Exercise Sheet 8

General remarks:

- **Due date:** December 14th (before the exercise class).
- You can hand in your solutions at the start of the exercise class or via L2P. Please remember to provide your matriculation number. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

Exercise 1 (Proving Almost-Sure Termination (I)) $\boxed{40\%}$

Consider the pGCL program P below:

$$
\begin{array}{l}
\texttt{while}\ (x > 0)\ \{ \\
\quad \texttt{if}\ (x = 1)\ \{ \\
\quad\quad \{x := 0\}\ [1/2]\ \{x := x + 1\} \\
\quad \}\ \texttt{else}\ \{ \\
\quad\quad \texttt{if}\ (x \geq 3)\ \{ \\
\quad\quad\quad x := 0 \\
\quad\quad \}\ \texttt{else}\ \{ \\
\quad\quad\quad x := x + 1 \\
\quad\quad \} \\
\quad \} \\
\}
\end{array}
$$

Use the proof rule for almost-sure termination from the lecture (lec. 14, slide 29) to show that P terminates almost-surely.

Exercise 2 (Proving Almost-Sure Termination (II)) $\boxed{40\%}$

Consider the pGCL program P below:

$$
\begin{array}{l}
\texttt{while}\ (x \neq 0)\ \{ \\
\quad \{x := x - 1\}\ [1/2]\ \{x := -x\} \\
\}
\end{array}
$$

Use the proof rule for almost-sure termination from the lecture (lec. 14, slide 29) to show that P terminates almost-surely.

Exercise 3 (Proving Positive Almost-Sure Termination?) $\boxed{\mathbf{20\%}}$

Consider the pGCL program P given by $\texttt{while}\,(G)\,\{P'\}$, where P' is a loop-free pGCL program. A clever student suggests the following scheme to prove positive almost-sure termination by weakest preexpectation reasoning: We first modify program P by introducing a fresh variable, say v, which is initialized with 0 and incremented for every loop iteration. Hence, the modified program $\hat{P}$ is given by $v := 0; \texttt{while}\,(G)\,\{v := v + 1; P'\}$. Show that $wp(\hat{P}, v)(s) < \infty$ does *not* necessarily imply that P terminates positively almost-surely on state s.

Does the implication hold if we addtionally know that P terminates almost-surely? Justify your answer.