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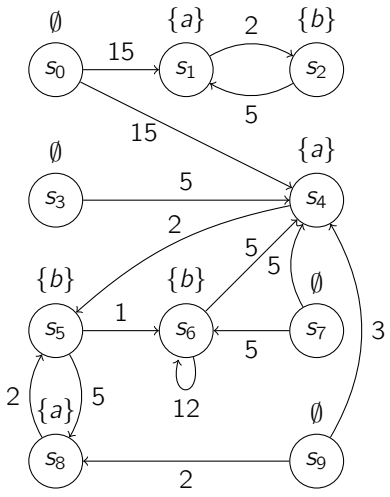
Modeling and Verification of Probabilistic Systems
— Exercise Sheet 8 —

Notes:

- The exercise sheets need to be solved in groups of 2 students.
- Hand in the solution until 12.12.2018 before the exercise class.
- We do not accept solutions via L2p or email.
- Write your names and matriculation numbers on the front page and staple all pages.

Exercise 1 (Bisimulation): (20 + 20 = 40 Points)

Consider the following CTMC $\mathcal{C}$:



a) Find the states that are probabilistically bisimilar, i.e., compute the relation $\sim_m$.

b) Find the states that are probabilistically weak bisimilar, i.e., compute the relation $\approx_m$.

Exercise 2 (Bisimulation Lemma): (20 Points)

Formally prove the following lemma from the lecture:
Let $\mathcal{C}$ be a CTMC and R an equivalence relation on S with $(s,t) \in R$, $\mathbf{P}(s,[s]_R) < 1$ and $\mathbf{P}(t,[t]_R) < 1$. Then the following two statements are equivalent:

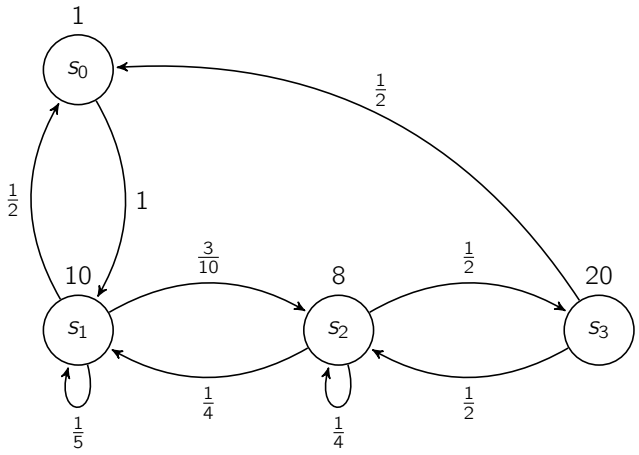
1. for all $C \in S \setminus R, C \neq [s]_R = [t]_R$:

$$\frac{\mathbf{P}(s,C)}{1-\mathbf{P}(s,[s]_R)} = \frac{\mathbf{P}(t,C)}{1-\mathbf{P}(t,[t]_R)} \text{ and } \mathbf{R}(s, S \setminus [s]_R) = \mathbf{R}(t, S \setminus [t]_R)$$

2. $\mathbf{R}(s,C) = \mathbf{R}(t,C)$ for all $C \in S \setminus R$ with $C \neq [s]_R = [t]_R$.

Exercise 3 (Uniformization): (15 Points)

Uniformize the following CTMC with rate r=20



Exercise 4 (Transient Probabilities): (5 + 10 + 10 = 25 Points)

A professor supervises three Ph.D. students who all need quite a bit of advice. When any of these students visits the professor, the time to the next visit has an exponential distribution with a mean of 8 hours. The time for the professor to advise the students has a mean value of 1/2 hour. (All times in this problem have exponential distributions). Students visit the professor one at a time. If the professor is busy, the students wait outside his/her office. And the students are treated in a first-come-first-served manner.

a) Construct the CTMC that describes the situation.

Hints:

- You may abstract away from which student is visiting the professor, so the CTMC consists of 4 states

b) Give the expression of computing the transient distribution of the CTMC at time T. Suppose initially the professor is free. Use the uniformised CTMC.

c) If now comes a new Ph.D. student, who has the same arrival rate as the others, but requires a mean time of one hour with the professor. Do **a)** again