Exercise Sheet 6

General remarks:

- **Please notice that the last task has been updated due to a change in the lecture slides.**
- **Due date:** November 30th (before the exercise class).
- You can hand in your solutions at the start of the exercise class or via L2P. Please remember to provide your matriculation number. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

Exercise 1 (Reasoning about Conditional Probabilities.) **35%**

Consider the following scenario: A telephone operator has forgotten what day of the week it is. However, she knows that she receives on average ten calls per hour in the week and three calls per hour at the weekend. She observes that she receives four calls in a given hour. Is it a week day?

(a) [10%] Write a `cpGCL` program P modeling the above scenario.

(b) [5%] Give an expectation f stating (for the program P) that it is a week day.

(c) [20%] Use the *cwp*-calculus to help the telephone operator to decide whether it is a week day. To this end, determine the probability that is a week day by computing $cwp(P, f)$.

Hint: We assume that probability of receiving k calls in an hour is given by a (discrete) poison distribution. That is, if we know that we receive on average r calls per hour, then the probability of receiving k calls in a given hour is $r^k/k! \cdot e^{-r}$.

Exercise 2 (Soundness of Conditional Weakest Pre-expectations) **25%**

Show the compatibility theorem for conditional *wp* in lecture 11, slide 30, for *loop-free* `cpGCL` programs. That is, show that for every loop-free `cpGCL` program P, state s and expectation f, we have

$$\frac{wp(P,f)(s)}{wlp(P,1)(s)} \;=\; ER^{[\![P]\!]}\left(s, (\Diamond\langle \text{sink}\rangle \mid \neg\Diamond\langle \lightning \rangle)\right) \;.$$

Exercise 3 (Removal of Conditioning) **15%**

Recall from lecture 11, slides 33–37, the program transformation $\hat{P}$, which removes observe statements of program P by means of "restarting". Prove or disprove that this program transformation is compositional. That is, for all `cpGCL` programs P, Q and expectations f, does it hold that

$$wp(\widehat{P;Q}, f) \;=\; wp(\hat{P};\hat{Q}, f) \;?$$

Exercise 4 (Hoisting) **25%**

Show that the hoisting transformation transformation presented in lecture 11, slide 41, is correct for loop-free programs. That is, show that for any *loop-free* `cpGCL` program P with at least one feasible run $f \in \mathbb{E}$, we have

$$\mathit{cwp}(P, (f, 1)) \;=\; \mathit{wp}(Q, f) \quad \text{with} \quad T(P, 1) = (Q, h) \;,$$

where h represents the probability that P satisfies all its observe statements.