Exercise Sheet 9

General remarks:

- **This is the last exercise sheet.**
- **Due date:** *February 1st* (before the exercise class).
- You can hand in your solutions at the start of the exercise class or via L2P. Please remember to provide your matriculation number. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

Exercise 1 (Basic Properties of the Expected Runtime Calculus) **30%**

Let P be a pGCL program, $t, t_1, t_2 \in \mathbb{T}$ be runtimes, and $k \in \mathbb{R}_{>0}$. Prove or disprove:

a) $ert(P, k \cdot t) \;=\; k \cdot ert(P, t)$.

b) $ert(P, t_1 + t_2) \;=\; ert(P, t_1) + ert(P, t_2)$.

c) $ert(P, k \cdot t) \;\geq\; t$.

Exercise 2 (Properties of Bayesian Networks) **20%**

Prove the decomposition result for Bayesian networks from the lecture (cf. slide 16). That is, show for disjoint sets of random vaiables $\mathbf{W}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$ that

$$I(\mathbf{X}, \mathbf{Z}, \mathbf{Y} \cup \mathbf{W}) \text{ implies } (I(\mathbf{X}, \mathbf{Z}, \mathbf{Y}) \text{ and } I(\mathbf{X}, \mathbf{Z}, \mathbf{W}))\ .$$

Exercise 3 (D-Separation) **20%**

Show that D-separation implies independence (cf. slide 22). That is, prove that

$$dsep_G(\mathbf{X}, \mathbf{Z}, \mathbf{Y}) \text{ implies } I(\mathbf{X}, \mathbf{Z}, \mathbf{Y})\ .$$

Exercise 4 (From pGCL programs to Bayesian Networks) 30%

Let B, E, A, J and M be Boolean variables. Moreover, consider the the following `pGCL` program:

$$
\begin{array}{l}
B :\approx 0.001 \cdot [\mathtt{true}\rangle + 0.999 \cdot [\mathtt{false}\rangle; \\
B :\approx 0.002 \cdot [\mathtt{true}\rangle + 0.998 \cdot [\mathtt{false}\rangle; \\
\mathtt{if}(B)\ \{ \\
\quad \mathtt{if}(E)\ \{ \\
\quad\quad A :\approx [\mathtt{true}\rangle \cdot 0.95 + [\mathtt{false}\rangle \cdot 0.05 \\
\quad \}\ \mathtt{else}\ \{ \\
\quad\quad A :\approx [\mathtt{true}\rangle \cdot 0.94 + [\mathtt{false}\rangle \cdot 0.06 \\
\quad \} \\
\}\ \mathtt{else}\ \{ \\
\quad \mathtt{if}(E)\ \{ \\
\quad\quad A :\approx [\mathtt{true}\rangle \cdot 0.29 + [\mathtt{false}\rangle \cdot 0.71 \\
\quad \}\ \mathtt{else}\ \{ \\
\quad\quad A :\approx [\mathtt{true}\rangle \cdot 0.01 + [\mathtt{false}\rangle \cdot 0.99 \\
\quad \} \\
\}; \\
\mathtt{if}(A)\ \{ \\
\quad J :\approx [\mathtt{true}\rangle \cdot 0.90 + [\mathtt{false}\rangle \cdot 0.10 \\
\quad M :\approx [\mathtt{true}\rangle \cdot 0.70 + [\mathtt{false}\rangle \cdot 0.30 \\
\}\ \mathtt{else}\ \{ \\
\quad J :\approx [\mathtt{true}\rangle \cdot 0.05 + [\mathtt{false}\rangle \cdot 0.95 \\
\quad M :\approx [\mathtt{true}\rangle \cdot 0.01 + [\mathtt{false}\rangle \cdot 0.99 \\
\}
\end{array}
$$

a) Translate the above program into a corresponding Bayesian network. (The Bayesian network must include the DAG describing the probabilistic dependences between variables as well as the corresponding conditional probability table for each of the variables.)

b) Using the Bayesian network constructed in the above exercise, determine probability

$$\Pr(\neg B \mid \neg E, A) .$$