Compiler Construction

Lecture 7: Syntax Analysis III ($LL(1)$ Parsing)

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$LL(k)$ Grammars I

$LL(k)$: reading of input from Left to right with k-lookahead, computing a Leftmost analysis

Definition ($LL(k)$ grammar)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ and $k \in \mathbb{N}$. Then G has the $LL(k)$ property (notation: $G \in LL(k)$) if for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \Rightarrow_l^* wx \\ \Rightarrow_l w\gamma\alpha \Rightarrow_l^* wy \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\mathrm{first}_k(x) \neq \mathrm{first}_k(y)$.

Thus: different productions must not yield the same lookahead.

$LL(k)$ Grammars II

Lemma (Characterisation of $LL(k)$)

$G \in LL(k)$ iff for all leftmost derivations of the form

$$S \Rightarrow_l^* wA\alpha \begin{cases} \Rightarrow_l w\beta\alpha \\ \Rightarrow_l w\gamma\alpha \end{cases}$$

such that $\beta \neq \gamma$, it follows that $\mathrm{first}_k(\beta\alpha) \cap \mathrm{first}_k(\gamma\alpha) = \emptyset$.

Proof.

omitted □

Remarks:

- If $G \in LL(k)$, then the A-production is determined by the lookahead sets $\mathrm{first}_k(\beta\alpha)$ (for every $A \to \beta \in P$).
- **Problem:** still infinitely many right contexts α to be considered (if β [or γ] "too short", i.e., $\mathrm{first}_k(\beta\alpha) \neq \mathrm{first}_k(\beta)$).
- **Idea:** α derives to "everything that follows A".

Recap: $LL(k)$ Grammars

The Case $k = 1$

Motivation:

- $k = 1$ sufficient to resolve nondeterminism in “most” practical applications
- Implementation of $LL(k)$ parsers for $k > 1$ rather involved (cf. ANTLR [ANother Tool for Language Recognition; formerly PCCTS] at http://www.antlr.org/)

Abbreviations: $\mathrm{fi} := \mathrm{first}_1$, $\mathrm{fo} := \mathrm{follow}_1$, $\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$

Corollary

1. *For every* $\alpha \in X^*$,

$$\mathrm{fi}(\alpha) = \{a \in \Sigma \mid \text{ex. } w \in \Sigma^* : \alpha \Rightarrow^* aw\} \cup \{\varepsilon \mid \alpha \Rightarrow^* \varepsilon\} \subseteq \Sigma_\varepsilon$$

2. *For every* $A \in N$,

$$\mathrm{fo}(A) = \{x \in \mathrm{fi}(\alpha) \mid \text{ex. } w \in \Sigma^*, \alpha \in X^* : S \Rightarrow_l^* wA\alpha\} \subseteq \Sigma_\varepsilon.$$

Lookahead Sets

Definition (Lookahead set)

Given $\pi = A \to \beta \in P$,

$$\text{la}(\pi) := \text{fi}(\beta \cdot \text{fo}(A)) \subseteq \Sigma_\varepsilon$$

is called the lookahead set of π (where $\text{fi}(\Gamma) := \bigcup_{\gamma \in \Gamma} \text{fi}(\gamma)$).

Corollary

1. *For all* $a \in \Sigma$,

$$a \in \text{la}(A \to \beta) \quad \textit{iff} \quad a \in \text{fi}(\beta) \textit{ or } (\beta \Rightarrow^* \varepsilon \textit{ and } a \in \text{fo}(A))$$

2. $\varepsilon \in \text{la}(A \to \beta)$ *iff* $\beta \Rightarrow^* \varepsilon$ *and* $\varepsilon \in \text{fo}(A)$

Characterisation of $LL(1)$

Theorem (Characterisation of $LL(1)$)

$G \in LL(1)$ iff for all pairs of rules $A \to \beta \mid \gamma \in P$ (where $\beta \neq \gamma$):

$$\mathrm{la}(A \to \beta) \cap \mathrm{la}(A \to \gamma) = \emptyset.$$

Proof.

later □

Remark: the above theorem generally does not hold if $k > 1$ (cf. exercises)

Computing Lookahead Sets I

(see Waite/Goos: *Compiler Construction*, p. 164f)

Lemma 7.1 (Computation of fi/fo)

$\mathrm{fi}(\alpha) \subseteq \Sigma_\varepsilon$ *(for* $\alpha \in X^*$*) and* $\mathrm{fo}(A) \subseteq \Sigma_\varepsilon$ *(for* $A \in N$*) are the least sets such that:*

1. $\mathrm{fi}(Y)$ *for* $Y \in X$*:*
   - $Y \in \Sigma \implies \mathrm{fi}(Y) = \{Y\}$
   - $Y \in N, Y \to \alpha \in P, x \in \mathrm{fi}(\alpha) \implies x \in \mathrm{fi}(Y)$
2. $\mathrm{fi}(Y_1 \ldots Y_n)$ *for* $n \in \mathbb{N}, Y_i \in X$*:*
   - $\varepsilon \in \mathrm{fi}(Y_1) \cap \ldots \cap \mathrm{fi}(Y_{k-1}), a \in \mathrm{fi}(Y_k)\ (k \in [n]) \implies a \in \mathrm{fi}(Y_1 \ldots Y_n)$
   - $\varepsilon \in \mathrm{fi}(Y_1) \cap \ldots \cap \mathrm{fi}(Y_n) \implies \varepsilon \in \mathrm{fi}(Y_1 \ldots Y_n)$
3. $\mathrm{fo}(A)$ *for* $A \in N$*:*
   - $\varepsilon \in \mathrm{fo}(S)$
   - $A \to \alpha B \beta \in P, a \in \mathrm{fi}(\beta) \implies a \in \mathrm{fo}(B)$
   - $A \to \alpha B \beta \in P, \varepsilon \in \mathrm{fi}(\beta), x \in \mathrm{fo}(A) \implies x \in \mathrm{fo}(B)$

Computing Lookahead Sets II

Corollary 7.2

1. $A \to a\alpha \in P \implies a \in \mathrm{fi}(A)$
2. $A \to B\alpha \in P, a \in \mathrm{fi}(B) \implies a \in \mathrm{fi}(A)$
3. $A \to \varepsilon \in P \implies \varepsilon \in \mathrm{fi}(A)$
4. $a \in \mathrm{fi}(A) \implies a \in \mathrm{fi}(A\alpha)$

Example 7.3

Grammar for arithmetic expressions (cf. Example 5.11):

$$\begin{aligned} G_{AE}: \quad E &\to E{+}T \mid T \\ T &\to T{*}F \mid F \\ F &\to (E) \mid \mathtt{a} \mid \mathtt{b} \end{aligned}$$

- $F \to \mathtt{a} \in P \implies \mathtt{a} \in \mathrm{fi}(F)$
- $T \to F \in P, \mathtt{a} \in \mathrm{fi}(F) \implies \mathtt{a} \in \mathrm{fi}(T)$
- $\mathtt{a} \in \mathrm{fi}(T)$ $\implies \mathrm{la}(T \to T{*}F) = \mathrm{fi}(T{*}F \cdot \mathrm{fo}(T)) \ni \mathtt{a}$
- $\mathtt{a} \in \mathrm{fi}(F)$ $\implies \mathrm{la}(T \to F) = \mathrm{fi}(F \cdot \mathrm{fo}(T)) \ni \mathtt{a}$

$\Rightarrow \mathtt{a} \in \mathrm{la}(T \to T{*}F) \cap \mathrm{la}(T \to F) \neq \emptyset$

$\Rightarrow G_{AE} \notin LL(1)$

Fixing the Problem (general methods later)

Example 7.4 (continuing Example 7.3)

Restructuring (such that $L(G'_{AE}) = L(G_{AE})$):

$$
\begin{aligned}
G'_{AE}:\; E &\to TE' \\
E' &\to +TE' \mid \varepsilon \\
T &\to FT' \\
T' &\to *FT' \mid \varepsilon \\
F &\to (E) \mid \mathtt{a} \mid \mathtt{b}
\end{aligned}
$$

$A \in N$	$\mathrm{fi}(A)$	$\mathrm{fo}(A)$
E	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{\varepsilon,)\}$
E'	$\{+, \varepsilon\}$	$\{\varepsilon,)\}$
T	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{+, \varepsilon,)\}$
T'	$\{*, \varepsilon\}$	$\{+, \varepsilon,)\}$
F	$\{(, \mathtt{a}, \mathtt{b}\}$	$\{*, +, \varepsilon,)\}$

Remember:

- $\varepsilon \in \mathrm{fo}(S)$
- $A \to \alpha B \beta \in P, a \in \mathrm{fi}(\beta)$
  $\implies a \in \mathrm{fo}(B)$
- $A \to \alpha B \beta \in P, \varepsilon \in \mathrm{fi}(\beta), x \in \mathrm{fo}(A)$
  $\implies x \in \mathrm{fo}(B)$

$A \to \beta \in P$	$\mathrm{la}(A \to \beta) = \mathrm{fi}(\beta \cdot \mathrm{fo}(A))$
$E \to TE'$	$\{(, \mathtt{a}, \mathtt{b}\}$
$E' \to +TE'$	$\{+\}$

The Approach

Deterministic top-down parsing

Given $G \in CFG_\Sigma$,

1. verify $G \in LL(1)$ by computing lookahead sets and checking alternatives for disjointness
2. start with nondeterministic top-down parsing automaton $NTA(G)$
3. use 1-symbol lookahead to trigger the choice of expanding productions:
   - $(aw, A\alpha, z) \vdash (aw, \beta\alpha, zi)$ if $\pi_i = A \to \beta$ and $a \in \mathrm{la}(\pi_i)$
   - $(\varepsilon, A\alpha, z) \vdash (\varepsilon, \beta\alpha, zi)$ if $\pi_i = A \to \beta$ and $\varepsilon \in \mathrm{la}(\pi_i)$
   - [matching steps as before: $(aw, a\alpha, z) \vdash (w, \alpha, z)$]

   $\Longrightarrow$ deterministic top-down parsing automaton $\mathrm{DTA}(G)$

Remarks:

- $\mathrm{DTA}(G)$ is actually not a pushdown automaton (a is inspected but not consumed).
  But: can be simulated using the finite control.
- Advantage of using lookahead is twofold:
  - removal of nondeterminism
  - earlier detection of syntax errors (in configurations $(aw, A\alpha, z)$ where $a \notin \bigcup_{A \to \beta \in P} \mathrm{la}(A \to \beta)$)

The Deterministic Top-Down Automaton I

Definition 7.5 (Deterministic top-down parsing automaton)

Let $G = \langle N, \Sigma, P, S \rangle \in LL(1)$. The deterministic top-down parsing automaton of G, $\mathrm{DTA}(G)$, is defined by the following components.

- Input alphabet Σ, pushdown alphabet X, output alphabet $[p]$
- Configurations $\Sigma^* \times X^* \times [p]^*$, initial configuration (w, S, ε), final configurations $\{\varepsilon\} \times \{\varepsilon\} \times [p]^*$ (as $NTA(G)$)
- Action function $\mathrm{act} : \Sigma_\varepsilon \times X_\varepsilon \to \{(\alpha, i) \mid \pi_i = A \to \alpha\} \cup \{\mathrm{pop}, \mathrm{accept}, \mathrm{error}\}$
  with $\mathrm{act}(x, A) := (\alpha, i)$ if $\pi_i = A \to \alpha$ and $x \in \mathrm{la}(\pi_i)$
  $\mathrm{act}(a, a) := \mathrm{pop}$
  $\mathrm{act}(\varepsilon, \varepsilon) := \mathrm{accept}$
  $\mathrm{act}(x, y) := \mathrm{error}$ otherwise
- Transitions for $x \in \Sigma_\varepsilon$, $w \in \Sigma^*$ (with $x \neq \varepsilon$ if $w \neq \varepsilon$), $Y \in X$, $\beta \in X^*$, and $z \in [p]^*$:

$$(xw, Y\beta, z) \vdash \begin{cases} (xw, \alpha\beta, zi) & \text{if } \mathrm{act}(x, Y) = (\alpha, i) \\ (w, \beta, z) & \text{if } \mathrm{act}(x, Y) = \mathrm{pop} \end{cases}$$

The Deterministic Top-Down Automaton II

Example 7.6 (cf. Example 7.4)

$$
\begin{array}{llll}
G'_{AE}: & E & \to TE' & (1) \\
 & E' & \to +TE' \mid \varepsilon & (2,3) \\
 & T & \to FT' & (4) \\
 & T' & \to *FT' \mid \varepsilon & (5,6) \\
 & F & \to (E) \mid \mathrm{a} \mid \mathrm{b} & (7,8,9)
\end{array}
$$

$A \to \beta \in P$	$\mathrm{la}(A \to \beta)$
$E \to TE'$	$\{(, \mathrm{a}, \mathrm{b}\}$
$E' \to +TE'$	$\{+\}$
$E' \to \varepsilon$	$\{\varepsilon,)\}$
$T \to FT'$	$\{(, \mathrm{a}, \mathrm{b}\}$
$T' \to *FT'$	$\{*\}$
$T' \to \varepsilon$	$\{+, \varepsilon,)\}$
$F \to (E)$	$\{(\}$
$F \to \mathrm{a}$	$\{\mathrm{a}\}$
$F \to \mathrm{b}$	$\{\mathrm{b}\}$

$\mathrm{act} : \Sigma_\varepsilon \times X_\varepsilon \to \{(\alpha, i) \mid \pi_i = A \to \alpha\} \cup \{\text{pop}, \text{accept}, \text{error}\}$ (empty $\hat{=}$ error)

act	E	E'	T	T'	F	a	b	(	)	$*$	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathrm{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathrm{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$((E), 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
$*$				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

The Deterministic Top-Down Automaton III

Example 7.6 (continued)

act	E	E'	T	T'	F	a	b	(	)	*	+	ε
a	$(TE', 1)$		$(FT', 4)$		$(\mathtt{a}, 8)$	pop						
b	$(TE', 1)$		$(FT', 4)$		$(\mathtt{b}, 9)$		pop					
(	$(TE', 1)$		$(FT', 4)$		$(\mathtt{(}E\mathtt{)}, 7)$			pop				
)		$(\varepsilon, 3)$		$(\varepsilon, 6)$					pop			
*				$(*FT', 5)$						pop		
+		$(+TE', 2)$		$(\varepsilon, 6)$							pop	
ε		$(\varepsilon, 3)$		$(\varepsilon, 6)$								accept

Leftmost analysis of `(a)*b`:

$(\mathtt{(a)*b}, E, \varepsilon)$	$\vdash (\mathtt{)*b}, E'\mathtt{)}T'E', 1471486)$
$\vdash (\mathtt{(a)*b}, TE', 1)$	$\vdash (\mathtt{)*b}, \mathtt{)}T'E', 14714863)$
$\vdash (\mathtt{(a)*b}, FT'E', 14)$	$\vdash (\mathtt{*b}, T'E', 14714863)$
$\vdash (\mathtt{(a)*b}, \mathtt{(}E\mathtt{)}T'E', 147)$	$\vdash (\mathtt{*b}, *FT'E', 147148635)$
$\vdash (\mathtt{a)*b}, E\mathtt{)}T'E', 147)$	$\vdash (\mathtt{b}, FT'E', 147148635)$
$\vdash (\mathtt{a)*b}, TE'\mathtt{)}T'E', 1471)$	$\vdash (\mathtt{b}, \mathtt{b}T'E', 1471486359)$
$\vdash (\mathtt{a)*b}, FT'E'\mathtt{)}T'E', 14714)$	$\vdash (\varepsilon, T'E', 1471486359)$
$\vdash (\mathtt{a)*b}, \mathtt{a}T'E'\mathtt{)}T'E', 147148)$	$\vdash (\varepsilon, E', 14714863596)$
$\vdash (\mathtt{)*b}, T'E'\mathtt{)}T'E', 147148)$	$\vdash (\varepsilon, \varepsilon, 147148635963)$