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Model Checking (Winter Term 2019/2020)

— Exercise Sheet 2 (due November 8^{th}) —

General Remarks

- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 10:30 or by dropping them into the "Model Checking" box at our chair *before 12:00*. Do *not* hand in your solutions via L2P or via e-mail.

Exercise 1

(a) Let P and P' be liveness properties over AP. Prove or disprove the following claims:

(i) $P \cup P'$ is a liveness property,

(ii) $P \cap P'$ is a liveness property.

(b) Answer the same questions for P and P' being safety properties.

Hint: you can use the distributivity of union over closure for LT properties P, P':

$$cl(P \cup P') = cl(P) \cup cl(P')$$

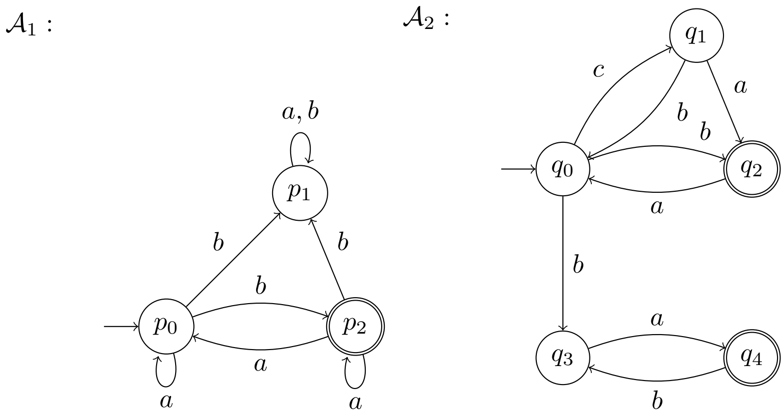
Exercise 2

Let P be an LT property. Prove: $pref(cl(P)) = pref(P)$.

Exercise 3★

In the following we have $\Sigma = \{a, b, c\}$.

(a) Consider the following NBA $\mathcal{A}_1, \mathcal{A}_2$.



For each NBA $\mathcal{A}_i$ give an ω-regular expression α_i which characterizes the language accepted by the NBA, i.e., $\mathcal{L}_\omega(\alpha_i) = \mathcal{L}_\omega(\mathcal{A}_i)$.

(b) Consider the following descriptions of ω-regular languages $\mathcal{L}_\omega^i$.

   (i) $\mathcal{L}_\omega^1$: a occurs infinitely many times. In between two successive a either

   - an odd number of b and no c, or
   - an even number of c and no b

   has to occur.

   (ii) $\mathcal{L}_\omega^2$:

   - If c occurs only finitely many times then a and b occur infinitely many times.
   - If c occurs infinitely many times then a and b occur only finitely many times.

   For each language $\mathcal{L}_\omega^i$ give an NBA $\mathcal{B}_i$ which accepts the language.

(c) Consider again the languages from (b). For each language $\mathcal{L}_\omega^i$ give a DBA $\mathcal{D}_i$ which accepts the language. If you can not find a DBA, justify why there exist no DBA accepting the language.

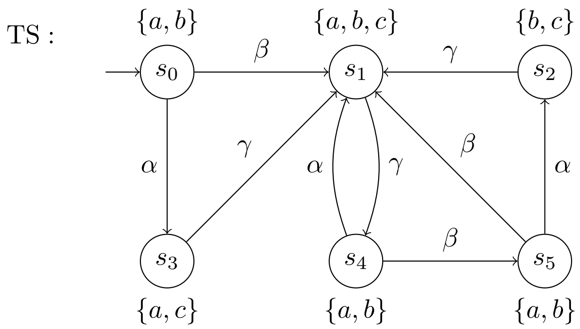
Exercise 4

2+3 Consider the transition system TS depicted below and the regular safety property

$$P_{safe} = \begin{array}{c}\text{“always if } a \text{ is valid and } b \wedge \neg c \text{ was valid somewhere before,} \\ \text{then neither } a \text{ nor } b \text{ holds thereafter at least until } c \text{ holds”}\end{array}$$

As an example, it holds:

$$\{b\}\,\emptyset\,\{a,b\}\,\{a,b,c\} \in \mathit{pref}(P_{safe})$$
$$\{a,b\}\,\{a,b\}\,\emptyset\,\{b,c\} \in \mathit{pref}(P_{safe})$$
$$\{b\}\,\{a,c\}\,\{a\}\,\{a,b,c\} \in \mathit{BadPref}(P_{safe})$$
$$\{b\}\,\{a,c\}\,\{a,c\}\,\{a\} \in \mathit{BadPref}(P_{safe})$$



(a) Define an NFA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = \mathit{MinBadPref}(P_{safe})$.

(b) Decide whether $\mathrm{TS} \models P_{safe}$ using the $\mathrm{TS} \otimes \mathcal{A}$ construction.
Provide a counterexample if $\mathrm{TS} \not\models P_{safe}$.

Exercise 5

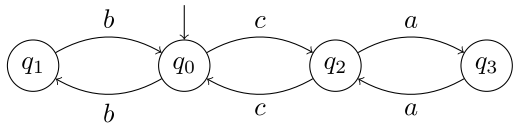
1+2 A nondeterministic Muller automaton is a quintuple $\mathcal{A} = (Q, \Sigma, \delta, Q_0, \mathcal{F})$ where Q, Σ, δ and Q_0 are as for NBA and $\mathcal{F} \subseteq 2^Q$. For an infinite run $\rho = q_0 q_1 q_2 \ldots$ of $\mathcal{A}$, let

$$\mathit{inf}(\rho) := \{q \in Q \mid \exists^\infty i \geq 0.\ q_i = q\}\,.$$

Let $\alpha \in \Sigma^\omega$.

$$\mathcal{A} \text{ accepts } \alpha \iff \text{exists infinite run } \rho \text{ of } \mathcal{A} \text{ on } \alpha \text{ s.t. } \mathit{inf}(\rho) \in \mathcal{F}.$$

(a) Consider the following Muller automaton $\mathcal{A}$ with $\mathcal{F} = \{\{q_2, q_3\}, \{q_1, q_3\}, \{q_0, q_2\}\}$:



Give the language accepted by $\mathcal{A}$ by means of an ω-regular expression.

(b) Show that every GNBA $\mathcal{G}$ can be transformed into a nondeterministic Muller automaton $\mathcal{A}$ such that $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$ by defining the corresponding transformation.