Compiler Construction

Lecture 13: Semantic Analysis II (Circularity Check)

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https://moves.rwth-aachen.de/teaching/ws-1819/cc/

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Outline of Lecture 13

Recap: Attribute Grammars

Checking Attribute Grammars for Circularity

The Circularity Check

Correctness and Complexity of the Circularity Check

Formal Definition of Attribute Grammars

Definition (Attribute grammar)

Let $G = \langle N, \Sigma, P, S\rangle \in CFG_\Sigma$ with $X := N \uplus \Sigma$.

- Let $Att = Syn \uplus Inh$ be a set of (synthesised or inherited) attributes, and let $V = \bigcup_{\alpha \in Att} V^\alpha$ be a collection of value sets.
- Let $\mathrm{att} : X \to 2^{Att}$ be an attribute assignment, and let $\mathrm{syn}(Y) := \mathrm{att}(Y) \cap Syn$ and $\mathrm{inh}(Y) := \mathrm{att}(Y) \cap Inh$ for every $Y \in X$.
- Every production $\pi = Y_0 \to Y_1 \dots Y_r \in P$ determines the set
$$Var_\pi := \{\alpha.i \mid \alpha \in \mathrm{att}(Y_i), i \in \{0, \dots, r\}\}$$
of attribute variables of π with the subsets of internal and external variables:
$$Int_\pi := \{\alpha.i \mid (i = 0, \alpha \in \mathrm{syn}(Y_i)) \text{ or } (i \in [r], \alpha \in \mathrm{inh}(Y_i))\} \quad Ext_\pi := Var_\pi \setminus Int_\pi$$
- A semantic rule of π is an equation of the form
$$\alpha_0.i_0 = f(\alpha_1.i_1, \dots, \alpha_n.i_n)$$
where $n \in \mathbb{N}$, $\alpha_0.i_0 \in Int_\pi$, $\alpha_j.i_j \in Ext_\pi$, and $f : V^{\alpha_1} \times \dots \times V^{\alpha_n} \to V^{\alpha_0}$.
- For each $\pi \in P$, let E_π be a set with exactly one semantic rule for every internal variable of π, and let $E := (E_\pi \mid \pi \in P)$.

Then $\mathfrak{A} := \langle G, E, V\rangle$ is called an attribute grammar: $\mathfrak{A} \in AG$.

Attribution of Syntax Trees

Definition (Attribution of syntax trees)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G with the set of nodes K.

- K determines the set of attribute variables of t:
$$Var_t := \{\alpha.k \mid k \in K \text{ labelled with } Y \in X, \alpha \in \mathrm{att}(Y)\}.$$
- Let $k_0 \in K$ be an (inner) node where production $\pi = Y_0 \to Y_1 \ldots Y_r \in P$ is applied, and let $k_1, \ldots, k_r \in K$ be the corresponding successor nodes. The attribute equation system E_{k_0} of k_0 is obtained from E_π by substituting every attribute index $i \in \{0, \ldots, r\}$ by k_i.
- The attribute equation system of t is given by
$$E_t := \bigcup \{E_k \mid k \text{ inner node of } t\}.$$

Solvability of Attribute Equation System

Definition (Solution of attribute equation system)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G. A solution of E_t is a mapping

$$v : Var_t \to V$$

such that, for every $\alpha_0.k_0 \in Var_t$ and $\alpha_0.k_0 = f(\alpha_1.k_1, \ldots, \alpha_n.k_n) \in E_t$,

$$v(\alpha_0.k_0) = f(v(\alpha_1.k_1), \ldots, v(\alpha_n.k_n)).$$

In general, the attribute equation system E_t of a given syntax tree t can have

- no solution,
- exactly one solution, or
- several solutions.

Circularity of Attribute Grammars

Goal: unique solvability of equation system
$\Longrightarrow$ avoid cyclic dependencies

Definition (Circularity)

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is called circular if there exists a syntax tree t such that the attribute equation system E_t is recursive (i.e., some attribute variable of t depends on itself). Otherwise it is called noncircular.

Remark: because of the division of Var_π into Int_π and Ext_π, cyclic dependencies cannot occur at production level.

Attribute Dependency Graphs I

Goal: graphical representation of attribute dependencies

Definition (Production dependency graph)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$. Every production $\pi \in P$ determines the dependency graph $D_\pi := \langle Var_\pi, \to_\pi \rangle$ where the set of edges $\to_\pi \subseteq Var_\pi \times Var_\pi$ is given by

$$x \to_\pi y \quad \text{iff} \quad y = f(\ldots, x, \ldots) \in E_\pi.$$

Corollary

The dependency graph of a production is acyclic (since $\to_\pi \subseteq Ext_\pi \times Int_\pi$ and $Ext_\pi \cap Int_\pi = \emptyset$).

Attribute Dependency Graphs II

Just as the attribute equation system E_t of a syntax tree t is obtained from the semantic rules of the contributing productions, the dependency graph of t is obtained by "glueing together" the dependency graphs of the productions.

Definition (Tree dependency graph)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$, and let t be a syntax tree of G.

- The dependency graph of t is defined by $D_t := \langle Var_t, \to_t \rangle$ where the set of edges, $\to_t \subseteq Var_t \times Var_t$, is given by

$$x \to_t y \quad \text{iff} \quad y = f(\ldots, x, \ldots) \in E_t.$$

- D_t is called cyclic if there exists $x \in Var_t$ such that $x \to_t^+ x$.

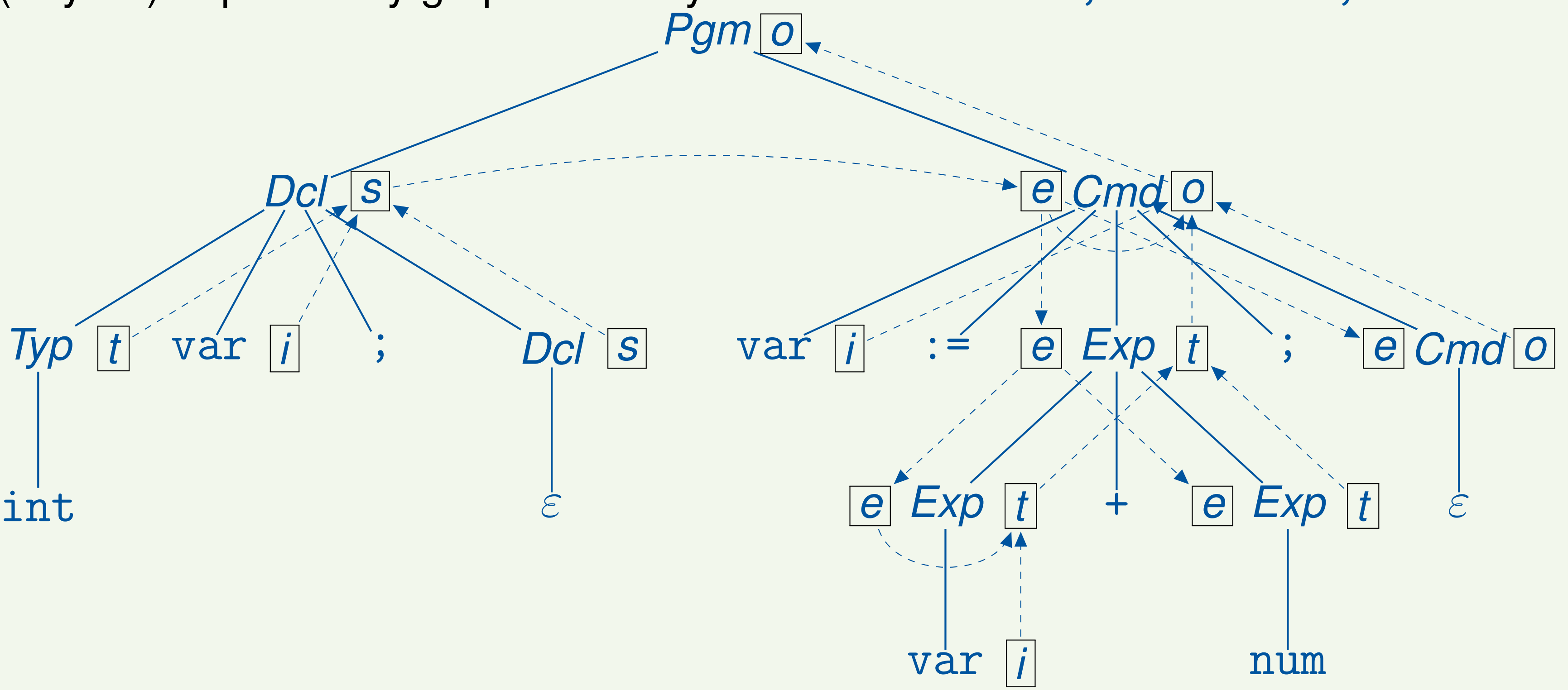
Corollary

An attribute grammar $\mathfrak{A} = \langle G, E, V \rangle \in AG$ is circular iff there exists a syntax tree t of G such that D_t is cyclic.

Attribute Dependency Graphs III

Example (cf. Example 12.1)

(Acyclic) dependency graph of the syntax tree for `int x; x := x+1;`:



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Attribute Dependency Graphs and Circularity I

Observation: a cycle in the dependency graph D_t of a given syntax tree t is caused by the occurrence of a "cover" production $\pi = A_0 \to w_0 A_1 w_1 \dots A_r w_r \in P$ in a node k_0 of t such that

- the dependencies in E_{k_0} yield the "upper end" of the cycle and
- for at least one $i \in [r]$, some attributes in $\mathrm{syn}(A_i)$ depend on attributes in $\mathrm{inh}(A_i)$.

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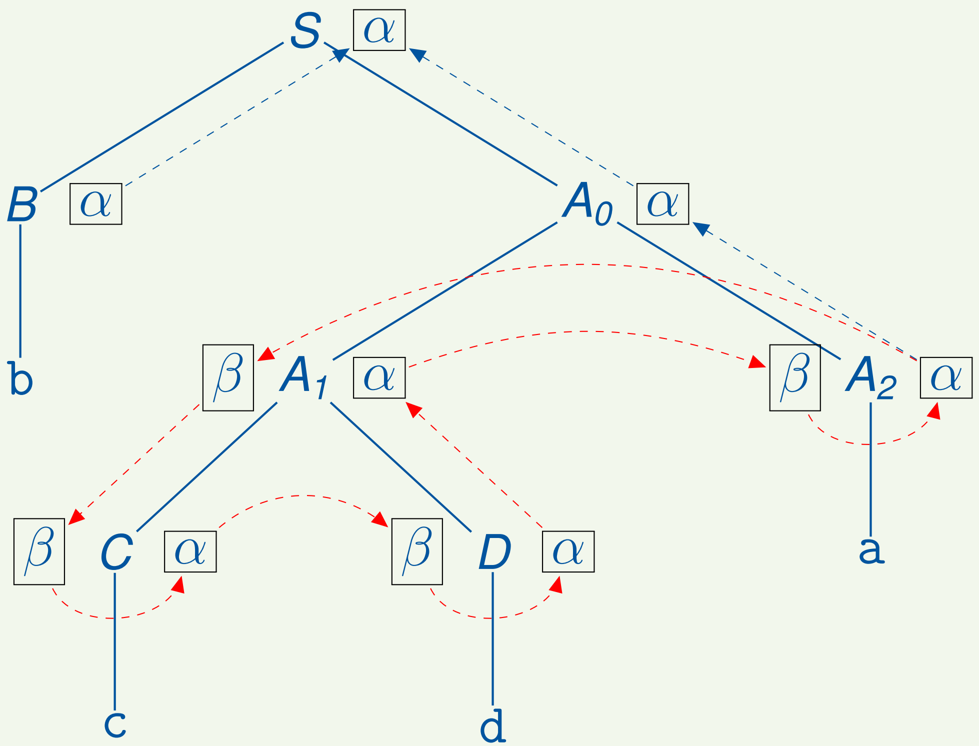
- for at least one $i \in [r]$, some attributes in $\mathrm{syn}(A_i)$ depend on attributes in $\mathrm{inh}(A_i)$.

To identify such "critical" situations we need to determine for each $i \in [r]$ the possible ways in which attributes in $\mathrm{syn}(A_i)$ can depend on attributes in $\mathrm{inh}(A_i)$.

Attribute Dependency Graphs and Circularity II

Example 13.1

Typical situation (with “cover” production $\pi = A_0 \rightarrow A_1 A_2 \in P$):



Attribute Dependency Graphs and Circularity III

Definition 13.2 (Attribute dependence)

Let $\mathfrak{A} = \langle G, E, V \rangle \in AG$ with $G = \langle N, \Sigma, P, S \rangle$.

- If t is a syntax tree with root label $A \in N$ and root node k, $\alpha \in \mathrm{syn}(A)$, and $\beta \in \mathrm{inh}(A)$ such that $\beta.k \to_t^+ \alpha.k$, then α is dependent on β below A in t (notation: $\beta \overset{A}{\hookrightarrow} \alpha$).

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- For every syntax tree t with root label $A \in N$,

$$is(A, t) := \{(\beta, \alpha) \in \mathrm{inh}(A) \times \mathrm{syn}(A) \mid \beta \overset{A}{\hookrightarrow} \alpha \text{ in } t\}.$$

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$$IS(A) := \{is(A, t) \mid t \text{ syntax tree with root label } A\} \subseteq 2^{Inh \times Syn}.$$

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Example 13.3

on the board

Outline of Lecture 13

The Circularity Check I

In the circularity check, the dependency systems $IS(A)$ are iteratively computed. The following notation is employed:

Definition 13.4

Given $\pi = A \to w_0 A_1 w_1 \dots A_r w_r \in P$ and $is_i \subseteq \mathrm{inh}(A_i) \times \mathrm{syn}(A_i)$ for each $i \in [r]$,

$$is[\pi; is_1, \dots, is_r] \subseteq \mathrm{inh}(A) \times \mathrm{syn}(A)$$

is defined by

$$is[\pi; is_1, \dots, is_r] :=$$
$$\left\{(\beta, \alpha) \mid (\beta.0, \alpha.0) \in \left(\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta'.p_i, \alpha'.p_i) \mid (\beta', \alpha') \in is_i\}\right)^+\right\}$$

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on the board

The Circularity Check II

Algorithm 13.6 (Circularity check for attribute grammars)

Input: $\mathfrak{A} = \langle G, E, V \rangle \in AG$ *with* $G = \langle N, \Sigma, P, S \rangle$

The Circularity Check II

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Procedure: 1. *for every* $A \in N$, *iteratively construct* $IS(A)$ *as follows:*

i. *if* $\pi = A \to w \in P$, *then*

$$is[\pi] \in IS(A)$$

ii. *if* $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$, *then*

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$$is[\pi; is_1, \ldots, is_r] \in IS(A)$$

2. test whether $\mathfrak{A}$ is circular by checking if there exist $\pi = A \to w_0 A_1 w_1 \ldots A_r w_r \in P$ and $is_i \in IS(A_i)$ for every $i \in [r]$ such that the following relation is cyclic:
$$\to_\pi \cup \bigcup_{i=1}^{r} \{(\beta.p_i, \alpha.p_i) \mid (\beta, \alpha) \in is_i\}$$
(where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$)

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2. *test whether* $\mathfrak{A}$ *is circular by checking if there exist* $\pi = A \to w_0 A_1 w_1 \dots A_r w_r \in P$ *and* $is_i \in IS(A_i)$ *for every* $i \in [r]$ *such that the following relation is cyclic:*

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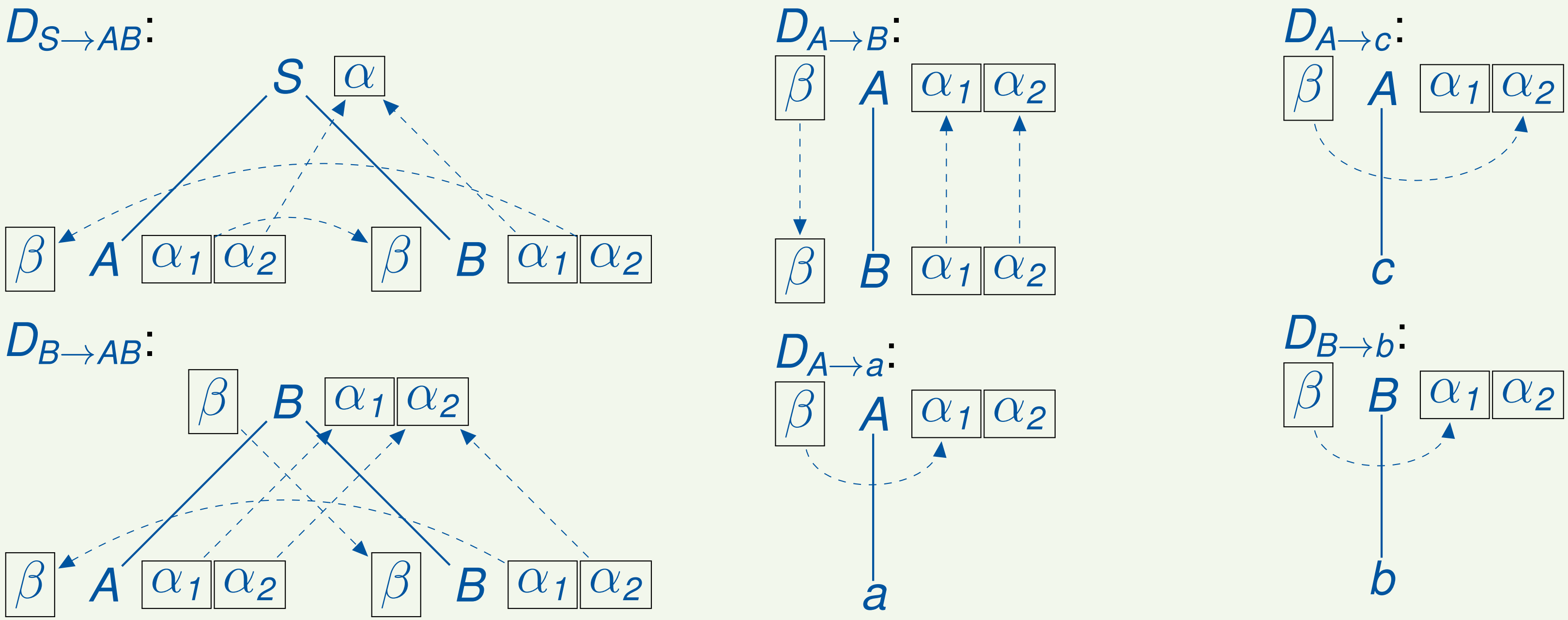
(where $p_i := \sum_{j=1}^{i} |w_{j-1}| + i$*)*

Output: *"yes" or "no"*

The Circularity Check III

Example 13.7



Application of Algorithm 13.6: on the board

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Theorem 13.8 (Correctness of circularity check)

An attribute grammar is circular iff Algorithm 13.6 yields the answer "yes"

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Lemma 13.9

The time complexity of the circularity check is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

Correctness and Complexity of Circularity Check

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Lemma 13.9

The time complexity of the circularity check is exponential in the size of the attribute grammar (= maximal length of right-hand sides of productions).

Proof.

by reduction of the word problem of alternating Turing machines (see M. Jazayeri: *A Simpler Construction for Showing the Intrinsically Exponential Complexity of the Circularity Problem for Attribute Grammars*, Comm. ACM 28(4), 1981, pp. 715–720) □