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Modeling and Verification of Probabilistic Systems
— Exercise Sheet 9 —

Notes:

- The exercise sheets need to be solved in groups of 2 students.
- Hand in the solution until 19.12.2018 before the exercise class.
- We do not accept solutions via L2p or email.
- Write your names and matriculation numbers on the front page and staple all pages.

Exercise 1 (CTMC Model Checking for $I = (t_1, t_2)$): (15 Points)

Show that for a CTMC with states S and $T \subset S$ and $0 < t_1 < t_2$ the following equation does **not** hold: $\Pr(\Diamond^{(t_1,t_2)} T) = \Pr(\Diamond^{\leq t_2} T) - \Pr(\Diamond^{\leq t_1} T)$.

Exercise 2 (Probability measure on CTMCs): (15 + 15 = 30 Points)

For a CTMC $\mathcal{C}$ consider the set Π of paths that stay at least one time unit in every state that they visit, i.e.,

$$\Pi = \{\pi \in \text{Paths}(\mathcal{C}) \mid \pi\langle i \rangle \geq 1 \text{ for all } i \geq 0\}$$

a) Formally prove that Π is measurable.

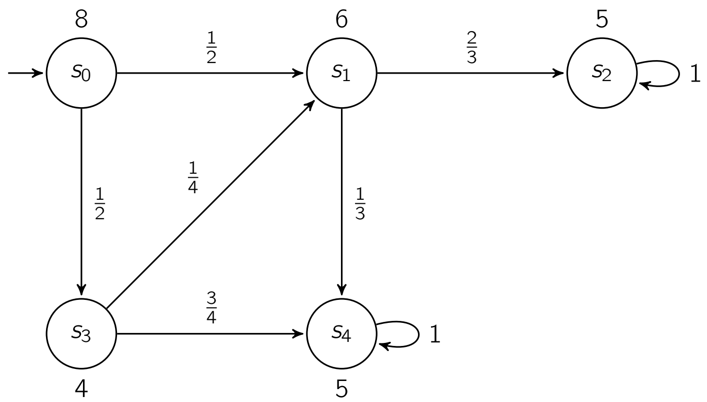
b) Formally prove that $\Pr(\Pi) = 0$. For the sake of simplicity, you may assume that the CTMC $\mathcal{C}$ is uniform.

Hints:

- Compute the probability that the first $n \in \mathbb{N}$ jumps take at least one time unit. Prove your claim, e.g., by induction over n.
- Argue why $\Pr(\Pi)$ is approached for $n \to \infty$.

Exercise 3 (Reachability Probabilities): (5 + 15 + 5 = 25 Points)

Consider the following CTMC $\mathcal{C}$:



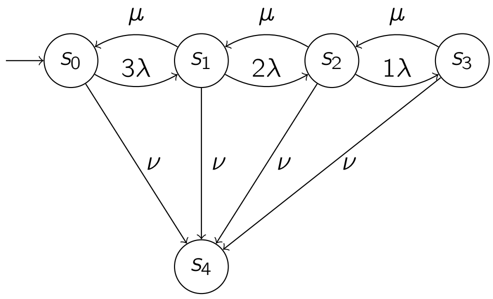
a) Calculate $\Pr^{\mathcal{C}}(s_0 \models \Diamond s_2)$

b) Calculate $\Pr^{\mathcal{C}}(Cyl(s_0, [2,3), s_1, [0,4], s_2))$

c) Provide the integral equations for: $\Pr^{\mathcal{C}}(s_0 \models \Diamond^{\leq 4} s_2)$

Exercise 4 (Computing Reachability Probabilities): (15 + 15 = 30 Points)

Consider the following CTMC $\mathcal{C}$.



a) Compute $\Pr^{\mathcal{C}}(\Diamond^{\leq 3}\{s_2\})$ using $\lambda = \frac{1}{10}$, $\mu = \frac{1}{20}$ and $\nu = \frac{1}{100}$. You may cut off an infinite sum after three terms.

b) Give the closed form expression for $\Pr^{\mathcal{C}}(\Diamond^{\leq t}\{s_4\})$.

Hints:

- The closed form is not that large.
- You may use weak-bisimulation.