General Remarks

- The exercises are to be solved in groups of three students.
- You may hand in your solutions for the exercises just before the exercise class starts at 10:30 or by dropping them into the “Model Checking” box at our chair before 10:25. Do not hand in your solutions via Moodle or via e-mail.

Exercise 1⋆ (3+2+3 Points)

Consider the following CTL formula \( \Phi \) and the fairness assumption \( sfair \):

\[
\Phi = \forall \Box \forall \Diamond a
\]

\[
sfair = \Box \Diamond (b \land \neg a) \rightarrow \Box \Diamond (b \lor (a \land \neg b))
\]

and transition system TS over \( AP = \{a, b\} \) which is given below.

Here, we abstract from the actions in TS as they are not relevant to the task.

(a) Determine \( \text{Sat}(\Phi_1) \) and \( \text{Sat}(\Psi_1) \) (without fairness). Justify your answer.
(b) Determine \( \text{Sat}_{sfair}(\exists \Box \text{true}) \). Justify your answer.
(c) Determine \( \text{Sat}_{sfair}(\Phi) \). Justify your answer.
Exercise 2  

Consider the CTL-formula $\Phi = \forall \square (a \rightarrow \forall \Diamond (b \land \neg a))$ together with the following CTL fairness assumption:

$$fair = \square \forall \Diamond (a \land \neg b) \rightarrow \square \forall \Diamond (b \land \neg a)$$

$$\land \Diamond \exists \Diamond b \rightarrow \square \Diamond b.$$

Check whether $TS \models fair \Phi$ for the transition system $TS$ below.

Exercise 3**  

For each of the following, give pairs of transition systems such that:

a) $T_1, T_2$ are simulation-equivalent, but not bisimilar.

b) $T_3, T_4$ are trace-equivalent, but not bisimilar.

c) $T_5, T_6$ are finite-trace equivalent, but not simulation-equivalent.

d) $T_7, T_8$ are finite-trace equivalent, but not trace-equivalent.

e) $T_9, T_{10}$ are trace-equivalent and simulation-equivalent.

Exercise 4  

Let $TS = (S, Act, \rightarrow, I, AP, L)$ be a transition system. The relations $\sim_n \subseteq S \times S$ are inductively defined by:

- $s_1 \sim_0 s_2$ iff $L(s_1) = L(s_2)$.
- $s_1 \sim_{n+1} s_2$ iff:
  - $L(s_1) = L(s_2)$,
  - for all $s'_1 \in Post(s_1)$ there exists $s'_2 \in Post(s_2)$ with $s'_1 \sim_n s'_2$,
  - for all $s'_2 \in Post(s_2)$ there exists $s'_1 \in Post(s_1)$ with $s'_1 \sim_n s'_2$.

Questions:

(a) Show that for finite $TS$ it holds that $\sim_{TS} = \cap_{n \geq 0} \sim_n$, i.e.,

$$s_1 \sim_{TS} s_2 \text{ iff } s_1 \sim_n s_2 \text{ for all } n \geq 0$$

(b) Does this also hold for infinite transition systems (provide a proof or a counterexample)?