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Model Checking (Winter Term 2019/2020)

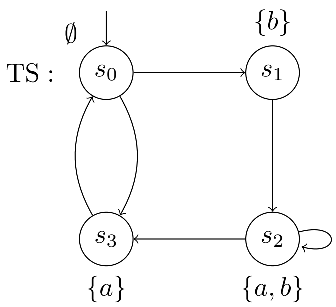
— Exercise Sheet 5 (due 29th November) —

General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 10:30 or by dropping them into the "Introduction to Model Checking" box at our chair *before 10:15*. Do *not* hand in your solutions via Moodle or via e-mail.

Exercise 1

Let $\varphi = \Box(a \rightarrow ((\neg b) \mathsf{U} (a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions. We are interested in checking whether $\text{TS} \models \varphi$ where TS is the following transition system:



(a) Convert $\neg\varphi$ into an equivalent LTL-formula ψ which is constructed according to the following grammar:

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi \,\mathsf{U}\, \varphi.$$

Derive $\mathit{closure}(\psi)$.

(b) Give *all* elementary sets wrt. $\mathit{closure}(\psi)$.

(c) Construct the GNBA $\mathcal{G}_\psi$ using the algorithm given in the lecture. It suffices to provide its initial states, its acceptance set and its transition relation.
Hint: Give the transition relation as a table where the rows and columns correspond to states of $\mathcal{G}_\psi$ and the entries are either empty (representing "no transition") or contain an element from 2^{AP} (representing the character that can be used for the transition).

(d) Now, construct a *non-blocking* NBA $\mathcal{A}_{\neg\varphi}$ **directly** from $\neg\varphi$, i.e. without relying on $\mathcal{G}_\psi$. Provide *an intuitive* explanation of why your automaton recognizes the right language.
Hint: Four states suffice. Consider rewriting $\neg\varphi$ using the release operator and recall that $\varphi \,\mathsf{R}\, \psi$ intuitively expresses that φ "releases" ψ. That is, ψ either holds all the time or at some point $\varphi \wedge \psi$ holds and at all previous positions ψ holds.

Exercise 2

Consider the following CTL formulae and the transition system TS outlined on the right:

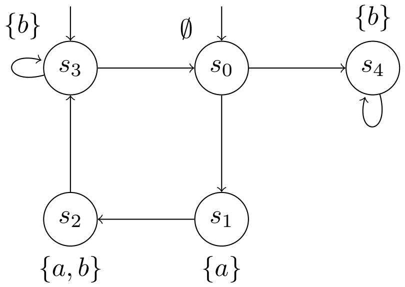
$\Phi_1 = \forall(a \mathsf{U} b) \vee \exists \bigcirc \forall\Box b$

$\Phi_2 = \forall\Box\forall(a \mathsf{U} b)$

$\Phi_3 = (a \wedge b) \rightarrow \exists\Box\exists\bigcirc\forall(b \mathsf{W} a)$

$\Phi_4 = \forall\Box\exists\Diamond\neg(a \vee b)$

TS :



Give the satisfaction sets $Sat(\Phi_i)$ for each CTL formula Φ_i, $1 \leq i \leq 4$.
Does $\text{TS} \models \Phi_i$ hold?

Exercise 3

(a) Using an appropriate theorem from the lecture, prove that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_1 = \forall\Diamond(a \wedge \exists\bigcirc a)$.

(b) Now prove directly (i.e. without the theorem from the lecture) that there does not exist an equivalent LTL-formula for the CTL-formula $\Phi_2 = \forall\Diamond\exists\bigcirc\forall\Diamond\neg a$.
Hint: Argue by contraposition. In particular, think about trace inclusion versus CTL-equivalence.

Exercise 4

Prove that $Sat(\exists(\Phi \mathsf{W} \Psi))$ is the largest set T such that

$$T \subseteq Sat(\Psi) \cup \{s \in Sat(\Phi) \mid \text{Post}(s) \cap T \neq \emptyset\}. \qquad (5.1)$$