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Model Checking (Winter Term 2019/2020)

— Exercise Sheet 3 (due 15th November) —

General Remarks

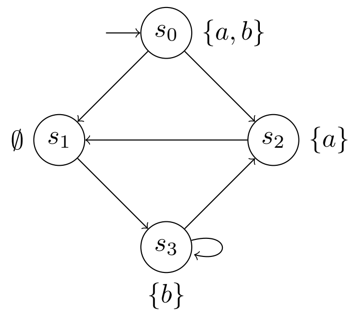
- The exercises are to be solved in groups of *three* students.
- You may hand in your solutions before the exercise class starts at 10:30 or by dropping them into the "Model Checking" box at our chair *before 10:20*. Do *not* hand in your solutions electronically.

Exercise 1 (5 Points)

Let the ω–regular LT properties P_1 and P_2 over the set of atomic propositions $\text{AP} = \{a, b\}$ be given by

$$P_1 := \text{"if } a \text{ holds infinitely often, then } b \text{ holds finitely often"}$$
$$P_2 := \text{"} a \text{ holds infinitely often and } b \text{ holds infinitely often"}$$

The model is given by the transition system TS as follows:



Algorithmically check whether $TS \models P_1$ and $TS \models P_2$. For this, proceed as follows.

a) Derive *suitable* NBA $\mathcal{A}_{P_1}$, $\mathcal{A}_{P_2}$, where suitable means "appropriate for part b)-d)".
Hint: For P_1 you can find an automaton with 3 states and for P_2 4 states suffice. Derive the automata directly.

b) Outline the reachable fragments of the product transition systems $\text{TS} \otimes \mathcal{A}_{P_1}$ and $\text{TS} \otimes \mathcal{A}_{P_2}$.

c) Decide whether $\text{TS} \models P_1$ by checking an appropriate persistence property via nested depth-first search on $\text{TS} \otimes \mathcal{A}_{P_1}$. Document *all* changes to the contents of U, V, π and ξ (the state sets and stacks of the nested depth-first search, see lecture). If the property is violated, provide a counterexample *based on the execution of the algorithm.*

d) Decide whether $\text{TS} \models P_2$ by checking an appropriate persistence property via SCC analysis on $\text{TS} \otimes \mathcal{A}_{P_2}$. If the property is violated, provide a counterexample *based on your analysis.*

Exercise 2 (3 Points)

Provide an example for a liveness property P_{live} that is *not* ω-regular. Show that P_{live} is indeed a liveness property and prove that P_{live} is not ω-regular.

Hint: Think about words of the form $\{a\}\,\{b\}\,\{a\}\,\{a\}\,\{b\}\,\{a\}\,\{a\}\,\{a\}\,\{b\}\ldots$.

Exercise 3 (2+2+2+1 Points)

(a) Provide NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ for the languages given by the ω-regular expressions $\alpha_1 = (AC + B)^* B^\omega$ and $\alpha_2 = (B^* AC)^\omega$.

(b) Apply the product construction to obtain a GNBA $\mathcal{G}$ with $\mathcal{L}_\omega(\mathcal{G}) = \mathcal{L}_\omega(\mathcal{A}_1) \cap \mathcal{L}_\omega(\mathcal{A}_2)$.

(c) Transform the GNBA $\mathcal{G}$ into an NBA $\mathcal{A}$ with $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$.

(d) Justify, why $\mathcal{L}_\omega(\mathcal{G}) = \emptyset$ on the level of the GNBA $\mathcal{G}$.

Hint: For a GNBA $\mathcal{G} = (Q, \Sigma, \delta, Q_0, \mathcal{F})$ with at least one element in $\mathcal{F} = \{F_1, \ldots, F_k\}$. Let $\mathcal{A} = (Q', \Sigma, \delta', Q'_0, F')$ be an NBA with

- $Q' = Q \times \{1, \ldots, k\}$,
- $Q'_0 = Q_0 \times \{1\}$,
- $F' = F_1 \times \{1\}$, and

for all $A \in \Sigma$ it is

$$\delta'(\langle q, i\rangle, A) = \begin{cases} \{\langle q', i\rangle \mid q' \in \delta(q, A)\} & \text{if } q \notin F_i \\ \{\langle q', (i \mod k) + 1\rangle \mid q' \in \delta(q, A)\} & \text{if } q \in F_i. \end{cases}$$

Then $\mathcal{L}_\omega(\mathcal{G}) = \mathcal{L}_\omega(\mathcal{A})$.

Exercise 4★ (4 Points)

Recall the following LT properties from exercise sheet 1.

(i) "**Winter is coming.**"
$P_1 = \emptyset^* \{winter\} (2^{\mathrm{AP}})^\omega$

(ii) "**Everything is awesome.**"
$P_2 = \{awesome\}^\omega$

(iii) "**I'll be back.**"
$P_3 = \{here\}^+ \emptyset^+ \{here\}^+ (2^{\mathrm{AP}})^\omega$

(iv) "**You either die a hero, or you live long enough to see yourself become the villain.**"
$P_4 = \{live, hero\}^+ \{hero\} (2^{\mathrm{AP}})^\omega + \{live, hero\}^+ \{live\} (2^{\mathrm{AP}})^\omega$

(v) "**By night one way, by day another
Thus shall be the norm
Till you receive true love's kiss
then, take love's true form.**"

$$P_5 = \big((\{form_1\} \{day, form_2\})^+ + \{form_1\} (\{day, form_2\} \{form_1\})^*\big) \{kiss, true\_form\} \\ (\{true\_form\} \{true\_form, day\})^\omega$$

(vi) "**A Lannister always pays his debts.**"
$P_6 = \emptyset^* (\{in\_debt\}^+ \emptyset^+)^* \emptyset^\omega$

(vii) "**Anything is possible [if you just believe]**"
$P_7 = (2^{\mathrm{AP}})^\omega$

(viii) "**It's gonna be legen... wait for it... dary!**"
$P_8 = \{legen\} \{wait\_for\_it\}^+ \{dary\} (2^{\mathrm{AP}})^\omega$

Express each property P_i as an LTL formula φ_i.