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Model Checking (Winter Term 2019/2020)

— Exercise Sheet 7 (due 20th December) —

General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 10:30 or by dropping them into the "Model Checking" box at our chair *before 10:25*. Do *not* hand in your solutions via Moodle or via e-mail.

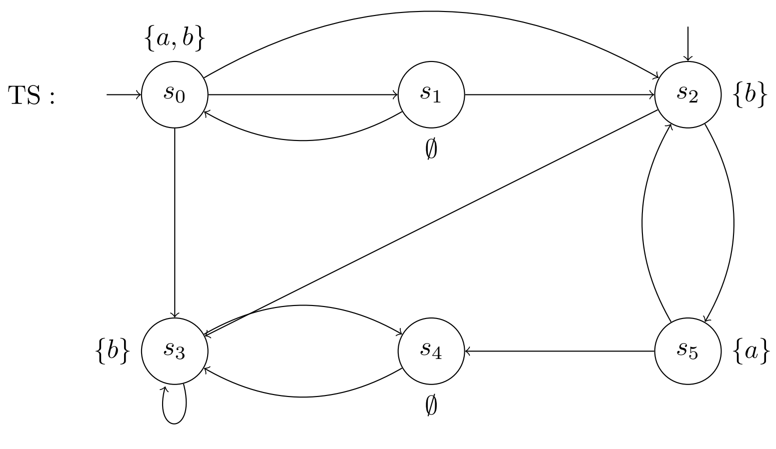
Exercise 1★ (3+2+3 Points)

Consider the following CTL formula Φ and the fairness assumption *sfair*:

$$\Phi = \forall\Box\forall\Diamond\, a$$

$$\mathit{sfair} = \Box\Diamond \underbrace{(b \wedge \neg a)}_{\Phi_1} \rightarrow \Box\Diamond \underbrace{\exists\Big(b \mathsf{~U~} (a \wedge \neg b)\Big)}_{\Psi_1}$$

and transition system TS over AP $= \{a, b\}$ which is given below.



Here, we abstract from the actions in TS as they are not relevant to the task.

(a) Determine $Sat(\Phi_1)$ and $Sat(\Psi_1)$ (without fairness). Justify your answer.

(b) Determine $Sat_{sfair}(\exists\Box\, true)$. Justify your answer.

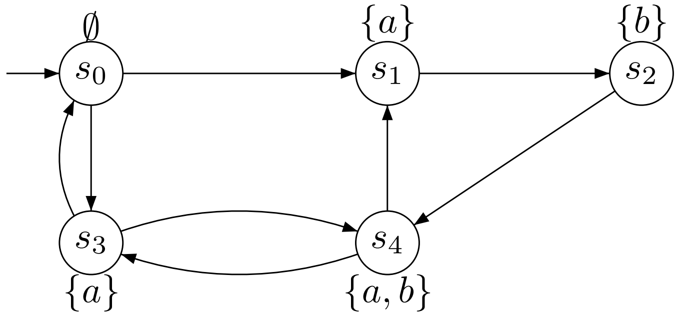
(c) Determine $Sat_{sfair}(\Phi)$. Justify your answer.

Exercise 2 (6 Points)

Consider the CTL-formula $\Phi = \forall\Box\left(a \rightarrow \forall\Diamond\,(b\wedge\neg a)\right)$ together with the following CTL fairness assumption

$$fair = \Box\Diamond\forall\bigcirc\ (a \wedge \neg b) \rightarrow \Box\Diamond\forall\bigcirc\ (b \wedge \neg a)$$
$$\wedge\ \Diamond\Box\exists\Diamond\, b \rightarrow \Box\Diamond\, b.$$

Check whether $TS \models_{fair} \Phi$ for the transition system TS below.



Exercise 3★ (3 Points)

For each of the following, give pairs of transition systems such that:

a) T_1, T_2 are simulation-equivalent, but not bisimular.

b) T_3, T_4 are trace-equivalent, but not bisimilar.

c) T_5, T_6 are finite-trace equivalent, but not simulation-equivalent.

d) T_7, T_8 are finite-trace equivalent, but not trace-equivalent.

e) T_9, T_10 are trace-equivalent and simulation-equivalent.

Exercise 4 (3 Points)

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. The relations $\sim_n \subseteq S \times S$ are inductively defined by:

- $s_1 \sim_0 s_2$ iff $L(s_1) = L(s_2)$.
- $s_1 \sim_{n+1} s_2$ iff:
  - $L(s_1) = L(s_2)$,
  - for all $s_1' \in Post(s_1)$ there exists $s_2' \in Post(s_2)$ with $s_1' \sim_n s_2'$,
  - for all $s_2' \in Post(s_2)$ there exists $s_1' \in Post(s_1)$ with $s_1' \sim_n s_2'$.

Questions:

(a) Show that for *finite* TS it holds that $\sim_{TS} = \bigcap_{n\geq 0} \sim_n$, i.e.,

$$s_1 \sim_{TS} s_2 \text{ iff } s_1 \sim_n s_2 \text{ for all } n \geq 0$$

(b) Does this also hold for infinite transition systems (provide a proof or a counterexample)?