

## Exercise Sheet 7

### General remarks:

- **Due date:** December 7<sup>th</sup> (before the exercise class).
- You can hand in your solutions at the start of the exercise class or via L2P. Please remember to provide your matriculation number. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

### Exercise 1 (Program Equivalence)

20%

We say that two deterministic programs are equivalent, if for every input state, both programs either do not terminate or terminate in the same state. Show that the problem of deciding whether two deterministic programs are equivalent is in  $\Pi_2^0$ .

### Exercise 2 (Complete Sets)

50%

For a (deterministic) program  $P$ , we define the set of all states  $s$  on which  $P$  terminates (see also lec. 12, slide 18) as

$$\mathcal{W}_P := \{s \mid (P, s) \in H\} .$$

- (a) [25%] Show that the set **NonEmp** =  $\{P \mid \mathcal{W}_P \neq \emptyset\}$  is  $\Sigma_1^0$ -complete.
- (b) [25%] Show that the set **Inf** =  $\{P \mid \mathcal{W}_P \text{ is infinite}\}$  is  $\Pi_2^0$ -complete.

### Exercise 3 (Hardness of Computing Expected Values)

30%

Recall from lecture 12, slide 26 the problem set  $EXP$ :

$$(P, s, f, q) \in EXP \quad \text{iff} \quad q = wp(P, f)(s) .$$

- (a) [10%] Show that  $EXP$  is in  $\Pi_2^0$ .
- (b) [20%] Show that  $EXP$  is  $\Pi_2^0$ -hard.