



Compiler Construction

Lecture 7: Syntax Analysis III ($LL(1)$ Parsing)

Winter Semester 2018/19

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<https://moves.rwth-aachen.de/teaching/ws-1819/cc/>

Recap: $LL(k)$ Grammars

Outline of Lecture 7

Recap: $LL(k)$ Grammars

Computing Lookahead Sets

Deterministic Top-Down Parsing

Recap: $LL(k)$ Grammars

$LL(k)$ Grammars I

$LL(k)$: reading of input from **L**eft to right with k -lookahead, computing a **L**eftmost analysis

Definition ($LL(k)$ grammar)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$ and $k \in \mathbb{N}$. Then G has the $LL(k)$ property (notation: $G \in LL(k)$) if for all leftmost derivations of the form

$$S \Rightarrow_i^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \Rightarrow_j^* wx \\ \Rightarrow_l w\gamma\alpha \Rightarrow_j^* wy \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\text{first}_k(x) \neq \text{first}_k(y)$.

Thus: different productions must not yield the same lookahead.

Recap: $LL(k)$ Grammars

$LL(k)$ Grammars II

Lemma (Characterisation of $LL(k)$)

$G \in LL(k)$ iff for all leftmost derivations of the form

$$S \Rightarrow_i^* wA\alpha \begin{cases} \Rightarrow_i w\beta\alpha \\ \Rightarrow_i w\gamma\alpha \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\text{first}_k(\beta\alpha) \cap \text{first}_k(\gamma\alpha) = \emptyset$.

Proof.

omitted □

Remarks:

- If $G \in LL(k)$, then the A -production is **determined by the lookahead sets** $\text{first}_k(\beta\alpha)$ (for every $A \rightarrow \beta \in P$).
- **Problem:** still **infinitely many right contexts** α to be considered (if β [or γ] “too short”, i.e., $\text{first}_k(\beta\alpha) \neq \text{first}_k(\beta)$).
- **Idea:** α derives to **“everything that follows A ”**.

Recap: $LL(k)$ Grammars

The Case $k = 1$

Motivation:

- $k = 1$ sufficient to resolve nondeterminism in “most” practical applications
- Implementation of $LL(k)$ parsers for $k > 1$ rather involved
(cf. ANTLR [ANother Tool for Language Recognition; formerly PCCTS] at <http://www.antlr.org/>)

Abbreviations: $fi := first_1$, $fo := follow_1$, $\Sigma_\epsilon := \Sigma \cup \{\epsilon\}$

Corollary

1. For every $\alpha \in X^*$,

$$fi(\alpha) = \{a \in \Sigma \mid \text{ex. } w \in \Sigma^* : \alpha \Rightarrow^* aw\} \cup \{\epsilon \mid \alpha \Rightarrow^* \epsilon\} \subseteq \Sigma_\epsilon$$

2. For every $A \in N$,

$$fo(A) = \{x \in fi(\alpha) \mid \text{ex. } w \in \Sigma^*, \alpha \in X^* : S \Rightarrow_j^* wA\alpha\} \subseteq \Sigma_\epsilon.$$

Recap: $LL(k)$ Grammars

Lookahead Sets

Definition (Lookahead set)

Given $\pi = A \rightarrow \beta \in P$,

$$\text{la}(\pi) := \text{fi}(\beta \cdot \text{fo}(A)) \subseteq \Sigma_\varepsilon$$

is called the **lookahead set** of π (where $\text{fi}(\Gamma) := \bigcup_{\gamma \in \Gamma} \text{fi}(\gamma)$).

Corollary

1. For all $a \in \Sigma$,

$$a \in \text{la}(A \rightarrow \beta) \quad \text{iff} \quad a \in \text{fi}(\beta) \text{ or } (\beta \Rightarrow^* \varepsilon \text{ and } a \in \text{fo}(A))$$

2. $\varepsilon \in \text{la}(A \rightarrow \beta)$ iff $\beta \Rightarrow^* \varepsilon$ and $\varepsilon \in \text{fo}(A)$

Recap: $LL(k)$ Grammars

Characterisation of $LL(1)$

Theorem (Characterisation of $LL(1)$)

$G \in LL(1)$ iff for all pairs of rules $A \rightarrow \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\text{la}(A \rightarrow \beta) \cap \text{la}(A \rightarrow \gamma) = \emptyset.$$

Proof.

later □

Remark: the above theorem generally does not hold if $k > 1$ (cf. exercises)

Computing Lookahead Sets

Outline of Lecture 7

Recap: $LL(k)$ Grammars

Computing Lookahead Sets

Deterministic Top-Down Parsing

Computing Lookahead Sets

Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

Lemma 7.1 (Computation of fi/fo)

$\text{fi}(\alpha) \subseteq \Sigma_\varepsilon$ (for $\alpha \in X^*$) and $\text{fo}(A) \subseteq \Sigma_\varepsilon$ (for $A \in N$) are the least sets such that:

1. $\text{fi}(Y)$ for $Y \in X$:

- $Y \in \Sigma \implies \text{fi}(Y) = \{Y\}$
- $Y \in N, Y \rightarrow \alpha \in P, x \in \text{fi}(\alpha) \implies x \in \text{fi}(Y)$

Computing Lookahead Sets

Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

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- $Y \in N, Y \rightarrow \alpha \in P, x \in \text{fi}(\alpha) \implies x \in \text{fi}(Y)$

2. $\text{fi}(Y_1 \dots Y_n)$ for $n \in \mathbb{N}, Y_i \in X$:

- $\varepsilon \in \text{fi}(Y_1) \cap \dots \cap \text{fi}(Y_{k-1}), a \in \text{fi}(Y_k) \ (k \in [n]) \implies a \in \text{fi}(Y_1 \dots Y_n)$
- $\varepsilon \in \text{fi}(Y_1) \cap \dots \cap \text{fi}(Y_n) \implies \varepsilon \in \text{fi}(Y_1 \dots Y_n)$

Computing Lookahead Sets

Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

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- $\varepsilon \in \text{fi}(Y_1) \cap \dots \cap \text{fi}(Y_n) \implies \varepsilon \in \text{fi}(Y_1 \dots Y_n)$

3. $\text{fo}(A)$ for $A \in N$:

- $\varepsilon \in \text{fo}(S)$
- $A \rightarrow \alpha B \beta \in P, a \in \text{fi}(\beta) \implies a \in \text{fo}(B)$
- $A \rightarrow \alpha B \beta \in P, \varepsilon \in \text{fi}(\beta), x \in \text{fo}(A) \implies x \in \text{fo}(B)$

Computing Lookahead Sets

Computing Lookahead Sets II

Corollary 7.2

1. $A \rightarrow a\alpha \in P \implies a \in \text{fi}(A)$

2. $A \rightarrow B\alpha \in P, a \in \text{fi}(B) \implies a \in \text{fi}(A)$

3. $A \rightarrow \varepsilon \in P \implies \varepsilon \in \text{fi}(A)$

4. $a \in \text{fi}(A) \implies a \in \text{fi}(A\alpha)$

Computing Lookahead Sets

Computing Lookahead Sets II

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Example 7.3

Grammar for
arithmetic expressions
(cf. Example 5.11):

$$\begin{aligned} G_{AE} : E &\rightarrow E+T \mid T \\ T &\rightarrow T*F \mid F \\ F &\rightarrow (E) \mid a \mid b \end{aligned}$$

Computing Lookahead Sets

Computing Lookahead Sets II

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Example 7.3

Grammar for
arithmetic expressions
(cf. Example 5.11):

- $F \rightarrow a \in P \implies a \in \text{fi}(F)$

$$G_{AE} : \begin{array}{l} E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F \\ F \rightarrow (E) \mid a \mid b \end{array}$$

Computing Lookahead Sets

Computing Lookahead Sets II

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- $F \rightarrow a \in P \implies a \in \text{fi}(F)$
- $T \rightarrow F \in P, a \in \text{fi}(F) \implies a \in \text{fi}(T)$

$$G_{AE} : \begin{array}{l} E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F \\ F \rightarrow (E) \mid a \mid b \end{array}$$

Computing Lookahead Sets

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- $T \rightarrow F \in P, a \in \text{fi}(F) \implies a \in \text{fi}(T)$
- $a \in \text{fi}(T)$
 $\implies \text{la}(T \rightarrow T*F) = \text{fi}(T*F \cdot \text{fo}(T)) \ni a$

Computing Lookahead Sets

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 $\implies \text{la}(T \rightarrow F) = \text{fi}(F \cdot \text{fo}(T)) \ni a$
- $\implies a \in \text{la}(T \rightarrow T*F) \cap \text{la}(T \rightarrow F) \neq \emptyset$
 $\implies G_{AE} \notin LL(1)$

Computing Lookahead Sets

Fixing the Problem (general methods later)

Example 7.4 (continuing Example 7.3)

Restructuring (such that $L(G'_{AE}) = L(G_{AE})$):

$$\begin{aligned} G'_{AE} : E &\rightarrow TE' \\ E' &\rightarrow +TE' \mid \varepsilon \\ T &\rightarrow FT' \\ T' &\rightarrow *FT' \mid \varepsilon \\ F &\rightarrow (E) \mid a \mid b \end{aligned}$$

Computing Lookahead Sets

Fixing the Problem (general methods later)

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$A \in N$	$fi(A)$
E	$\{ (, a, b \}$
E'	$\{ +, \varepsilon \}$
T	$\{ (, a, b \}$
T'	$\{ *, \varepsilon \}$
F	$\{ (, a, b \}$

Computing Lookahead Sets

Fixing the Problem (general methods later)

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$A \in N$	$\text{fi}(A)$	$\text{fo}(A)$
E	$\{ (, a, b \}$	$\{ \varepsilon,) \}$
E'	$\{ +, \varepsilon \}$	$\{ \varepsilon,) \}$
T	$\{ (, a, b \}$	$\{ +, \varepsilon,) \}$
T'	$\{ *, \varepsilon \}$	$\{ +, \varepsilon,) \}$
F	$\{ (, a, b \}$	$\{ *, +, \varepsilon,) \}$

Remember:

- $\varepsilon \in \text{fo}(S)$
- $A \rightarrow \alpha B \beta \in P, a \in \text{fi}(\beta)$
 $\implies a \in \text{fo}(B)$
- $A \rightarrow \alpha B \beta \in P, \varepsilon \in \text{fi}(\beta), x \in \text{fo}(A)$
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Computing Lookahead Sets

Fixing the Problem (general methods later)

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 \end{aligned}$$

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T	$\{ (, a, b \}$	$\{ +, \varepsilon,) \}$
T'	$\{ *, \varepsilon \}$	$\{ +, \varepsilon,) \}$
F	$\{ (, a, b \}$	$\{ *, +, \varepsilon,) \}$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta) = fi(\beta \cdot fo(A))$
$E \rightarrow TE'$	$\{ (, a, b \}$
$E' \rightarrow +TE'$	$\{ + \}$
$E' \rightarrow \varepsilon$	$\{ \varepsilon,) \}$
$T \rightarrow FT'$	$\{ (, a, b \}$
$T' \rightarrow *FT'$	$\{ * \}$
$T' \rightarrow \varepsilon$	$\{ +, \varepsilon,) \}$
$F \rightarrow (E)$	$\{ (\}$
$F \rightarrow a$	$\{ a \}$
$F \rightarrow b$	$\{ b \}$

Computing Lookahead Sets

Fixing the Problem (general methods later)

Example 7.4 (continuing Example 7.3)

Restructuring (such that $L(G'_{AE}) = L(G_{AE})$):

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 \end{aligned}$$

$A \in N$	$fi(A)$	$fo(A)$
E	$\{ (, a, b \}$	$\{ \varepsilon,) \}$
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F	$\{ (, a, b \}$	$\{ *, +, \varepsilon,) \}$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta) = fi(\beta \cdot fo(A))$
$E \rightarrow TE'$	$\{ (, a, b \}$
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$F \rightarrow (E)$	$\{ (\}$
$F \rightarrow a$	$\{ a \}$
$F \rightarrow b$	$\{ b \}$

$\implies G'_{AE} \in LL(1)$

Deterministic Top-Down Parsing

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Deterministic Top-Down Parsing

The Approach

Deterministic top-down parsing

Given $G \in CFG_{\Sigma}$,

1. verify $G \in LL(1)$ by computing lookahead sets and checking alternatives for disjointness

Deterministic Top-Down Parsing

The Approach

Deterministic top-down parsing

Given $G \in CFG_{\Sigma}$,

1. verify $G \in LL(1)$ by computing lookahead sets and checking alternatives for disjointness
2. start with nondeterministic top-down parsing automaton $NTA(G)$

Deterministic Top-Down Parsing

The Approach

Deterministic top-down parsing

Given $G \in CFG_{\Sigma}$,

1. verify $G \in LL(1)$ by computing lookahead sets and checking alternatives for disjointness
2. start with nondeterministic top-down parsing automaton $NTA(G)$
3. use **1-symbol lookahead** to trigger the choice of expanding productions:
 - $(aw, A\alpha, z) \vdash (aw, \beta\alpha, zi)$ if $\pi_i = A \rightarrow \beta$ and $a \in \text{la}(\pi_i)$
 - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$ if $\pi_i = A \rightarrow \beta$ and $\varepsilon \in \text{la}(\pi_i)$
 - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$] \implies **deterministic top-down parsing automaton** $DTA(G)$

Deterministic Top-Down Parsing

The Approach

Deterministic top-down parsing

Given $G \in CFG_{\Sigma}$,

1. verify $G \in LL(1)$ by computing lookahead sets and checking alternatives for disjointness
2. start with nondeterministic top-down parsing automaton $NTA(G)$
3. use **1-symbol lookahead** to trigger the choice of expanding productions:
 - $(aw, A\alpha, z) \vdash (aw, \beta\alpha, zi)$ if $\pi_i = A \rightarrow \beta$ and $a \in \text{la}(\pi_i)$
 - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$ if $\pi_i = A \rightarrow \beta$ and $\varepsilon \in \text{la}(\pi_i)$
 - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$] \implies **deterministic top-down parsing automaton** $DTA(G)$

Remarks:

- $DTA(G)$ is actually **not a pushdown automaton** (a is inspected but not consumed).
But: can be simulated using the finite control.
- Advantage of using lookahead is **twofold**:
 - removal of nondeterminism
 - earlier detection of syntax errors (in configurations $(aw, A\alpha, z)$ where $a \notin \bigcup_{A \rightarrow \beta \in P} \text{la}(A \rightarrow \beta)$)

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton I

Definition 7.5 (Deterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LL(1)$. The **deterministic top-down parsing automaton** of G , $DTA(G)$, is defined by the following components.

- **Input alphabet** Σ , **pushdown alphabet** X , **output alphabet** $[p]$
- **Configurations** $\Sigma^* \times X^* \times [p]^*$, **initial configuration** (w, S, ε) , **final configurations** $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$ (as $NTA(G)$)
- **Action function** $\text{act} : \Sigma_\varepsilon \times X_\varepsilon \rightarrow \{(\alpha, i) \mid \pi_i = A \rightarrow \alpha\} \cup \{\text{pop}, \text{accept}, \text{error}\}$
with $\text{act}(x, A) := (\alpha, i)$ if $\pi_i = A \rightarrow \alpha$ and $x \in \text{la}(\pi_i)$
 $\text{act}(a, a) := \text{pop}$
 $\text{act}(\varepsilon, \varepsilon) := \text{accept}$
 $\text{act}(x, y) := \text{error}$ otherwise
- **Transitions** for $x \in \Sigma_\varepsilon$, $w \in \Sigma^*$ (with $x \neq \varepsilon$ if $w \neq \varepsilon$), $Y \in X$, $\beta \in X^*$, and $z \in [p]^*$:

$$(xw, Y\beta, z) \vdash \begin{cases} (xw, \alpha\beta, zi) & \text{if } \text{act}(x, Y) = (\alpha, i) \\ (w, \beta, z) & \text{if } \text{act}(x, Y) = \text{pop} \end{cases}$$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton II

Example 7.6 (cf. Example 7.4)

$$\begin{aligned} G'_{AE} : E &\rightarrow TE' && (1) \\ E' &\rightarrow +TE' \mid \varepsilon && (2, 3) \\ T &\rightarrow FT' && (4) \\ T' &\rightarrow *FT' \mid \varepsilon && (5, 6) \\ F &\rightarrow (E) \mid a \mid b && (7, 8, 9) \end{aligned}$$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta)$
$E \rightarrow TE'$	$\{ (, a, b \}$
$E' \rightarrow +TE'$	$\{ + \}$
$E' \rightarrow \varepsilon$	$\{ \varepsilon,) \}$
$T \rightarrow FT'$	$\{ (, a, b \}$
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Deterministic Top-Down Parsing

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 \end{aligned}$$

$A \rightarrow \beta \in P$	$la(A \rightarrow \beta)$
$E \rightarrow TE'$	$\{(, a, b\}$
$E' \rightarrow +TE'$	$\{+\}$
$E' \rightarrow \varepsilon$	$\{\varepsilon,)\}$
$T \rightarrow FT'$	$\{(, a, b\}$
$T' \rightarrow *FT'$	$\{*\}$
$T' \rightarrow \varepsilon$	$\{+, \varepsilon,)\}$
$F \rightarrow (E)$	$\{($
$F \rightarrow a$	$\{a\}$
$F \rightarrow b$	$\{b\}$

act : $\Sigma_\varepsilon \times X_\varepsilon \rightarrow \{(\alpha, i) \mid \pi_i = A \rightarrow \alpha\} \cup \{\text{pop, accept, error}\}$ (empty $\hat{=}$ error)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$	pop						
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$$\begin{aligned} & ((a)*b, E, \epsilon) \\ \vdash & ((a)*b, TE', 1) \end{aligned}$$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT'E', 14)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$					pop		
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

- $((a)*b, E, \epsilon)$
- $\vdash ((a)*b, TE', 1)$
- $\vdash ((a)*b, FT'E', 14)$
- $\vdash ((a)*b, (E)T'E', 147)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

- $((a)*b, E, \epsilon)$
- $\vdash ((a)*b, TE', 1)$
- $\vdash ((a)*b, FT'E', 14)$
- $\vdash ((a)*b, (E)T'E', 147)$
- $\vdash (a)*b, E)T'E', 147)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT'E', 14)$
 $\vdash ((a)*b, (E)T'E', 147)$
 $\vdash (a)*b, (E)T'E', 147)$
 $\vdash (a)*b, TE')T'E', 1471)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

- $((a)*b, E, \epsilon)$
- $\vdash ((a)*b, TE', 1)$
- $\vdash ((a)*b, FT'E', 14)$
- $\vdash ((a)*b, (E)T'E', 147)$
- $\vdash (a)*b, E)T'E', 147)$
- $\vdash (a)*b, TE')T'E', 1471)$
- $\vdash (a)*b, FT'E')T'E', 14714)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

- $((a)*b, E, \epsilon)$
- $\vdash ((a)*b, TE', 1)$
- $\vdash ((a)*b, FT'E', 14)$
- $\vdash ((a)*b, (E)T'E', 147)$
- $\vdash ((a)*b, E)T'E', 147)$
- $\vdash ((a)*b, TE')T'E', 1471)$
- $\vdash ((a)*b, FT'E')T'E', 14714)$
- $\vdash ((a)*b, aT'E')T'E', 147148)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

- $((a)*b, E, \epsilon)$
- $\vdash ((a)*b, TE', 1)$
- $\vdash ((a)*b, FT'E', 14)$
- $\vdash ((a)*b, (E)T'E', 147)$
- $\vdash (a)*b, (E)T'E', 147)$
- $\vdash (a)*b, TE')T'E', 1471)$
- $\vdash (a)*b, FT'E')T'E', 14714)$
- $\vdash (a)*b, aT'E')T'E', 147148)$
- $\vdash ()*b, T'E')T'E', 147148)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon) \vdash ((a)*b, E') T' E', 1471486)$
 $\vdash ((a)*b, TE', 1)$
 $\vdash ((a)*b, FT' E', 14)$
 $\vdash ((a)*b, (E) T' E', 147)$
 $\vdash ((a)*b, E) T' E', 147)$
 $\vdash ((a)*b, TE') T' E', 1471)$
 $\vdash ((a)*b, FT' E') T' E', 14714)$
 $\vdash ((a)*b, aT' E') T' E', 147148)$
 $\vdash (() *b, T' E') T' E', 147148)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon) \vdash ((a)*b, E') T' E', 1471486)$
 $\vdash ((a)*b, TE', 1) \vdash ((a)*b,) T' E', 14714863)$
 $\vdash ((a)*b, FT' E', 14)$
 $\vdash ((a)*b, (E) T' E', 147)$
 $\vdash ((a)*b, E) T' E', 147)$
 $\vdash ((a)*b, TE') T' E', 1471)$
 $\vdash ((a)*b, FT' E') T' E', 14714)$
 $\vdash ((a)*b, aT' E') T' E', 147148)$
 $\vdash (() *b, T' E') T' E', 147148)$

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$\vdash ((a)*b, E, \epsilon)$	$\vdash (()*b, E')T'E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash (()*b,)T'E', 14714863)$
$\vdash ((a)*b, FT'E', 14)$	$\vdash (*b, T'E', 14714863)$
$\vdash ((a)*b, (E)T'E', 147)$	
$\vdash (a)*b, E)T'E', 147)$	
$\vdash (a)*b, TE')T'E', 1471)$	
$\vdash (a)*b, FT'E')T'E', 14714)$	
$\vdash (a)*b, aT'E')T'E', 147148)$	
$\vdash ()*b, T'E')T'E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	
$\vdash (a)*b, TE') T' E', 1471)$	
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, aT' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$	pop						
($(TE', 1)$		$(FT', 4)$		$((E), 7)$		pop					
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$				pop				
*				$(*FT', 5)$					pop			
+	$(+TE', 2)$			$(\varepsilon, 6)$						pop		
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, aT' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, bT' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	
$\vdash (a)*b, aT' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ϵ
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\epsilon, 3)$		$(\epsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+	$(+TE', 2)$			$(\epsilon, 6)$							pop	
ϵ		$(\epsilon, 3)$		$(\epsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \epsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, bT' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\epsilon, T' E', 1471486359)$
$\vdash (a)*b, aT' E') T' E', 147148)$	
$\vdash () *b, T' E') T' E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, bT' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\varepsilon, T' E', 1471486359)$
$\vdash (a)*b, aT' E') T' E', 147148)$	$\vdash (\varepsilon, E', 14714863596)$
$\vdash () *b, T' E') T' E', 147148)$	

Deterministic Top-Down Parsing

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	()	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(a, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(b, 9)$		pop					
($(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of $(a)*b$:

$((a)*b, E, \varepsilon)$	$\vdash ((a)*b, E') T' E', 1471486)$
$\vdash ((a)*b, TE', 1)$	$\vdash ((a)*b,) T' E', 14714863)$
$\vdash ((a)*b, FT' E', 14)$	$\vdash (*b, T' E', 14714863)$
$\vdash ((a)*b, (E) T' E', 147)$	$\vdash (*b, *FT' E', 147148635)$
$\vdash (a)*b, E) T' E', 147)$	$\vdash (b, FT' E', 147148635)$
$\vdash (a)*b, TE') T' E', 1471)$	$\vdash (b, bT' E', 1471486359)$
$\vdash (a)*b, FT' E') T' E', 14714)$	$\vdash (\varepsilon, T' E', 1471486359)$
$\vdash (a)*b, aT' E') T' E', 147148)$	$\vdash (\varepsilon, E', 14714863596)$
$\vdash () *b, T' E') T' E', 147148)$	$\vdash (\varepsilon, \varepsilon, 147148635963)$