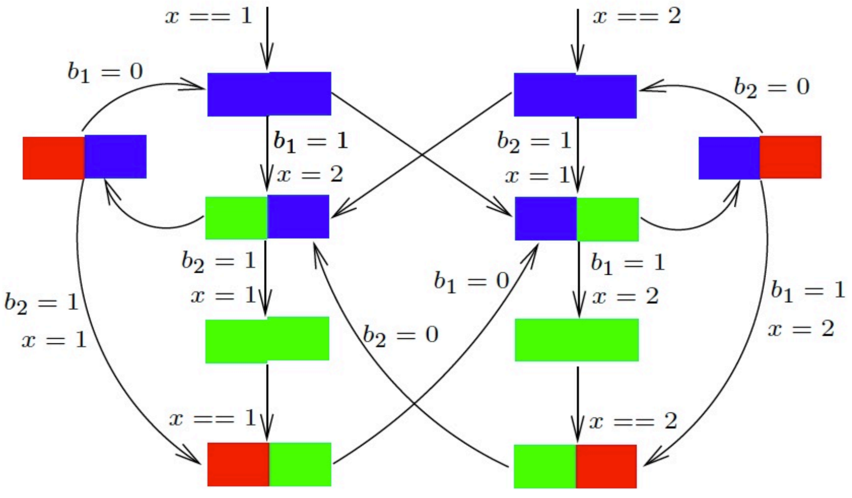
```
      while true {
          ......
nc :      ⟨b1, x = true, 2; ⟩
wt :      wait until(x == 1 || ¬ b2) {
cs :          ...@account...}
          b1 = false;
          ......
      }
```

Thread Right behaves as follows:

```
      while true {
          ......
nc :      ⟨b2, x = true, 1; ⟩
wt :      wait until(x == 2 || ¬ b1) {
cs :          ...@account...}
          b2 = false;
          ......
      }
```

Does only one thread at a time has access to the bank account?

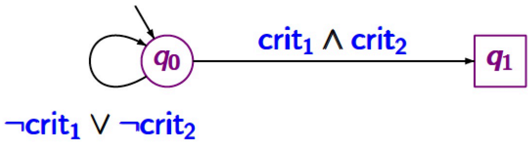
Peterson's Transition System



Manual inspection reveals that mutual exclusion is guaranteed

Verifying Mutual Exclusion

- Mutual exclusion = no simultaneous access to the account
- Bad prefix NFA $\mathfrak{A}$:



- Checking mutual exclusion:

$$\underbrace{Traces_{fin}(TS_{Pet})}_{\text{finite traces}} \cap \underbrace{BadPref(E_{safe})}_{\mathfrak{L}(\mathfrak{A})} = \varnothing?$$

- Intersection, complementation and emptiness of $\underbrace{\text{finite automata}}_{\text{accept finite words}}$

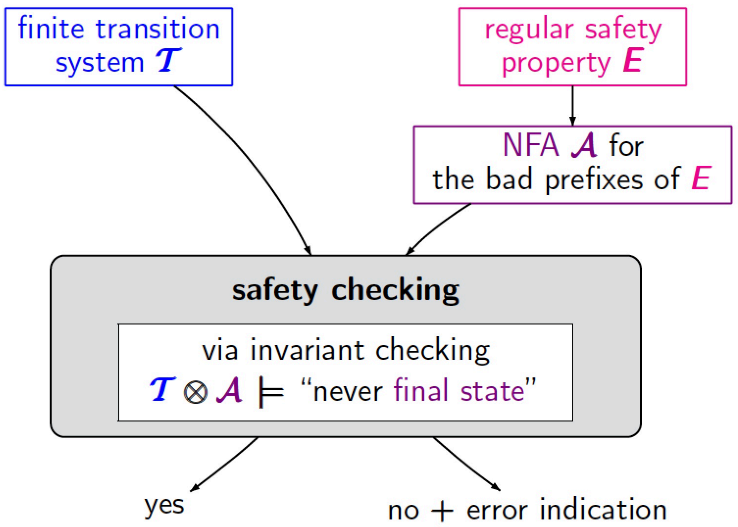
Problem Statement

Let

1. E_{safe} be a regular safety property over AP
2. $\mathfrak{A}$ be an NFA (or DFA) recognizing the bad prefixes of E_{safe}
   - with $\varepsilon \notin \mathfrak{L}(\mathfrak{A})$
   - otherwise all finite words over 2^{AP} are bad prefixes and $E_{safe} = \varnothing$
3. TS be a finite transition system (over AP) without terminal states

How to establish whether $TS \models E_{safe}$?

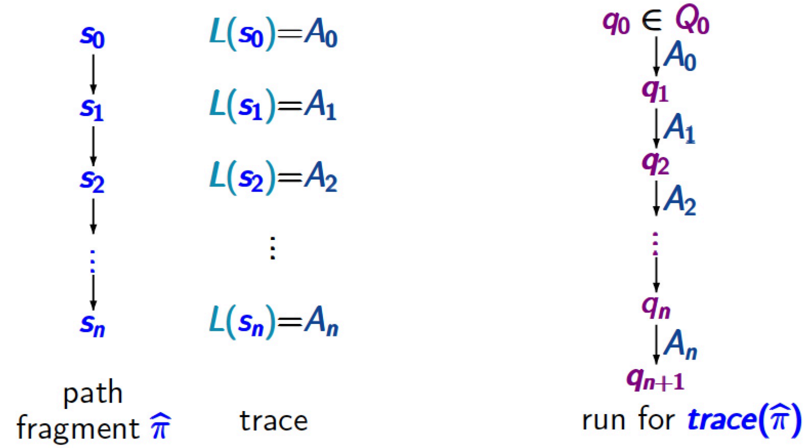
Verifying Regular Safety Properties



Product: Idea (1)

finite transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

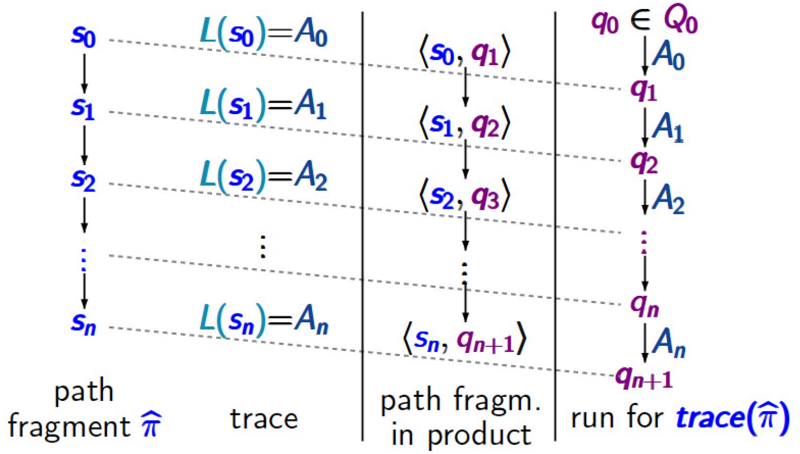
NFA for bad prefixes $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$



Product: Idea (2)

finite transition system $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

NFA for bad prefixes $\mathcal{A} = (Q, 2^{AP}, \delta, Q_0, F)$



Synchronous Product

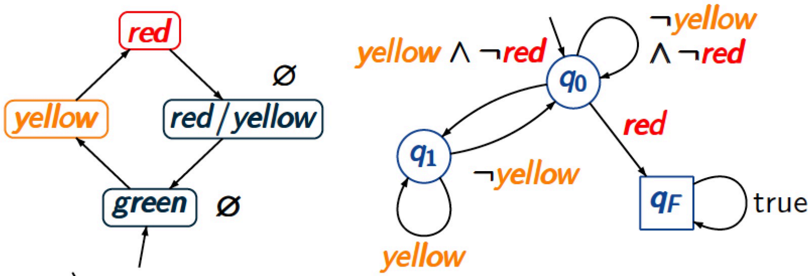
Definition: synchronous product of TS and NFA

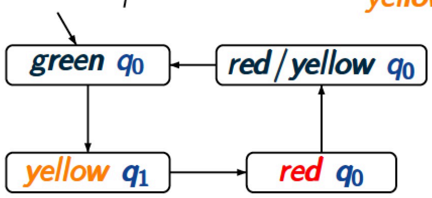
Let transition system $TS = (S, Act, \rightarrow, I, AP, L)$ without terminal states and $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ an NFA with $\Sigma = 2^{AP}$ and $Q_0 \cap F = \varnothing$. The product of TS and $\mathfrak{A}$ is the transition system:

$$TS \otimes \mathfrak{A} \; = \; (S', Act, \rightarrow', I', AP', L') \qquad \text{where}$$

- $S' = S \times Q$, $AP' = Q$ and $L'(\langle s, q\rangle) \; = \; \{ q \}$
- $\rightarrow'$ is the smallest relation defined by: $\dfrac{s \xrightarrow{\alpha} t \;\wedge\; q \xrightarrow{L(t)} p}{\langle s, q\rangle \xrightarrow{\alpha}{}' \langle t, p\rangle}$
- $I' = \{ \langle s_0, q\rangle \mid s_0 \in I \;\wedge\; \exists q_0 \in Q_0.\; q_0 \xrightarrow{L(s_0)} q \}$.

Example





product-TS $\mathcal{T} \otimes \mathcal{A}$

$4 * 3 = 12$ states, but just 4 reachable states

A Note on Terminal States

It may be safely assumed that $TS \otimes \mathfrak{A}$ has no terminal states

- Although TS has no terminal state, $TS \otimes \mathfrak{A}$ may have one
- This can only occur if $\delta(q, A) = \varnothing$ for some $A \subseteq AP$
- Let NFA $\mathfrak{A}$ with some reachable state q with $\delta(q, A) = \varnothing$
- Obtain an equivalent NFA $\mathfrak{A}'$ as follows:
  - introduce new state $q_{trap} \notin Q$
  - if $\delta(q, A) = \varnothing$ let $\delta'(q, A) = \{ q_{trap} \}$
  - set $\delta'(q_{trap}, A) = \{ q_{trap} \}$ for all $A \subseteq AP$
  - keep all other transitions that are present in $\mathfrak{A}$
- It follows $\mathfrak{L}(\mathfrak{A}) \; = \; \mathfrak{L}(\mathfrak{A}')$

Verifying Regular Safety Properties

Theorem

Let TS over AP, E_{safe} a safety property such that $\mathfrak{L}(\mathfrak{A}) = BadPref(E_{safe})$ for some NFA $\mathfrak{A}$. Then:

$$TS \models E_{safe} \;\text{ iff }\; Traces_{fin}(TS) \cap \mathfrak{L}(\mathfrak{A}) = \varnothing \;\text{ iff }\; TS \otimes \mathfrak{A} \models \underbrace{\text{always } \neg F}_{\text{invariant}}$$

where F stands for $\bigvee_{q \in F} q$.

Proof

Example

Counterexamples

For each initial path fragment $\langle s_0, q_1 \rangle \ldots \langle s_n, q_{n+1} \rangle$ of $TS \otimes \mathfrak{A}$:

$$q_1, \ldots, q_n \notin F \text{ and } q_{n+1} \in F \quad \Rightarrow \quad \underbrace{trace(s_0\, s_1 \ldots s_n)}_{\text{bad prefix for } E_{safe}} \in \mathfrak{L}(\mathfrak{A}).$$

Complexity of Verifying Regular Safety Properties

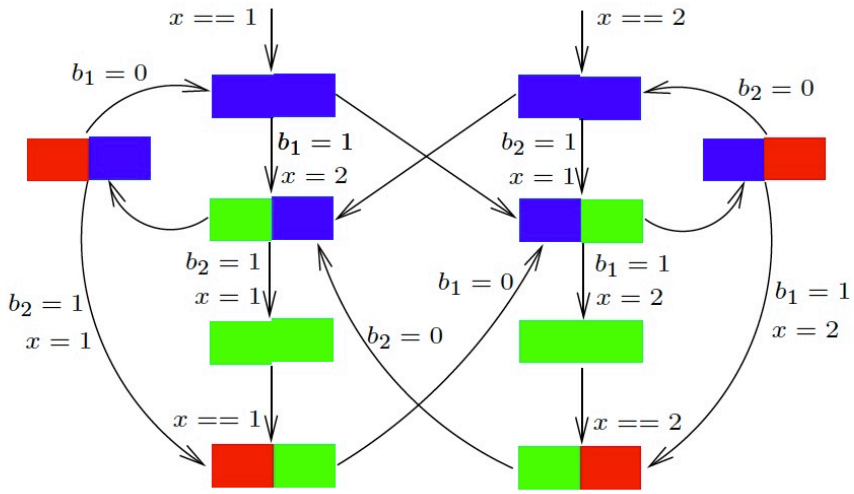
The time and space complexity of checking $TS \models E_{safe}$ is in $O(|TS| \cdot |\mathfrak{A}|)$ where $\mathfrak{A}$ is an NFA with $\mathfrak{L}(\mathfrak{A}) = BadPref(E_{safe})$ and $|\mathfrak{A}|$ is the size of $\mathfrak{A}$.

The size of NFA $\mathfrak{A}$ is the number of states and transitions in $\mathfrak{A}$:

$$|\mathfrak{A}| \;=\; |Q| + \sum_{q \in Q} \sum_{A \in \Sigma} |\, \delta(q, A) \,|$$

Overview

Peterson's Transition System



If a thread wants to update the account, does it ever get the opportunity to do so?

always (req_L → eventually @$account_L$) ∧ always (req_R → eventually @$account_R$)

Verifying Starvation Freedom

- Starvation freedom = when a thread wants access to account, it eventually gets it
- "Infinite bad prefix" automaton: once a thread wants access to the account, it never gets it
- Checking starvation freedom:

$$\underbrace{Traces(TS_{Pet})}_{\text{infinite traces}} \cap \mathfrak{L}(\overline{E_{live}}) \;=\; \varnothing ?$$

- Intersection, complementation and emptiness of Büchi $\underbrace{\text{automata}}_{\text{accept infinite words}}$

ω-Regular Expressions: Syntax

Definition: ω-regular expression

An ω-regular expression G over the alphabet Σ has the form:

$$G = E_1.F_1^\omega + \ldots + E_n.F_n^\omega \quad \text{for } n \in \mathbb{N}_{>0}$$

where E_i, F_i are regular expressions over Σ with $\varepsilon \notin \mathfrak{L}(F_i)$.

- ω-Regular expressions denote languages of infinite words
- Examples over the alphabet $\Sigma = \{A, B\}$:
  - language of all words with infinitely many As: $(B^*.A)^\omega$
  - language of all words with finitely many As: $(A+B)^*.B^\omega$
  - the empty language $\varnothing^\omega$

ω-Regular Expressions: Semantics

Definition: semantics of ω-regular expressions

The semantics of ω-regular expression

$$G = E_1.F_1^\omega + \ldots + E_n.F_n^\omega$$

is the language $\mathfrak{L}(G) \subseteq \Sigma^\omega$ defined by:

$$\mathfrak{L}_\omega(G) = \mathfrak{L}(E_1).\mathfrak{L}(F_1)^\omega \cup \ldots \cup \mathfrak{L}(E_n).\mathfrak{L}(F_n)^\omega.$$

where for $\mathfrak{L} \subseteq \Sigma^*$, we have $\mathfrak{L}^\omega = \{ w_1 w_2 w_3 \ldots \mid \forall i \geq 0.\, w_i \in \mathfrak{L} \}$.

The ω-regular expression G_1 and G_2 are equivalent, denoted $G_1 \equiv G_2$, if $\mathfrak{L}_\omega(G_1) = \mathfrak{L}_\omega(G_2)$.

ω-Regular Properties

Definition: ω-regular language

The set $\mathfrak{L}$ of infinite works over the alphabet Σ is ω-regular if $\mathfrak{L} = \mathfrak{L}_\omega(G)$ for some ω-regular expression G over Σ.

Definition: ω-regular properties

LT property E over AP is ω-regular if E is an ω-regular language over 2^{AP}.

We will see that this is equivalent to:

LT property E over AP is ω-regular if E is accepted by a non-deterministic Büchi automaton (over the alphabet 2^{AP}).

But not by a deterministic Büchi automaton.

Example ω-Regular Properties

- Any invariant E is an ω-regular property
  - Φ^ω describes E with invariant condition Φ
- Any regular safety property E is an ω-regular property
  - $\overline{E} = BadPref(E).\left(2^{AP}\right)^\omega$ is ω-regular
  - and ω-regular languages are closed under complement
- Let $\Sigma = \{a, b\}$ Then:
  - Infinitely often a:

$$\left((\varnothing + \{b\})^*.(\{a\} + \{a, b\})\right)^\omega$$

  - eventually a:

$$\left(2^{AP}\right)^\omega.(\{a\} + \{a, b\}).\left(2^{AP}\right)$$

Shorthand Notation

Examples for $AP = \{a, b\}$

- invariant with invariant condition $a \vee \neg b$

$$(a \vee \neg b)^\omega \quad \widehat{=} \quad (\emptyset + \{a\} + \{a, b\})^\omega$$

- "infinitely often a"

$$((\neg a)^*.a)^\omega \quad \widehat{=} \quad ((\emptyset + \{b\})^*.(\{a\} + \{a, b\}))^\omega$$

- "from some moment on a":

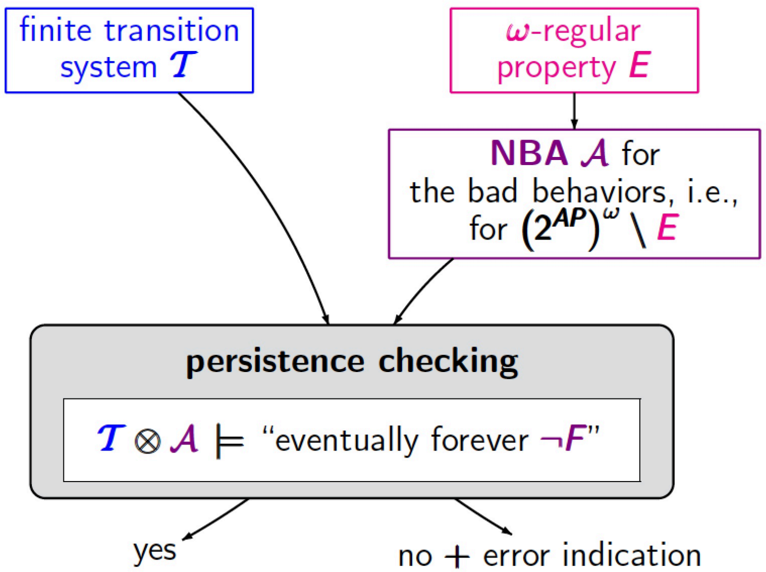
$$\mathit{true}^*.a^\omega$$

- "whenever a then b will hold somewhen later"

$$((\neg a)^*.a.\mathit{true}^*.b)^*.(\neg a)^\omega + ((\neg a)^*.a.\mathit{true}^*.b)^\omega$$

Overview

Verifying ω-Regular Properties



Julius Richard Büchi

Julius Richard Büchi (1924 – †1984)

Nondeterministic Büchi automata

Definition: Nondeterministic Büchi automaton

A nondeterministic Büchi automaton (NBA) $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ with:

- Q is a finite set of states
- Σ is an alphabet
- $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function
- $Q_0 \subseteq Q$ a set of initial states
- $F \subseteq Q$ is a set of accept (or: final) states.

This definition is the same as for NFA.

The acceptance condition of NBA is different though.

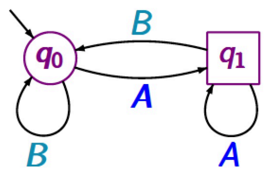
Language of an Büchi Automaton

- NBA $\mathfrak{A} = (Q, \Sigma, \delta, Q_0, F)$ and infinite word $w = A_1\, A_2 \ldots \in \Sigma^\omega$
- A run for w in $\mathfrak{A}$ is an infinite sequence $q_0\, q_1 \ldots \in Q^\omega$ such that:
  - $q_0 \in Q_0$ and $q_i \xrightarrow{A_{i+1}} q_{i+1}$ for all $0 \leq i$
- Run $q_0\, q_1 \ldots$ is accepting if $q_i \in F$ for infinitely many i
- The accepted language of $\mathfrak{A}$:

$$\mathfrak{L}_\omega(\mathfrak{A}) = \left\{ w \in \Sigma^\omega \mid \mathfrak{A} \text{ has an accepting run for } w \right\}$$

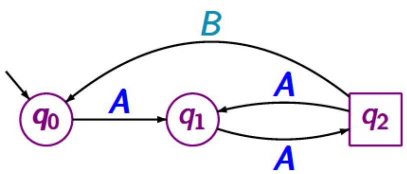
  - $w \in \Sigma^*$ is accepted by $\mathfrak{A}$ if $\mathfrak{A}$ has an accepting run for w
- NBA $\mathfrak{A}$ and $\mathfrak{A}'$ are equivalent if $\mathfrak{L}_\omega(\mathfrak{A}) = \mathfrak{L}_\omega(\mathfrak{A}')$

Examples



accepted language:
set of all infinite words that contain infinitely many ***A***'s

$(B^*.A)^\omega$



accepted language:
"every ***B*** is preceded by a positive even number of ***A***'s"

$((A.A)^+.B)^\omega + ((A.A)^+.B)^*.A^\omega$

NBA for LT Properties

NBA versus NFA

Deterministic Büchi Automata

Definition: Deterministic Büchi automaton

Büchi automaton $\mathfrak{A}$ is deterministic if

$$|Q_0| \leq 1 \quad \text{and} \quad |\delta(q, A)| \leq 1 \quad \text{for all } q \in Q \text{ and } A \in \Sigma.$$

A DBA is total if both inequalities are equalities.

A total DBA has a unique run for each input word.

DBA Are Less Expressive Than NBA

There is no DBA that accepts $\mathcal{L}_\omega((A + B)^* B^\omega)$.

NFA and DFA are equally expressive but NBA and DBA are not!

LT Properties Need Nondeterminism

Overview

NBA and ω-Regular Languages

Theorem

1. For every NBA $\mathfrak{A}$, the language $\mathfrak{L}_\omega(\mathfrak{A})$ is ω-regular.
2. For every ω-regular language L, there is an NBA $\mathfrak{A}$ with $L = \mathfrak{L}_\omega(\mathfrak{A})$.

Proof.

Next lecture. □

Verifying ω-Regular Safety Properties

Theorem

Let TS over AP, E an ω-regular property and NBA $\mathfrak{A}$ with $\mathfrak{L}(\mathfrak{A}) = \overline{E}$. Then:

$$TS \models E \text{ iff } Traces(TS) \cap \mathfrak{L}_\omega(\mathfrak{A}) = \varnothing \text{ iff } TS \otimes \mathfrak{A} \models \underbrace{\text{eventually forever} \neg F}_{\text{persistence property}}$$

where F stands for $\bigvee_{q \in F} q$.

Proof.

Next lecture. □

Next Lecture

Friday October 25, 14:30