Compiler Construction

Lecture 2: Lexical Analysis I (Simple Matching Problem)

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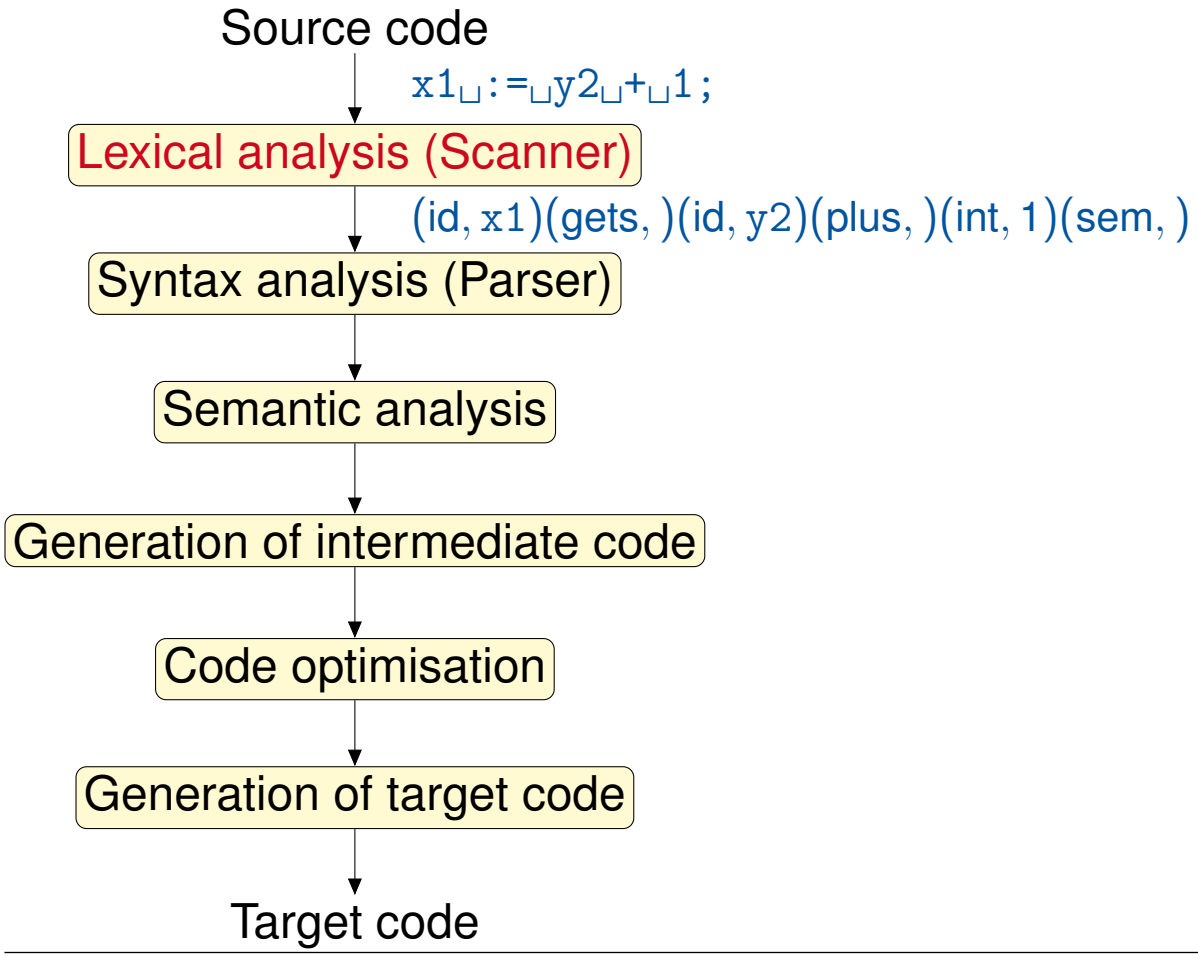
Conceptual Structure of a Compiler



regular expressions/
finite automata

Lexical Structures

From Merriam-Webster's Online Dictionary

Lexical: of or relating to words or the vocabulary of a language as distinguished from its grammar and construction

- **Starting point:** source program P as a character sequence
  - Ω (finite) character set (e.g., ASCII, ISO Latin-1, Unicode, ...)
  - $a, b, c, \ldots \in \Omega$ characters (= lexical atoms)
  - $P \in \Omega^*$ source program

  (of course, not every $w \in \Omega^*$ is a valid program)
- P exhibits lexical structures:
  - natural language for keywords, identifiers, ...
  - textual notation for numbers, formulae, ... (e.g., $x^2 \rightsquigarrow$ `x**2` or $2.9979 \cdot 10^8 \rightsquigarrow$ `2.9979D+8`)
  - spaces, line breaks, indentation
  - comments and compiler directives (pragmas)
- Translation of P follows its hierarchical structure (later)

Lexical as Part of Syntax Analysis

Remark: lexical analysis could be made integral part of syntax analysis (as regular languages are a proper subclass of context-free languages – cf. `ANTLR` approach)

Reasons for keeping lexical and syntax analysis separate

Efficiency: scanner may do simple parts of the work faster than a more general parser

Modularity: syntax definition not cluttered with low-level details such as white spaces or comments

Tradition: language standards typically separate lexical and syntactical elements

Problem Statement

Observations

1. Syntactic atoms (called symbols) are represented as sequences of input characters, called lexemes

First goal of lexical analysis

Decomposition of program text into a sequence of lexemes

2. Differences between similar lexemes are (mostly) irrelevant for syntax analysis (e.g., identifiers do not need to be distinguished)
   - lexemes grouped into symbol classes
     - e.g., identifiers, numbers, ...
   - symbol classes abstractly represented by tokens
   - symbols identified by additional attributes
     - e.g., identifier names, numerical values, ...; required for semantic analysis and code generation

$\Rightarrow$ symbol = (token, attribute)

Second goal of lexical analysis

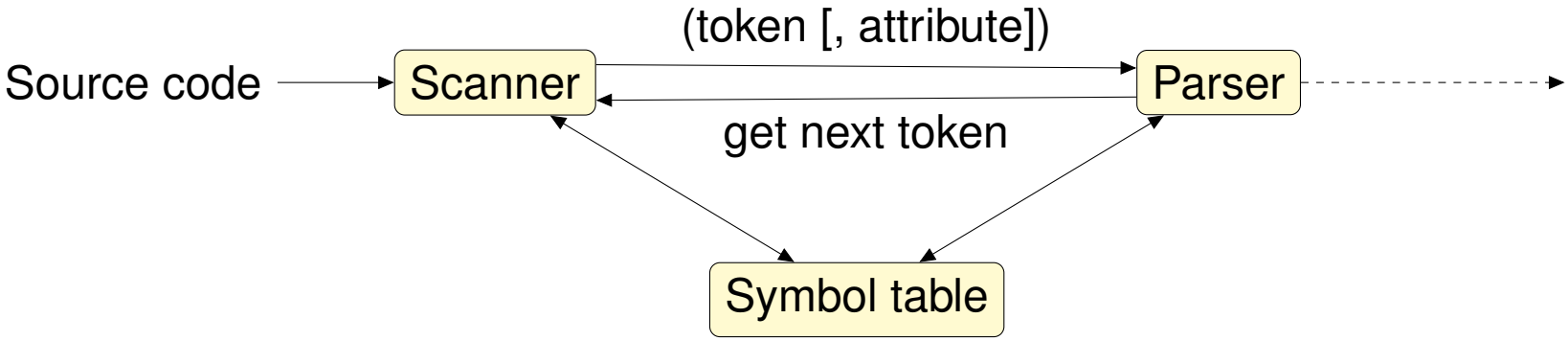
Transformation of a sequence of lexemes into a sequence of symbols

Lexical Analysis

Definition 2.1

The goal of lexical analysis is the decomposition a source program into a sequence of lexemes and their transformation into a sequence of symbols.

The corresponding program is called a scanner (or lexer):



Example:

$$\ldots\,␣\texttt{x1}␣\texttt{:=y2+}␣\texttt{1}␣\texttt{;}␣\ldots$$

$$\Downarrow$$

$$\ldots(\mathrm{id}, p_1)(\mathrm{gets},)(\mathrm{id}, p_2)(\mathrm{plus},)(\mathrm{int}, 1)(\mathrm{sem},)\ldots$$

Important Symbol Classes

Identifiers:
- for naming variables, constants, types, procedures, classes, ...
- usually a sequence of letters and digits (and possibly special symbols), starting with a letter
- keywords usually forbidden; length possibly restricted

Keywords:
- identifiers with a predefined meaning
- for representing control structures (`while`), operators (`and`), ...

Numerals: certain sequences of digits, `+`, `-`, `.`, letters (for exponent and hexadecimal representation)

Special symbols:
- one special character, e.g., `+`, `*`, `<`, `(`, `;`, ...
- ... or two or more special characters, e.g., `:=`, `**`, `<=`, ...
- each makes up a symbol class (plus, gets, ...)
- ... or several combined into one class (arithOp)

White spaces:
- blanks, tabs, line breaks, ...
- generally for separating symbols (exception: FORTRAN)
- usually not represented by token (but just removed)

Problem Statement

Specification and Implementation of Scanners

Representation of symbols: symbol = (token, attribute)

Token: denotation of symbol class (id, gets, plus, ...)

Attribute: additional information required in later compilation phases

- reference to symbol table,
- value of numeral,
- concrete arithmetic/relational/Boolean operator, ...
- usually unused for singleton symbol classes

Observation: symbol classes are regular sets

$\Longrightarrow$

- specification by regular expressions
- recognition by finite automata
- enables automatic generation of scanners (`[f]lex`)

Regular Expressions I

Definition 2.2 (Syntax of regular expressions)

Given some alphabet Ω, the set of regular expressions over Ω, RE_Ω, is the least set with

- $\emptyset \in RE_\Omega$,
- $\Omega \subseteq RE_\Omega$, and
- whenever $\alpha, \beta \in RE_\Omega$, also $\alpha \mid \beta, \alpha \cdot \beta, \alpha^* \in RE_\Omega$.

Remarks:

- abbreviations: $\alpha^+ := \alpha \cdot \alpha^*$, $\varepsilon := \emptyset^*$
- $\alpha \cdot \beta$ often written as $\alpha\beta$
- Binding priority: $^* > \cdot > \mid$ (i.e., $a \mid b \cdot c^* := a \mid (b \cdot (c^*))$)

Regular Expressions II

Regular expressions specify regular languages:

Definition 2.3 (Semantics of regular expressions)

The semantics of a regular expression is defined by the mapping $[\![.]\!] : RE_\Omega \to 2^{\Omega^*}$:

$$
\begin{aligned}
[\![\emptyset]\!] &:= \emptyset \\
[\![a]\!] &:= \{a\} \\
[\![\alpha \mid \beta]\!] &:= [\![\alpha]\!] \cup [\![\beta]\!] \\
[\![\alpha \cdot \beta]\!] &:= [\![\alpha]\!] \cdot [\![\beta]\!] \\
[\![\alpha^*]\!] &:= [\![\alpha]\!]^*
\end{aligned}
$$

Remarks: for formal languages $L, M \subseteq \Omega^*$, we have

- $L \cdot M := \{vw \mid v \in L, w \in M\}$
- $L^* := \bigcup_{n=0}^{\infty} L^n$ where $L^0 := \{\varepsilon\}$ and $L^{n+1} := L \cdot L^n$
  - thus $L^* = \{w_1 w_2 \ldots w_n \mid n \in \mathbb{N}, \forall 1 \leq i \leq n : w_i \in L\}$ and $\varepsilon \in L^*$
- $[\![\emptyset^*]\!] = [\![\emptyset]\!]^* = \emptyset^* = \{\varepsilon\}$

Regular Expressions III

Example 2.4

1. A keyword: `begin`
2. Identifiers:

$$(\texttt{a} \mid \ldots \mid \texttt{z} \mid \texttt{A} \mid \ldots \mid \texttt{Z})(\texttt{a} \mid \ldots \mid \texttt{z} \mid \texttt{A} \mid \ldots \mid \texttt{Z} \mid \texttt{0} \mid \ldots \mid \texttt{9} \mid \texttt{\$} \mid \texttt{\_} \mid \ldots)^*$$

3. (Unsigned) Integer numbers: $(\texttt{0} \mid \ldots \mid \texttt{9})^+$
4. (Unsigned) Fixed-point numbers:

$$((\texttt{0} \mid \ldots \mid \texttt{9})^+.(\texttt{0} \mid \ldots \mid \texttt{9})^*) \mid ((\texttt{0} \mid \ldots \mid \texttt{9})^*.(\texttt{0} \mid \ldots \mid \texttt{9})^+)$$

The Simple Matching Problem I

Problem 2.5 (Simple matching problem)

Given $\alpha \in RE_\Omega$ and $w \in \Omega^*$, decide whether $w \in [\![\alpha]\!]$ or not.

This problem can be solved using the following concept:

Definition 2.6 (Finite automaton)

A nondeterministic finite automaton (NFA) is of the form $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle$ where

- Q is a finite set of states
- Ω denotes the input alphabet
- $\delta : Q \times \Omega_\varepsilon \to 2^Q$ is the transition function with $\Omega_\varepsilon := \Omega \cup \{\varepsilon\}$ (write $q \xrightarrow{x} q'$ for $q' \in \delta(q, x)$)
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is the set of final states

The set of all NFA over Ω is denoted by NFA_Ω.
If $\delta(q, \varepsilon) = \emptyset$ and $|\delta(q, a)| = 1$ for every $q \in Q$ and $a \in \Omega$ (i.e., $\delta : Q \times \Omega \to Q$), then $\mathfrak{A}$ is called deterministic (DFA). Notation: DFA_Ω

The Simple Matching Problem II

Definition 2.7 (Acceptance condition)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$ and $w = a_1 \dots a_n \in \Omega^*$.

- A w-labelled $\mathfrak{A}$-run from q_1 to q_2 is a sequence of transitions

$$q_1 \xrightarrow{\varepsilon}{}^* \xrightarrow{a_1} \xrightarrow{\varepsilon}{}^* \xrightarrow{a_2} \xrightarrow{\varepsilon}{}^* \dots \xrightarrow{\varepsilon}{}^* \xrightarrow{a_n} \xrightarrow{\varepsilon}{}^* q_2$$

- $\mathfrak{A}$ accepts w if there is a w-labelled $\mathfrak{A}$-run from q_0 to some $q \in F$
- The language recognised by $\mathfrak{A}$ is

$$L(\mathfrak{A}) := \{w \in \Omega^* \mid \mathfrak{A} \text{ accepts } w\}$$

- A language $L \subseteq \Omega^*$ is called NFA-recognisable if there exists a NFA $\mathfrak{A}$ such that $L(\mathfrak{A}) = L$

Example 2.8

NFA for $a^*b \mid a^*$ (on the board)

The Simple Matching Problem III

Remarks:

- NFA as specified in Definition 2.6 are sometimes called NFA with ε-transitions (ε-NFA).
- For $\mathfrak{A} \in DFA_\Omega$, the acceptance condition yields $\delta^* : Q \times \Omega^* \to Q$ with $\delta^*(q, \varepsilon) = q$ and $\delta^*(q, aw) = \delta^*(\delta(q, a), w)$, and

$$L(\mathfrak{A}) = \{w \in \Omega^* \mid \delta^*(q_0, w) \in F\}.$$

The DFA Method I

Known from *Formal Systems, Automata and Processes*:

Algorithm 2.9 (DFA method)

Input: regular expression $\alpha \in RE_\Omega$, input string $w \in \Omega^*$

Procedure: 1. using Kleene's Theorem, construct $\mathfrak{A}_\alpha \in NFA_\Omega$ such that $L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$

2. apply powerset construction (cf. Definition 2.11) to obtain $\mathfrak{A}'_\alpha = \langle Q', \Omega, \delta', q'_0, F' \rangle \in DFA_\Omega$ with $L(\mathfrak{A}'_\alpha) = L(\mathfrak{A}_\alpha) = [\![\alpha]\!]$
3. solve the matching problem by deciding whether $\delta'^*(q'_0, w) \in F'$

Output: "yes" or "no"

Kleene's Theorem

Working principle: on the board

The DFA Method II

The powerset construction involves the following concept:

Definition 2.10 (ε-closure)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in \mathit{NFA}_\Omega$. The ε-closure $\varepsilon(T) \subseteq Q$ of a subset $T \subseteq Q$ is the least set with (1) $T \subseteq \varepsilon(T)$ and (2) if $q \in \varepsilon(T)$, then $\delta(q, \varepsilon) \subseteq \varepsilon(T)$

Definition 2.11 (Powerset construction)

Let $\mathfrak{A} = \langle Q, \Omega, \delta, q_0, F \rangle \in \mathit{NFA}_\Omega$. The powerset automaton $\mathfrak{A}' = \langle Q', \Omega, \delta', q_0', F' \rangle \in \mathit{DFA}_\Omega$ is defined by

- $Q' := 2^Q$
- $q_0' := \varepsilon(\{q_0\})$
- $\forall T \subseteq Q, a \in \Omega : \delta'(T, a) := \varepsilon\left(\bigcup_{q \in T} \delta(q, a)\right)$
- $F' := \{T \subseteq Q \mid T \cap F \neq \emptyset\}$

Example 2.12

Powerset construction for Example 2.8 (on the board)

Complexity Analysis of Simple Matching

Complexity of DFA Method

1. in construction phase:
   - Kleene method: time and space $\mathcal{O}(|\alpha|)$
     (where $|\alpha| :=$ length of α)
   - Powerset construction: time and space $\mathcal{O}(2^{|\mathfrak{A}_\alpha|}) = \mathcal{O}(2^{|\alpha|})$
     (where $|\mathfrak{A}_\alpha| :=$ # of states of $\mathfrak{A}_\alpha$)
2. at runtime:
   - Word problem:
     - time $\mathcal{O}(|w|)$ (where $|w| :=$ length of w)
     - space $\mathcal{O}(1)$ (but $\mathcal{O}(2^{|\alpha|})$ for storing DFA)

$\Rightarrow$ nice runtime behaviour but memory requirements very high
(and exponential time in construction phase)

Complexity Analysis of Simple Matching

The NFA Method

Idea: reduce memory requirements by applying powerset construction at runtime, i.e., only "to the run of w through $\mathfrak{A}_\alpha$"

Algorithm 2.13 (NFA method)

Input: automaton $\mathfrak{A}_\alpha = \langle Q, \Omega, \delta, q_0, F \rangle \in NFA_\Omega$, input string $w \in \Omega^*$

Variables: $T \subseteq Q$, $a \in \Omega$

Procedure: $T := \varepsilon(\{q_0\})$;

while $w \neq \varepsilon$ **do**

$a := \mathbf{head}(w)$;

$T := \varepsilon\left(\bigcup_{q \in T} \delta(q, a)\right)$;

$w := \mathbf{tail}(w)$

od

Output: *if* $T \cap F \neq \emptyset$ *then "yes" else "no"*

Complexity Analysis of Simple Matching

Complexity Analysis

For NFA method at runtime:

- Space: $\mathcal{O}(|\alpha|)$ (for storing NFA and T)
- Time: $\mathcal{O}(|\alpha| \cdot |w|)$ (in the loop's body, $|T|$ states need to be considered)

Comparison:

Method	Space	Time (for "$w \in [\![\alpha]\!]$?")
DFA	$\mathcal{O}(2^{\|\alpha\|})$	$\mathcal{O}(\|w\|)$
NFA	$\mathcal{O}(\|\alpha\|)$	$\mathcal{O}(\|\alpha\| \cdot \|w\|)$

$\Longrightarrow$ trades exponential space for increase in time

In practice:

- Exponential blowup of DFA method does usually not occur in practice
  ($\Longrightarrow$ used in `[f]lex`)
- Improvement of NFA method: caching of transitions $\delta'(T, a)$
  $\Longrightarrow$ combination of both methods