Lehrstuhl für Informatik 2 Softwaremodellierung und Verifikation
RWTH AACHEN UNIVERSITY

Exercise Sheet 5

General remarks:

- **Due date:** November 23$^{\text{th}}$ (before the exercise class).
- You can hand in your solutions at the start of the exercise class or via L2P. Please remember to provide your matriculation number. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

Exercise 1 (Probabilistic Specifications) 21%

Let $F \in \mathbb{P}$. Match each of the following formal specifications of program P

(a) [3%] $[F] = wlp(P, 1)$
(b) [3%] $[F] = wp(P, 1)$
(c) [3%] $[F] \leq wp(P, 1)$
(d) [3%] $1 = wlp(P, 0)$
(e) [3%] $1 \leq wlp(P, 0)$
(f) [3%] $1 = wp(P, 0)$
(g) [3%] $0 \leq wp(P, [F])$

with their corresponding colloquial description below. Notice that there may be more than one formal specification in (a)-(g) with the same colloquial description. There may also be "unmatched" colloquial interpretations.

1. Program P diverges almost surely for all initial states.
2. The program never finishes in a final state satisfying F.
3. The specification does not say anything about program P; it is logically equivalent to `true`.
4. The specification is logically equivalent to `false`.
5. None of the other ones. Provide yourself the interpretation of the specification.
6. Program P terminates almost surely whenever executed in an initial state that satisfies F.
7. Program P terminates almost surely when executed in an initial state that satisfies F and diverges almost surely when executed in an initial state that satisfies $\neg F$.

Exercise 2 (Invariants) **9%**

For each of the following statements, prove or disprove whether it holds for arbitrary pGCL loops $P = \texttt{while}\,(G)\,\{\,P'\,\}$ and expectations $f \in \mathbb{E}$.

(a) [4%] There exists a *wp*–superinvariant of P with respect to f.

(b) [5%] For every *wp*–ω–superinvariant $(I_n)_{n\in\mathbb{N}}$ of P with respect to f, $\sup_{n\in\mathbb{N}} I_n$ is a *wp*–superinvariant of P with respect to f.

Exercise 3 (Proving Termination) **30%**

Consider the following probabilistic program P:

```
1    x := 1
2    while (x = 1) {
3        { x := 0 } [1/3] { y:=y+1 }
4    }
```

Prove that $wp(P, 1) = 1$.

Exercise 4 (Faulty Programs (optional)) **40%**

Consider the following probabilistic program P_k for some integer constant $k > 0$ and some rational $q \in (0, 1)$:

```
1    n := k;
2    x := 1;
3    while (n ≠ 0) {
4        x := x * n;
5        { n := n + 1 } [q] { n := n − 1 }
6    }
```

Determine a reasonable, i.e. depending on k and q, *lower* bound on $wp(P_k, [k! = x])$, where $k!$ denotes the factorial of k.