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Model Checking (Winter Term 2019/2020)

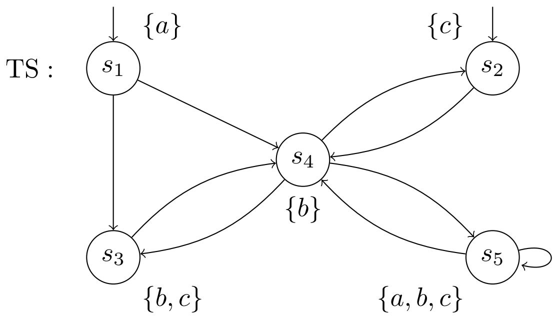
— Exercise Sheet 4 (due 22th November) —

General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions before the exercise class starts at 10:30 or by dropping them into the "Model Checking" box at our chair *before 10:20*. Do *not* hand in your solutions electronically.

Exercise 1★ (6 Points)

Consider the following transition system TS where we omit the transition labels, because they are all τ.



For each of the LTL formulae φ_i below, decide whether $\text{TS} \models \varphi_i$. Justify your answer and, in particular, provide a path $\pi_i \in \mathit{Paths}(\text{TS})$ such that $\pi_i \not\models \varphi_i$ in case you find $\text{TS} \not\models \varphi_i$.

- $\varphi_1 = \Diamond \Box c$,
- $\varphi_2 = \Box \Diamond c$,
- $\varphi_3 = \bigcirc \neg c \rightarrow \bigcirc \bigcirc c$,
- $\varphi_4 = \Box a$,
- $\varphi_5 = a \,\mathsf{U}\, \Box (b \vee c)$,
- $\varphi_6 = (\bigcirc \bigcirc b) \,\mathsf{U}\, (b \vee c)$,
- $\varphi_7 = c \,\mathsf{R}\, b$,

where the *release operator* $\varphi \,\mathsf{R}\, \psi$ for two LTL formulae φ, ψ is defined by $\varphi \,\mathsf{R}\, \psi \equiv \neg(\neg\varphi \,\mathsf{U}\, \neg\psi)$.

Exercise 2* (2+2+2+2 Points)

Let φ, ψ, π be arbitrary LTL formulae. For each of the following pairs of LTL formulae, determine in which relation they are. More specifically, determine whether they are equivalent, one of them subsumes the other or they are incomparable. Prove your claims.

(a) $\Diamond\Box\varphi$ and $\Box\Diamond\varphi$

(b) $\Diamond\Box\varphi \wedge \Diamond\Box\psi$ and $\Diamond(\Box\varphi \wedge \Box\psi)$

(c) $\varphi \wedge \Box(\varphi \rightarrow \bigcirc\Diamond\varphi)$ and $\Box\Diamond\varphi$

(d) $(\varphi \,\mathsf{U}\, \psi) \,\mathsf{U}\, \pi$ and $\varphi \,\mathsf{U}\, (\psi \,\mathsf{U}\, \pi)$

General Notation

In the following we transform LTL formulae into the corresponding GNBAs. As an example consider the LTL formula $\varphi = a \,\mathsf{U}\, (\neg a \wedge b)$. We order the subformulae of φ from the innermost formulae to the outermost, and from left to right. In our example we get the subformulae a, b, $\neg a \wedge b$ and φ. The elementary sets are given in the following table where we order the sets by their binary encoding:

B	a	b	$\neg a \wedge b$	φ
B_1	0	0	0	0
B_2	0	1	1	1
B_3	1	0	0	0
B_4	1	0	0	1
B_5	1	1	0	0
B_6	1	1	0	1

Moreover, for the GNBA $\mathcal{G}_\varphi$ the transition relation can be given as a table where the rows and columns correspond to states of $\mathcal{G}_\varphi$ and the entries are either empty (representing "no transition") or contain an element from 2^{AP} (representing the character that can be used for the transition).

For example, an extract of the transition relation for the GNBA $\mathcal{G}_\varphi$ is given in the following.

	B_1	B_2	B_3	B_4	B_5	B_6
B_1	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
B_2			...			
B_3	$\{a\}$		$\{a\}$		$\{a\}$	
...			...			

Exercise 3* (1+3+3 Points)

Let $\mathrm{AP} = \{a, b\}$. Let $\varphi = (a \rightarrow \bigcirc\neg b) \,\mathsf{W}\, (a \wedge b)$ as in exercise sheet 7.2.

(a) Transform $\neg\varphi$ into an equivalent LTL formula φ' (i.e., $\mathit{Words}(\neg\varphi) = \mathit{Words}(\varphi')$) which is constructed according to the following grammar:

$$\varphi ::= \mathit{true} \mid \mathit{false} \mid a \mid b \mid \varphi \wedge \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi \,\mathsf{U}\, \varphi.$$

(b) Compute all elementary sets with respect to $\mathit{closure}(\varphi')$.

(c) Construct the GNBA $\mathcal{G}_{\varphi'}$ according to the algorithm from the lecture such that $\mathcal{L}_\omega(\mathcal{G}_{\varphi'}) = \mathit{Words}(\varphi')$. It suffices to provide the initial states, the acceptance set and the transition relation of $\mathcal{G}_{\varphi'}$ as a table.

Exercise 4★ (3+3 Points)

Let φ and ψ be LTL formulae. Consider the following new operators.

- "At next": $\varphi \,\mathsf{AX}\, \psi \iff$ the next time at which ψ holds, φ also holds.
- "While": $\varphi \,\mathsf{WH}\, \psi \iff \varphi$ holds at least as long as ψ holds.
- "Before": $\varphi \,\mathsf{B}\, \psi \iff$ if ψ holds at some point, φ does so (strictly) before.

(a) Formalize the semantics of these operators on infinite words $\sigma \in (2^{\mathrm{AP}})^\omega$.

(b) Show that these operators are LTL-definable by providing equivalent LTL formulae.
Hint: You may use both the until and weak until operator.