

## Exercise Sheet 2

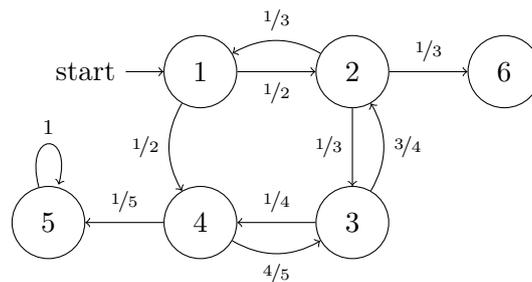
### General remarks:

- **Due date:** November 2<sup>nd</sup> (before the exercise class).
- You can hand in your solutions right before the exercise class starts or via L2P. Please remember to write your matriculation number on your submission. We kindly ask you to hand in your solutions in groups of **three**.
- Solutions must be written in English.
- While we will publish sketches of exercise solutions, we do *not* guarantee that these sketches contain all details that are necessary to properly solve an exercise. Hence, it is recommended to attend the exercise classes.
- If you have any questions regarding the lecture or the exercise, feel free to write us an email or visit us at the chair.

### Exercise 1 (Reachability Probabilities)

25%

Consider the following Markov chain :



- (a) [10%] Give the underlying equation system for the above Markov chain the set of target states  $\{6\}$ .
- (b) [15%] Compute the probability of reaching the target state 6 from initial state 1, i.e. compute  $\Pr(1 \models \diamond 6)$ .

### Exercise 2 (Operational Semantics of pGCL)

20%

Apply the operational semantics of pGCL programs to derive the reachable fragment of the Markov chain corresponding of the program below for an arbitrary, but fixed initial state.

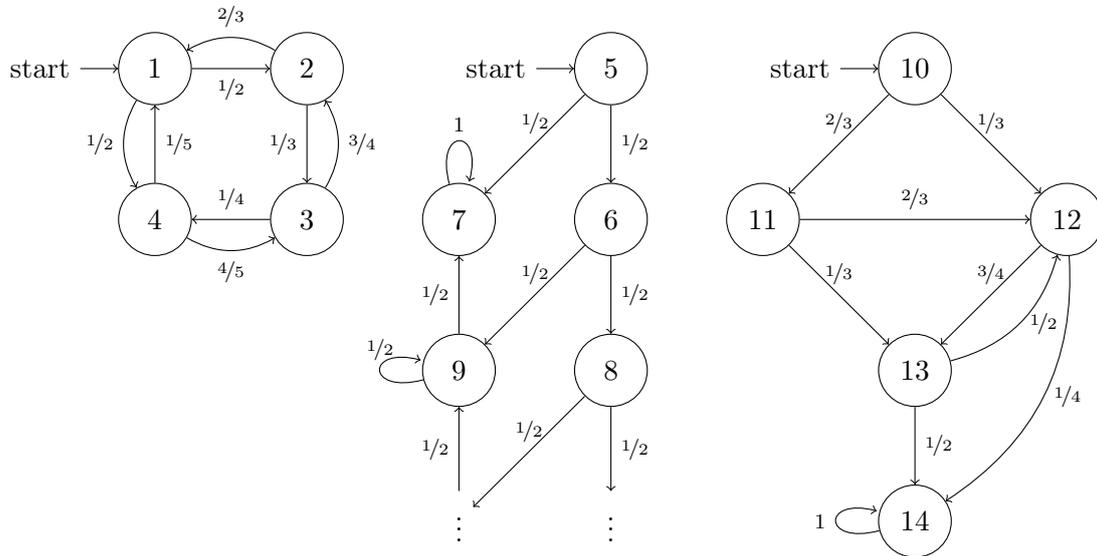
```

x := 1;
while(x = 1) {
  { x := 0 } [0.5] { x := 2 };
  if(x > 0) {
    { x := 3 - x } [1/x] { x := x - 2 }
  }
}
    
```

**Exercise 3 (Recurrent and Periodic States)**

**30%**

Consider the three Markov chains below:



- (a) [5%] Which states in the above Markov chains are *positive recurrent*? Briefly justify your answer.
- (b) [5%] Which states in the above Markov chains are *null recurrent*? Briefly justify your answer.
- (c) [5%] Which states in the above Markov chains are *transient*? Briefly justify your answer.
- (d) [15%] Decide for each state in the above Markov chains whether it is periodic or aperiodic. If it is periodic, also provide a suitable period  $d$ .

**Exercise 4 (Null Recurrence)**

**25%**

Prove that a finite Markov Chain does not contain any null recurrent state.