



Compiler Construction

Lecture 9: Syntax Analysis V ($LR(k)$ Grammars and $LR(0)$ Parsing)

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<https://moves.rwth-aachen.de/teaching/ws-1819/cc/>

Recap: Bottom-Up Parsing

Outline of Lecture 9

Recap: Bottom-Up Parsing

Nondeterminism in $NBA(G)$

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Examples of $LR(0)$ Conflicts

$LR(0)$ Parsing Functions

Recap: Bottom-Up Parsing

Bottom-Up Parsing

Approach:

1. Given $G \in CFG_{\Sigma}$, construct a **nondeterministic bottom-up parsing automaton** (NBA) which accepts $L(G)$ and which additionally computes corresponding (reversed) rightmost analyses
 - input alphabet: Σ
 - pushdown alphabet: X
 - output alphabet: $[p]$ (where $p := |P|$)
 - state set: omitted
 - transitions:
 - shift**: shifting input symbols onto the pushdown
 - reduce**: replacing the right-hand side of a production by its left-hand side (= inverse expansion step)
2. Remove nondeterminism by allowing **lookahead** on the input:
 $G \in LR(k)$ iff $L(G)$ recognisable by deterministic bottom-up parsing automaton with lookahead of k symbols

Recap: Bottom-Up Parsing

The Nondeterministic Bottom-Up Automaton

Definition (Nondeterministic bottom-up parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$. The **nondeterministic bottom-up parsing automaton** of G , $NBA(G)$, is defined by the following components.

- **Input alphabet:** Σ
- **Pushdown alphabet:** X
- **Output alphabet:** $[p]$
- **Configurations:** $\Sigma^* \times X^* \times [p]^*$ (top of pushdown to the right)
- **Transitions** for $w \in \Sigma^*$, $\alpha \in X^*$, and $z \in [p]^*$:
 - shifting steps: $(aw, \alpha, z) \vdash (w, \alpha a, z)$ if $a \in \Sigma$
 - reduction steps: $(w, \alpha\beta, z) \vdash (w, \alpha A, zi)$ if $\pi_i = A \rightarrow \beta$
- **Initial configuration** for $w \in \Sigma^*$: $(w, \varepsilon, \varepsilon)$
- **Final configurations:** $\{\varepsilon\} \times \{S\} \times [p]^*$

Recap: Bottom-Up Parsing

Correctness of $NBA(G)$

Theorem (Correctness of $NBA(G)$)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $NBA(G)$ as before. Then, for every $w \in \Sigma^*$ and $z \in [p]^*$,

$(w, \varepsilon, \varepsilon) \vdash^* (\varepsilon, S, z)$ iff \overleftarrow{z} is a rightmost analysis of w

Proof.

similar to the top-down case (Theorem 6.1) □

Nondeterminism in $NBA(G)$

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Nondeterminism in $NBA(G)$

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Observation: $NBA(G)$ is generally **nondeterministic**

- **Shift or reduce?** Example:

$$(bw, \alpha a, z) \vdash \begin{cases} (w, \alpha ab, z) \\ (bw, \alpha A, zi) \end{cases} \quad \text{if } \pi_j = A \rightarrow a$$

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- If reduce: **which “handle” β ?** Example:

$$(w, \alpha ab, z) \vdash \begin{cases} (w, \alpha A, zi) \\ (w, \alpha aB, zj) \end{cases} \quad \text{if } \pi_i = A \rightarrow ab \text{ and } \pi_j = B \rightarrow b$$

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- **When to terminate parsing?** Example:

$$\underbrace{(\varepsilon, S, z)}_{\text{final}} \vdash (\varepsilon, A, zi) \quad \text{if } \pi_i = A \rightarrow S$$

Resolving Termination Nondeterminism

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Resolving Termination Nondeterminism

Resolving Termination Nondeterminism I

General assumption to avoid nondeterminism of last type:
every grammar is start separated

Definition 9.1 (Start separation)

A grammar $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ is called **start separated** if S only occurs in productions of the form $S \rightarrow A$ where $A \neq S$.

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Remarks:

- Start separation always possible by adding $S' \rightarrow S$ with **new start symbol** S'
- From now on consider only **reduced** grammars of this form (and let $\pi_0 := S' \rightarrow S$)

Resolving Termination Nondeterminism

Resolving Termination Nondeterminism II

Start separation removes “When to terminate parsing?” nondeterminism:

Lemma 9.2

*If $G \in CFG_{\Sigma}$ is start separated, then no successor of a final configuration (ε, S', z) in $NBA(G)$ is again a final configuration.
(Thus parsing should be stopped in the **first final configuration**.)*

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Proof.

- To (ε, S', z) , only reductions by ε -productions can be applied:

$$(\varepsilon, S', z) \vdash (\varepsilon, S'A, zi) \quad \text{if } \pi_j = A \rightarrow \varepsilon$$

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- Thereafter, only reductions by productions of the form $A_0 \rightarrow A_1 \dots A_n$ ($n \geq 0$) applicable
- Every resulting configuration is of the (non-final) form

$$(\varepsilon, S'B_1 \dots B_k, z) \quad \text{where } k \geq 1$$



$LR(k)$ Grammars

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LR(k) Grammars

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Goal: resolve remaining nondeterminism of $NBA(G)$ by supporting lookahead of $k \in \mathbb{N}$ symbols on the input

\implies LR(k): reading of input from left to right with k -lookahead, computing a rightmost analysis

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\implies $LR(k)$: reading of input from **left to right** with k -lookahead, computing a **rightmost analysis**

Definition 9.3 (LR(k) grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated and $k \in \mathbb{N}$. Then G has the **LR(k) property** (notation: $G \in LR(k)$) if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

such that $\text{first}_k(w) = \text{first}_k(y)$, it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

LR(k) Grammars

LR(k) Grammars II

Remarks:

- If $G \in LR(k)$, then the reduction of $\alpha\beta w$ to αAw is already determined by $\text{first}_k(w)$.
- Therefore $NBA(G)$ in configuration $(w, \alpha\beta, z)$ can decide to reduce and how to reduce.

LR(k) Grammars II

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- **Computation of $NBA(G)$ for $S \Rightarrow_r^* \alpha Aw \Rightarrow_r \alpha\beta w$ ($\pi_i = A \rightarrow \beta$):**

$$(w'w, \varepsilon, \varepsilon) \vdash^* (w, \alpha\beta, z) \stackrel{\text{red } i}{\vdash} (w, \alpha A, zi) \vdash \dots$$

where $\pi_i = A \rightarrow \beta$

LR(k) Grammars II

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- If $G \in LR(k)$, then the reduction of $\alpha\beta w$ to $\alpha A w$ is already determined by $\text{first}_k(w)$.
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- **Computation of $NBA(G)$ for $S \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w$ ($\pi_i = A \rightarrow \beta$):**

$$(w'w, \varepsilon, \varepsilon) \vdash^* (w, \alpha\beta, z) \stackrel{\text{red } i}{\vdash} (w, \alpha A, zi) \vdash \dots$$

where $\pi_i = A \rightarrow \beta$

- **Computation of $NBA(G)$ for $S \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha\beta y$:**
 - with immediate reduction ($y = x, \alpha\beta = \gamma\delta, \pi_j = B \rightarrow \delta$):

$$(y'y, \varepsilon, \varepsilon) \vdash^* (y, \alpha\beta, z') \stackrel{\text{red } j}{\vdash} (x, \gamma B, z'j) \vdash \dots$$

LR(k) Grammars II

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- with previous shifts ($y = x'x, x' \neq \varepsilon, \alpha\beta x' = \gamma\delta, \pi_j = B \rightarrow \delta$):

$$(y'y, \varepsilon, \varepsilon) \vdash^* (y, \alpha\beta, z') \stackrel{\text{shift}^+}{\vdash} (x'x, \alpha\beta, z') \stackrel{\text{red } j}{\vdash} (x, \gamma\delta, z') \vdash (x, \gamma B, z'j) \vdash \dots$$

LR(0) Grammars

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LR(0) Grammars

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The case $k = 0$ is relevant (in contrast to $LL(0)$): here the decision is just based on the contents of the pushdown, **without any lookahead**.

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Corollary 9.4 ($LR(0)$ grammar)

$G \in CFG_{\Sigma}$ has the **$LR(0)$ property** if for all rightmost derivations of the form

$$S \begin{cases} \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta w \\ \Rightarrow_r^* \gamma B x \Rightarrow_r \alpha \beta y \end{cases}$$

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it follows that $\alpha = \gamma$, $A = B$, and $x = y$.

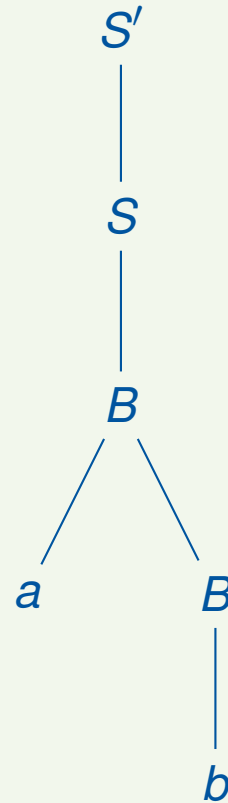
Goal: derive a **finite information** from the pushdown which suffices to resolve the nondeterminism (similar to abstraction of right context in LL parsing by fo-sets)

LR(0) Grammars

LR(0) Items and Sets I

Example 9.5

$G : S' \rightarrow S \quad (0)$
 $S \rightarrow B \mid C \quad (1, 2)$
 $B \rightarrow aB \mid b \quad (3, 4)$
 $C \rightarrow aC \mid c \quad (5, 6)$



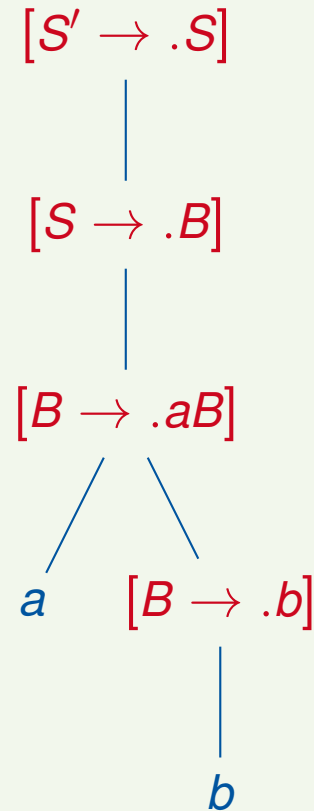
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$NBA(G)$:
($ab, \varepsilon, \varepsilon$)



LR(0) Grammars

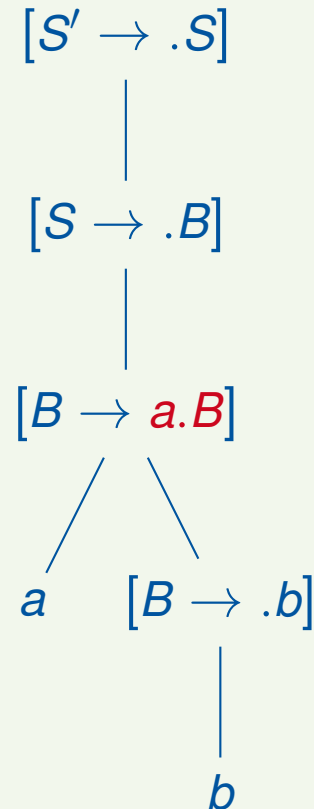
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 $\vdash (b, a, \varepsilon)$



LR(0) Grammars

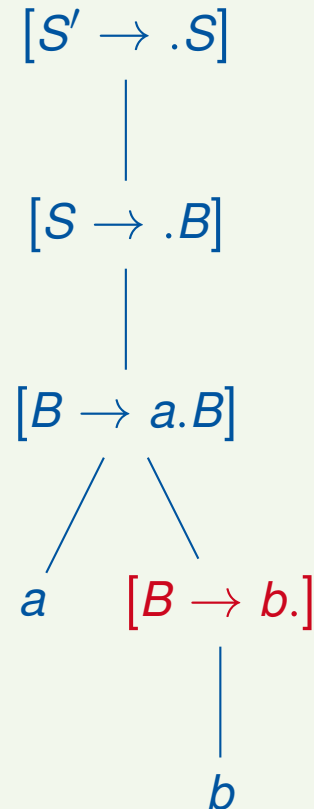
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LR(0) Grammars

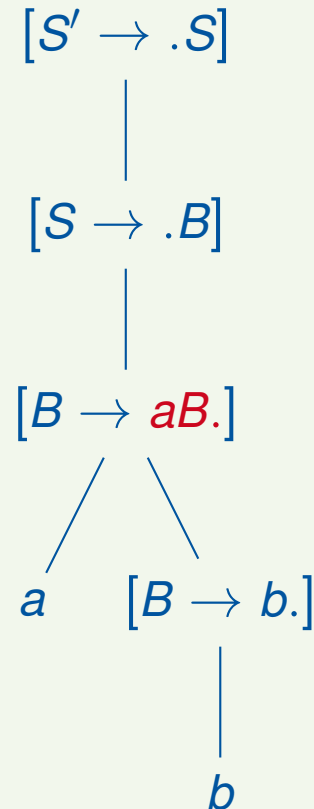
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LR(0) Grammars

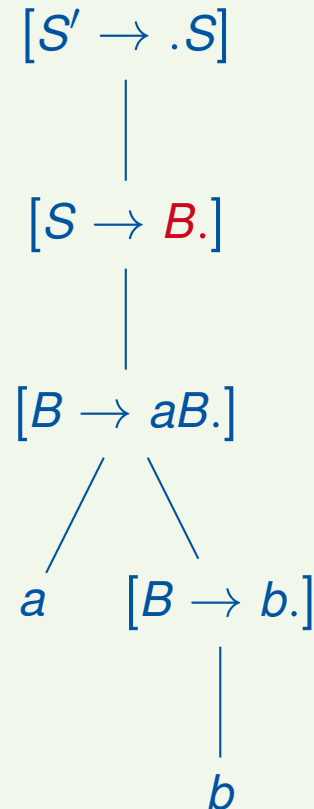
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LR(0) Grammars

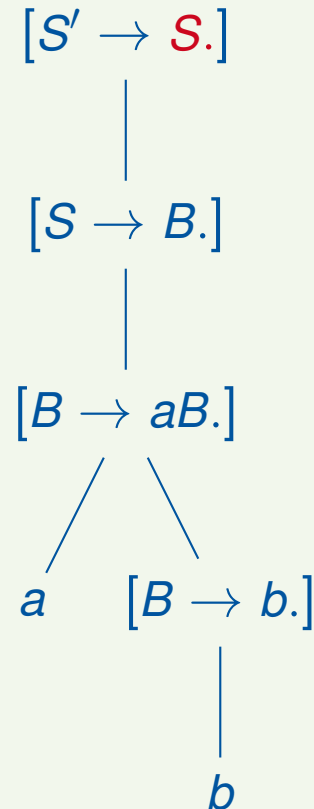
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LR(0) Grammars

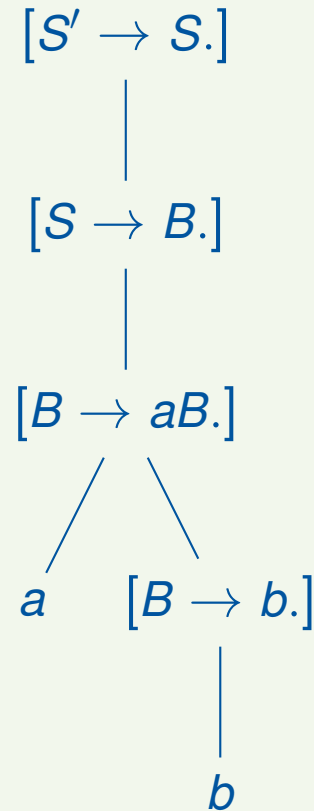
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LR(0) Items and Sets II

Definition 9.6 (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **LR(0) item** for $\alpha \beta_1$.

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- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all **LR(0) items** for γ , called the **LR(0) set** (or: **LR(0) information**) of γ .

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Definition 9.6 (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

- $[A \rightarrow \beta_1 \cdot \beta_2]$ is called an **LR(0) item** for $\alpha \beta_1$.
- Given $\gamma \in X^*$, $LR(0)(\gamma)$ denotes the set of all **LR(0) items** for γ , called the **LR(0) set** (or: **LR(0) information**) of γ .
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$.

Corollary 9.7

1. For every $\gamma \in X^*$, $LR(0)(\gamma)$ is finite.
2. $LR(0)(G)$ is finite.
3. The item $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$ indicates the possible **reduction** $(w, \alpha \beta, z) \vdash (w, \alpha A, zi)$ where $\pi_i = A \rightarrow \beta$ and $\gamma = \alpha \beta$.

LR(0) Items and Sets II

Definition 9.6 (LR(0) items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$ (i.e., $A \rightarrow \beta_1 \beta_2 \in P$).

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4. The item $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$ indicates an **incomplete handle** β_1 (to be completed by shift operations or ε -reductions).

LR(0) Grammars

LR(0) Conflicts

Definition 9.8 (LR(0) conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $I \in LR(0)(G)$.

- I has a **shift/reduce conflict** if there exist $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$ such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

LR(0) Grammars

LR(0) Conflicts

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- I has a **reduce/reduce conflict** if there exist $A \rightarrow \alpha, B \rightarrow \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

LR(0) Grammars

LR(0) Conflicts

Definition 9.8 (LR(0) conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $I \in LR(0)(G)$.

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$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

Lemma 9.9

$G \in LR(0)$ iff no $I \in LR(0)(G)$ contains conflicting items.

Proof.

omitted □

Computing LR(0) Sets I

Theorem 9.10 (Computing LR(0) sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

1. $LR(0)(\varepsilon)$ is the least set such that
 - $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
 - if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$, then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.

Computing LR(0) Sets I

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Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ be start separated by $S' \rightarrow S$ and reduced.

1. $LR(0)(\varepsilon)$ is the least set such that

- $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$ and
- if $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$.

2. $LR(0)(\alpha Y)$ ($\alpha \in X^*$, $Y \in X$) is the least set such that

- if $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$,
then $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$ and
- if $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$ and $B \rightarrow \beta \in P$,
then $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$.

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$G: S' \rightarrow S \quad S \rightarrow B \mid C$$
$$B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$ $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S]$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{array}{lll} G: S' \rightarrow S & S \rightarrow B \mid C & [A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P \\ & B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon) \\ I_0 := LR(0)(\varepsilon) : & [S' \rightarrow \cdot S] & [S \rightarrow \cdot B] \end{array}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{array}{l} G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \cdot B \gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P \\ \quad B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon) \\ I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \end{array}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{array}{l} G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P \\ \quad B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon) \\ I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \end{array}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon), B \rightarrow \beta \in P$

$B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$

$I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G: S' \rightarrow S \quad S \rightarrow B \mid C & \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \\ I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : [S' \rightarrow S \cdot] \end{aligned}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{array}{l} G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ \quad B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \\ \\ I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : [S' \rightarrow S \cdot] \\ I_2 := LR(0)(B) : [S \rightarrow B \cdot] \end{array}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G: S' \rightarrow S \quad S \rightarrow B \mid C & \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \\ \\ I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : [S' \rightarrow S \cdot] \\ I_2 := LR(0)(B) : [S \rightarrow B \cdot] \\ I_3 := LR(0)(C) : [S \rightarrow C \cdot] \end{aligned}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G: S' \rightarrow S \quad S \rightarrow B \mid C & \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \\ \\ l_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ l_1 := LR(0)(S) : [S' \rightarrow S \cdot] \\ l_2 := LR(0)(B) : [S \rightarrow B \cdot] \\ l_3 := LR(0)(C) : [S \rightarrow C \cdot] \\ l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \end{aligned}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$I_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$I_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$I_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot B \gamma_2] \in LR(0)(\alpha Y), B \rightarrow \beta \in P$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$I_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$I_3 := LR(0)(C) : [S \rightarrow C \cdot]$

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Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{aligned} G: S' \rightarrow S \quad S \rightarrow B \mid C & \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{aligned}$$
$$\begin{aligned} I_0 := LR(0)(\varepsilon) : & \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_1 := LR(0)(S) : & \quad [S' \rightarrow S \cdot] \\ I_2 := LR(0)(B) : & \quad [S \rightarrow B \cdot] \\ I_3 := LR(0)(C) : & \quad [S \rightarrow C \cdot] \\ I_4 := LR(0)(a) : & \quad [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \\ I_5 := LR(0)(b) : & \quad [B \rightarrow b \cdot] \end{aligned}$$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c \quad \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$

$I_0 := LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$I_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$I_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$I_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$I_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$I_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$I_6 := LR(0)(c) : [C \rightarrow c \cdot]$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$$\begin{array}{lll} G: S' \rightarrow S & S \rightarrow B \mid C & [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha) \\ & B \rightarrow aB \mid b \quad C \rightarrow aC \mid c & \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y) \end{array}$$
$$I_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$$
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Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C \quad [A \rightarrow \gamma_1 \cdot Y \gamma_2] \in LR(0)(\alpha)$
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$l_0 := LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$

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$l_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

$(LR(0)(aa) = LR(0)(a) = l_4, LR(0)(ab) = LR(0)(b) = l_5, LR(0)(ac) = LR(0)(c) = l_6, \dots,$

$l_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases, e.g., for $\gamma = bB$)

Computing LR(0) Sets II

Example 9.11 (cf. Example 9.5)

$G: S' \rightarrow S \quad S \rightarrow B \mid C$
 $B \rightarrow aB \mid b \quad C \rightarrow aC \mid c$

$l_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

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$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

$(LR(0)(aa) = LR(0)(a) = l_4, LR(0)(ab) = LR(0)(b) = l_5, LR(0)(ac) = LR(0)(c) = l_6, \dots,$

$l_9 := LR(0)(\gamma) = \emptyset$ in all remaining cases, e.g., for $\gamma = bB$)

No conflicts $\implies G \in LR(0)$ (but $G \notin LL(1)$)

Examples of $LR(0)$ Conflicts

Outline of Lecture 9

Recap: Bottom-Up Parsing

Nondeterminism in $NBA(G)$

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Examples of $LR(0)$ Conflicts

$LR(0)$ Parsing Functions

Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

Example 9.12

$G : S' \rightarrow S$

$S \rightarrow aA$

$A \rightarrow S \mid \varepsilon$

Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

Example 9.12

$$G : S' \rightarrow S$$
$$S \rightarrow aA$$
$$A \rightarrow S \mid \varepsilon$$
$$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aA]$$
$$LR(0)(S) : [S' \rightarrow S \cdot]$$
$$LR(0)(a) : [S \rightarrow a \cdot A] \quad [A \rightarrow \cdot S] \quad [A \rightarrow \cdot] \quad [S \rightarrow \cdot aA]$$
$$LR(0)(aA) : [S \rightarrow aA \cdot]$$

Examples of $LR(0)$ Conflicts

Shift/Reduce Conflicts

Example 9.12

$G : S' \rightarrow S$

$S \rightarrow aA$

$A \rightarrow S \mid \varepsilon$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot aA]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(a) : [S \rightarrow a \cdot A] \quad [A \rightarrow \cdot S] \quad [A \rightarrow \cdot] \quad [S \rightarrow \cdot aA]$

$LR(0)(aA) : [S \rightarrow aA \cdot]$

Note: G is unambiguous (and $G \in LL(1)$)

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

Example 9.13

$G : S' \rightarrow S$

$S \rightarrow Aa \mid Bb$

$A \rightarrow a$

$B \rightarrow a$

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

Example 9.13

$G : S' \rightarrow S$

$S \rightarrow Aa \mid Bb$

$A \rightarrow a$

$B \rightarrow a$

$LR(0)(\epsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(A) : [S \rightarrow A \cdot a]$

$LR(0)(B) : [S \rightarrow B \cdot b]$

$LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$

$LR(0)(Aa) : [S \rightarrow Aa \cdot]$

$LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Examples of $LR(0)$ Conflicts

Reduce/Reduce Conflicts

Example 9.13

$G : S' \rightarrow S$

$S \rightarrow Aa \mid Bb$

$A \rightarrow a$

$B \rightarrow a$

$LR(0)(\varepsilon) : [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot Aa] \quad [S \rightarrow \cdot Bb] \quad [A \rightarrow \cdot a] \quad [B \rightarrow \cdot a]$

$LR(0)(S) : [S' \rightarrow S \cdot]$

$LR(0)(A) : [S \rightarrow A \cdot a]$

$LR(0)(B) : [S \rightarrow B \cdot b]$

$LR(0)(a) : [A \rightarrow a \cdot] \quad [B \rightarrow a \cdot]$

$LR(0)(Aa) : [S \rightarrow Aa \cdot]$

$LR(0)(Bb) : [S \rightarrow Bb \cdot]$

Note: G is unambiguous (and $G \notin LL(1)$)

$LR(0)$ Parsing Functions

Outline of Lecture 9

Recap: Bottom-Up Parsing

Nondeterminism in $NBA(G)$

Resolving Termination Nondeterminism

$LR(k)$ Grammars

$LR(0)$ Grammars

Examples of $LR(0)$ Conflicts

$LR(0)$ Parsing Functions

LR(0) Parsing Functions

The goto Function I

Observation: if $G \in LR(0)$, then $LR(0)(\gamma)$ yields **deterministic shift/reduce decision** for $NBA(G)$ in a configuration with pushdown γ
 \implies **new pushdown alphabet:** $LR(0)(G)$ in place of X

LR(0) Parsing Functions

The goto Function I

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Moreover $LR(0)(\gamma Y)$ is determined by $LR(0)(\gamma)$ and Y but **independent from γ** in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

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The goto Function I

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Definition 9.14 ($LR(0)$ goto function)

The function **goto** : $LR(0)(G) \times X \rightarrow LR(0)(G)$ is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

LR(0) Parsing Functions

The goto Function II

Example 9.15 (cf. Example 9.11: $S' \rightarrow S, S \rightarrow B \mid C, B \rightarrow aB \mid b, C \rightarrow aC \mid c$)

$$\begin{aligned}l_0 &:= LR(0)(\varepsilon) : \begin{array}{l} [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot B] \quad [S \rightarrow \cdot C] \\ [B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b] \\ [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array} \\l_1 &:= LR(0)(S) : [S' \rightarrow S \cdot] \\l_2 &:= LR(0)(B) : [S \rightarrow B \cdot] \\l_3 &:= LR(0)(C) : [S \rightarrow C \cdot] \\l_4 &:= LR(0)(a) : \begin{array}{l} [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C] \quad [B \rightarrow \cdot aB] \\ [B \rightarrow \cdot b] \quad [C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c] \end{array} \\l_5 &:= LR(0)(b) : [B \rightarrow b \cdot] \\l_6 &:= LR(0)(c) : [C \rightarrow c \cdot] \\l_7 &:= LR(0)(aB) : [B \rightarrow aB \cdot] \\l_8 &:= LR(0)(aC) : [C \rightarrow aC \cdot] \\l_9 &:= \emptyset\end{aligned}$$

LR(0) Parsing Functions

The goto Function II

Example 9.15 (cf. Example 9.11: $S' \rightarrow S, S \rightarrow B \mid C, B \rightarrow aB \mid b, C \rightarrow aC \mid c$)

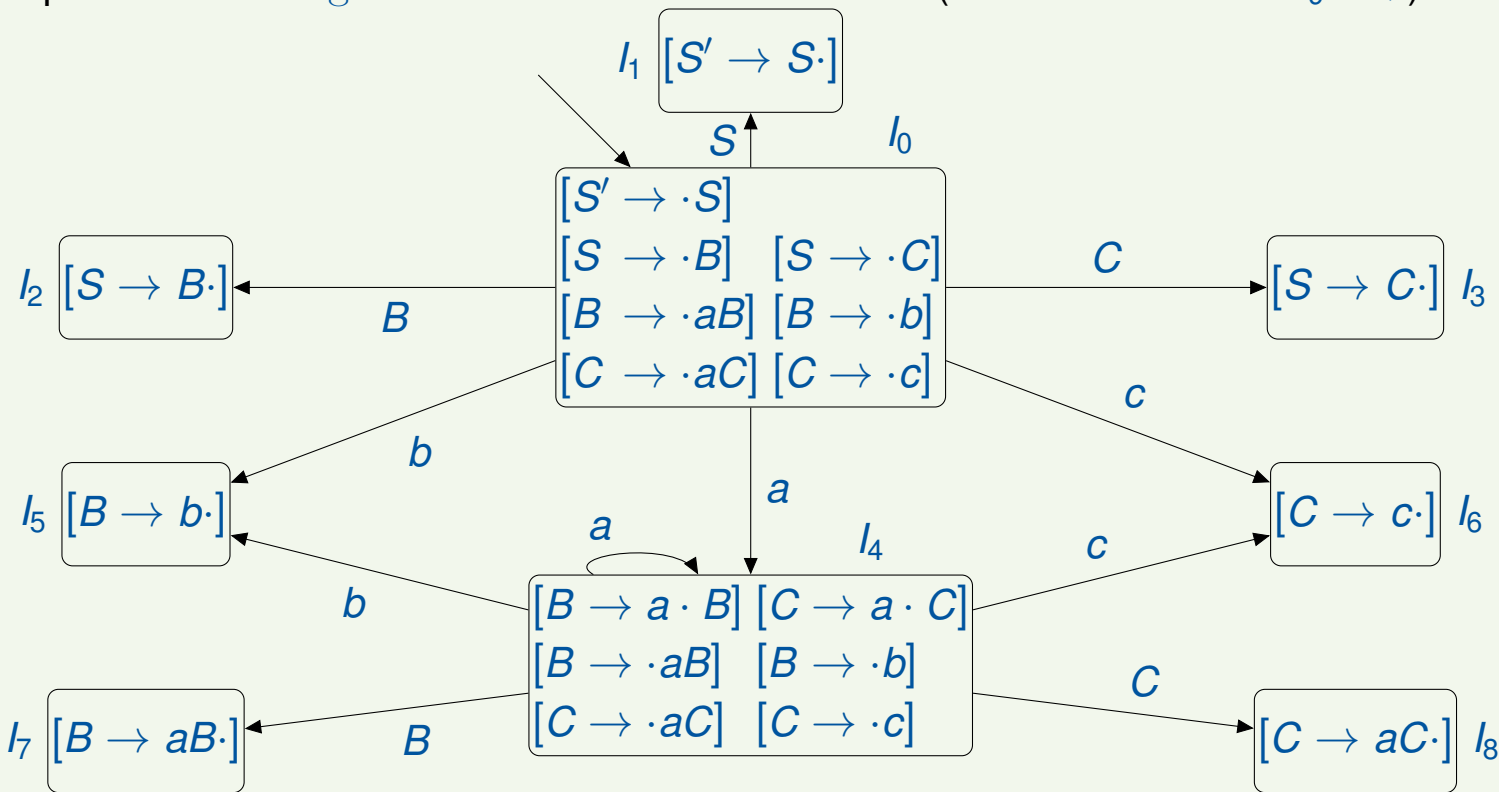
$l_0 := LR(0)(\varepsilon) :$	$[S' \rightarrow \cdot S]$	$[S \rightarrow \cdot B]$	$[S \rightarrow \cdot C]$	goto	S	B	C	a	b	c
	$[B \rightarrow \cdot aB]$	$[B \rightarrow \cdot b]$		l_0	l_1	l_2	l_3	l_4	l_5	l_6
	$[C \rightarrow \cdot aC]$	$[C \rightarrow \cdot c]$		l_1						
$l_1 := LR(0)(S) :$	$[S' \rightarrow S \cdot]$			l_2						
$l_2 := LR(0)(B) :$	$[S \rightarrow B \cdot]$			l_3						
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$l_4 := LR(0)(a) :$	$[B \rightarrow a \cdot B]$	$[C \rightarrow a \cdot C]$	$[B \rightarrow \cdot aB]$	l_5						
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$l_5 := LR(0)(b) :$	$[B \rightarrow b \cdot]$			l_7						
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$l_7 := LR(0)(aB) :$	$[B \rightarrow aB \cdot]$			l_9						
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$l_9 := \emptyset$										

(empty = l_9)

The goto Function III

Example 9.15 (continued)

Representation of goto function as finite automaton (omitted: sink state $l_9 = \emptyset$):



LR(0) Parsing Functions

The LR(0) Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision (reminder: $\pi_0 = S' \rightarrow S$).

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Definition 9.16 (LR(0) action function)

The **LR(0) action function** $\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$ is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

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Corollary 9.17

For every $G \in CFG_\Sigma$, $G \in LR(0)$ iff act is well defined.

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For every $G \in CFG_{\Sigma}$, $G \in LR(0)$ iff act is well defined.

Together, act and goto form the **LR(0) parsing table** of G .