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Modeling and Verification of Probabilistic Systems

— Exercise Sheet 10 —

Notes:

- The exercise sheets need to be solved in groups of 2 students.
- Hand in the solution until 16.01.2019 before the exercise class.
- We do not accept solutions via L2p or email.
- Write your names and matriculation numbers on the front page and staple all pages.

Exercise 1 (Modelling): (15 Points)

Model the following scenario using the syntax for describing PAs (Lecture 19-20 page 23). Actions are *emphasized*. A computer tries to find a yet unknown prime number. After *computing a candidate*, the number is *checked*. Either the number is a prime or it is not a prime. If it is not a prime, the computer retries. If it is a prime, then the computer *sends* this result to a news agency and waits for an *acknowledgement*. With probability 0.6, the news agency *receives* the big news and sends an *acknowledgement*. The *acknowledgement arrives* with probability 0.7. After the acknowledgement arrives, the computer restarts the procedure.

Exercise 2 (Probabilistic Automata): (5+10 Points)

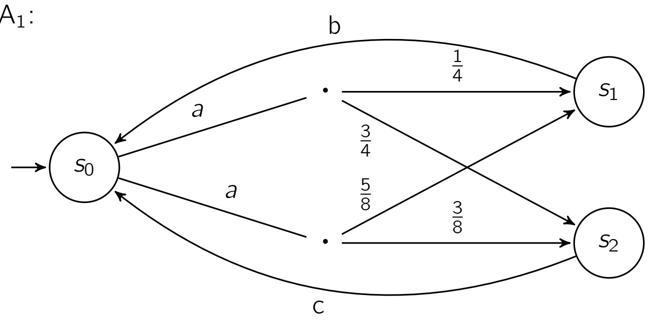
a) Give PAs for the following three processes.
$S = \text{send}.(\text{send}.S + \text{send}.\underline{0})$
$T = \text{send}.(\text{receive}.\text{T} \oplus_{0.6} \text{T})$
$R = \text{receive}.\text{ringbell}.R$

b) Give the PA for $S||T||R$. Assume that all shared actions are synchroninzing.

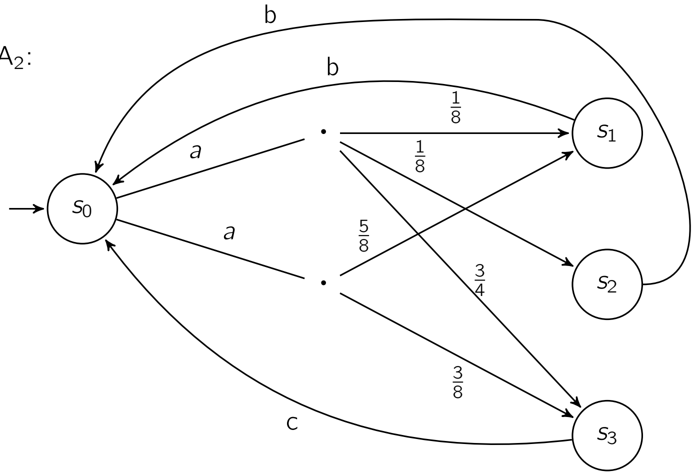
Exercise 3 (Strong Bisimulation): (5+5+5+5=20 Points)

Consider the following PAs

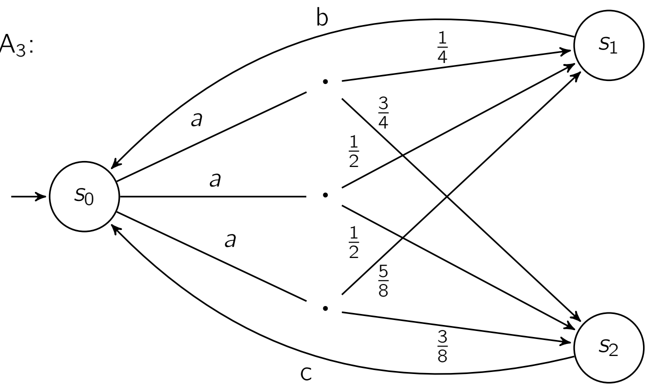
PA_1:



PA_2:



PA_3:



a) Prove or disprove the following statements:

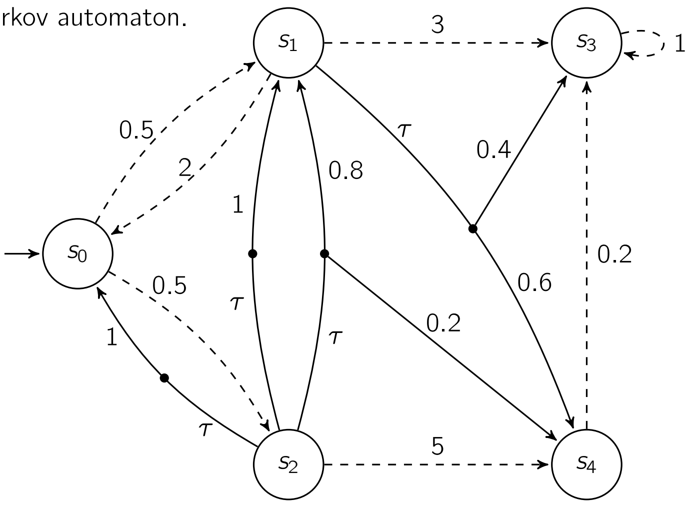
- $PA_1 \sim_p PA_2$
- $PA_1 \sim_p PA_3$

b) Prove or disprove the following statements:

- $PA_1 \sim_{cp} PA_2$
- $PA_1 \sim_{cp} PA_3$

Exercise 4 (Markov Automata): (5+15=20 Points)

Consider the following Markov automaton.



a) Depict the Markov automaton that results from applying the maximal progress assumption.

b) Compute $eT^{\min}(s_0, \lozenge\{s_3\})$, i.e., the minimal expected time until reaching s_3 from s_0 for the Markov automaton from **a)**. To this end, repeatedly apply the Bellman operator (Lecture 21-22 page 51) with starting values $v(s) = 0$ for all states s until a fixpoint is reached. Hint: This should not take more than 8 iterations.

Exercise 5 (Zeno Behaviour): (15 Points)

Let $M = (S, Act, \rightarrow, \dashrightarrow, s_0)$ be a Markov automaton. A state $s \in S$ is called *Markovian* iff there exists $r \in \mathbb{R}_{>0}$ and $s' \in S$ with $(s, r, s') \in \dashrightarrow$. Let $MS \subseteq S$ denote the set of Markovian states. We say that M has *Zeno behaviour* iff there is a state s with $\Pr^{\min}(s \models \lozenge MS) < 1$.
Show that the fixpoint of the Bellman operator (Lecture 21-22 page 51) is in general not unique for Markov automata with Zeno behavior.

Exercise 6 (Generalized Stochastic Petri Nets): (15 Points)

Depict the Markov automaton for the following generalized stochastic Petri net (GSPN). Do not apply the maximum progress assumption.