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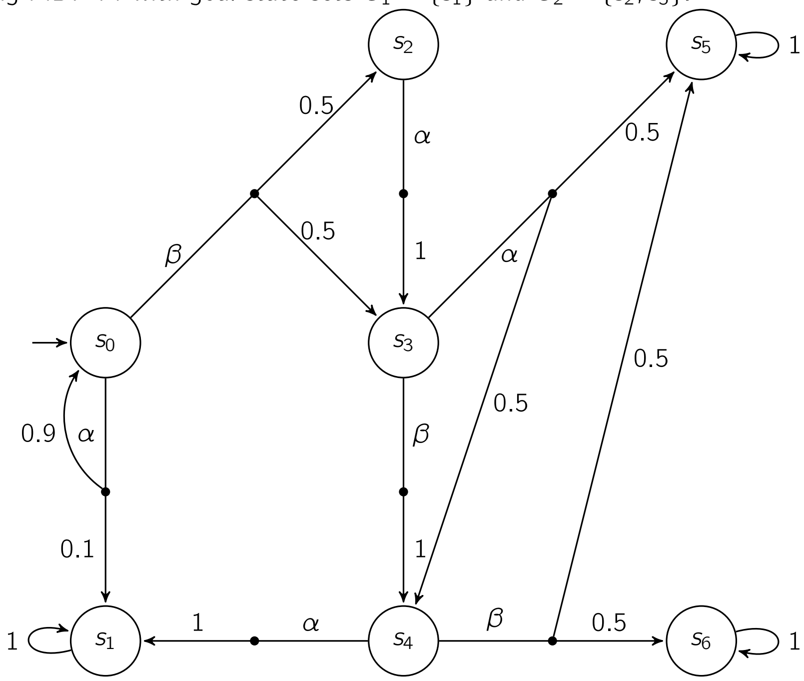
Modeling and Verification of Probabilistic Systems
— Exercise Sheet 11 —

Notes:

- The exercise sheets need to be solved in groups of 2 students.
- Hand in the solution until 30.01.2019 before the exercise class.
- We do not accept solutions via L2p or email.
- Write your names and matriculation numbers on the front page and staple all pages.
- **This is the last exercise sheet.**

Exercise 1 (Multi-objective Model Checking): (11+15+10=36 Points)

Consider the following MDP M with goal state sets $G_1 = \{s_1\}$ and $G_2 = \{s_2, s_5\}$.



a) Provide the goal-unfolding M_U for M, G_1, and G_2. It suffices to depict the reachable fragment of M_U.

b) Check for each of the following points $\mathbf{p}_j$ whether they are achievable, i.e., check whether $\mathfrak{p}_j \in \mathfrak{A}(\langle \Diamond G_i, \geq \rangle_2)$ holds. Justify your answer by either providing a **positional** (randomized) policy for M_U that achieves the corresponding point or by explaining why no such policy exists.

i) $\mathbf{p}_1 = (1, \frac{1}{2})$

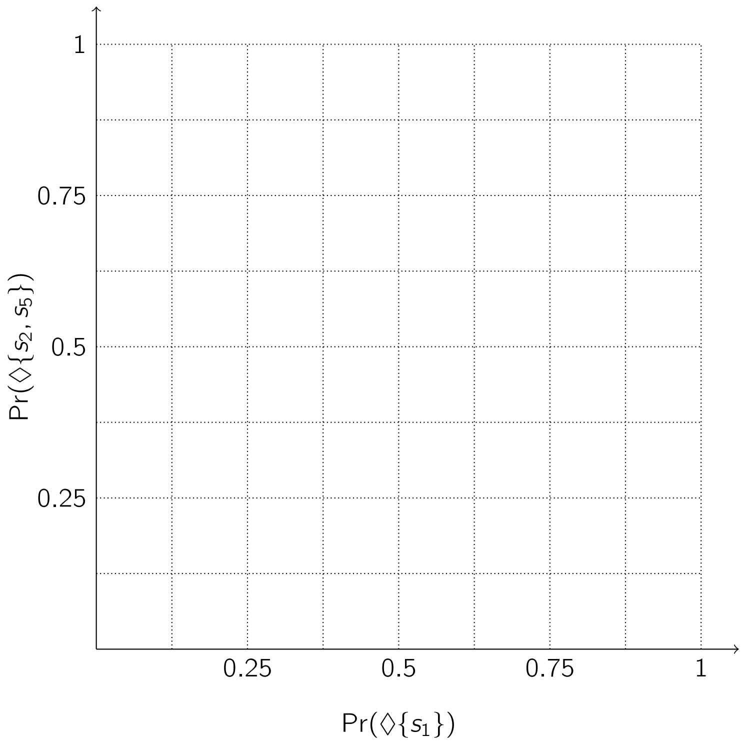
ii) $\mathbf{p}_2 = (1, 1)$

iii) $\mathbf{p}_3 = (\frac{3}{4}, \frac{3}{4})$

iv) $\mathbf{p}_4 = (\frac{1}{2}, \frac{7}{8})$

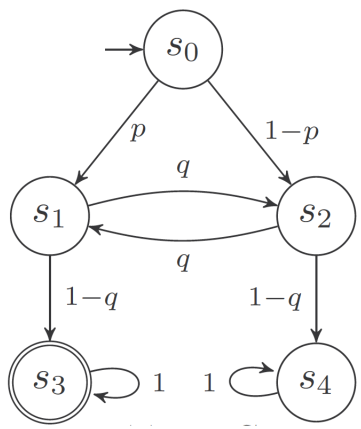
v) $\mathbf{p}_5 = (\frac{7}{8}, \frac{5}{8})$

c) Draw the Pareto curve $\mathfrak{P}(\langle \Diamond G_i, \geq \rangle_2)$ in the following plot.



Exercise 2 (State Elimination): (25 Points)

Consider the following pMC $\mathcal{D}$:



Apply state elimination on $\mathcal{D}$, first eliminate s_2 then s_1, show the steps.

Exercise 3 (Valuations): (5+5+5=15 Points)

Consider the following definitions for well-defined and graph preserving valuations

Definition 1 (Well-defined valuation)**.** *A total valuation u is well-defined for pMC $\mathcal{D} = (S, Var, \mathbf{P}, s_i, AP, L)$ if for the induced pMC $\mathcal{D}_u = (S_u, \mathbf{P}_u, s_i, AP, L)$ it holds that:*

$$\mathbf{P}_u \colon S_u \times S_u \to [0,1] \text{ with for all } s \in S_u, \sum_{t \in S_u} \mathbf{P}(s,t) = 1$$

Definition 2 (Graph preserving valuation)**.** *A total valuation u for pMC $\mathcal{D}$ is called graph preserving if it is well-defined and it holds that:*

$$\forall s, s' \in S. \mathbf{P}(s,s') \neq 0 \to \mathbf{P}(s,s')[u] > 0$$

For pMC $\mathcal{D}$ of exercise 2 provide

a) a *not well-defined valuation*,

b) a *well-defined and not graph preserving valuation*, and

c) a *graph preserving valuation*.

Exercise 4 (Parameter Lifting): (8+8+8=24 Points)

Consider the pMC $\mathcal{D}$ of exercise 2, with region $R = [0.1, 0.8] \times [0.4, 0.7]$

a) Give $rel(\mathcal{D})$ (relaxation of $\mathcal{D}$), also provide $rel(R)$ (region for $rel(\mathcal{D})$).

b) Apply parameter-substitution for R

c) Give an upperbound of $\Pr(\Diamond s_3)$ of $\mathcal{D}$ in region R using parameter lifting.