Compiler Construction

Lecture 11: Syntax Analysis VII (Practical Issues)

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Outline of Lecture 11

Recap: $LR(1)$ Parsing

$LR(1)$ Items and Sets

Observation: not every element of $\mathrm{fo}(A)$ can follow every occurrence of A

$\Longrightarrow$ refinement of $LR(0)$ items by adding possible lookahead symbols

Definition ($LR(1)$ items and sets)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ be start separated by $S' \to S$.

- If $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha\beta_1\beta_2 a w$, then $[A \to \beta_1 \cdot \beta_2, a]$ is called an $LR(1)$ item for $\alpha\beta_1$.
- If $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha\beta_1\beta_2$, then $[A \to \beta_1 \cdot \beta_2, \varepsilon]$ is called an $LR(1)$ item for $\alpha\beta_1$.
- Given $\gamma \in X^*$, $LR(1)(\gamma)$ denotes the set of all $LR(1)$ items for γ, called the $LR(1)$ set (or: $LR(1)$ information) of γ.
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$.

$LR(1)$ Conflicts

Definition ($LR(1)$ conflicts)

Let $G = \langle N, \Sigma, P, S \rangle \in CFG_\Sigma$ and $I \in LR(1)(G)$.

- I has a shift/reduce conflict if there exist $A \to \alpha_1 a \alpha_2, B \to \beta \in P$ and $x \in \Sigma_\varepsilon$ such that

$$[A \to \alpha_1 \cdot a\alpha_2, x], [B \to \beta\cdot, a] \in I.$$

- I has a reduce/reduce conflict if there exist $x \in \Sigma_\varepsilon$ and $A \to \alpha, B \to \beta \in P$ with $A \neq B$ or $\alpha \neq \beta$ such that

$$[A \to \alpha\cdot, x], [B \to \beta\cdot, x] \in I.$$

Lemma

$G \in LR(1)$ iff no $I \in LR(1)(G)$ contains conflicting items.

The $LR(1)$ Action Function

Definition ($LR(1)$ action function)

The $LR(1)$ action function

$$\mathrm{act} : LR(1)(G) \times \Sigma_\varepsilon \to \{\text{red } i \mid i \in [p]\} \cup \{\text{shift}, \text{accept}, \text{error}\}$$

is defined by

$$\mathrm{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \to \alpha \text{ and } [A \to \alpha\cdot, x] \in I \\ \text{shift} & \text{if } [A \to \alpha_1 \cdot x\alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \to S\cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Corollary

For every $G \in CFG_\Sigma$, $G \in LR(1)$ iff its $LR(1)$ action function is well defined.

Outline of Lecture 11

$LALR(1)$ Parsing

- **Motivation:** resolving conflicts using $LR(1)$ too expensive
- Example 10.11/10.17: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$

$LALR(1)$ Parsing

- **Motivation:** resolving conflicts using $LR(1)$ too expensive
- Example 10.11/10.17: $|LR(0)(G_{LR})| = 11$, $|LR(1)(G_{LR})| = 15$
- Empirical evaluations:
  - A. Johnstone, E. Scott: *Generalised Reduction Modified LR Parsing for Domain Specific Language Prototyping*, HICSS'02, IEEE, 2002
  - X. Chen, D. Pager: *Full LR(1) Parser Generator Hyacc and Study on the Performance of LR(1) Algorithms*, C3S2E'11, ACM, 2011

Grammar	$\|LR(0)(G)\|$	$\|LR(1)(G)\|$
Pascal	368	1395
Ansi-C	381	1788
C++	1236	9723

$LR(0)$ Equivalence

Observation: potential redundancy by containment of $LR(0)$ sets in $LR(1)$ sets (cf. Corollary 10.13)

Definition 11.1 ($LR(0)$ equivalence)

Let $\mathrm{lr}_0 : LR(1)(G) \to LR(0)(G)$ be defined by

$$\mathrm{lr}_0(I) := \{[A \to \beta_1 \cdot \beta_2] \mid [A \to \beta_1 \cdot \beta_2, x] \in I\}.$$

Two sets $I_1, I_2 \in LR(1)(G)$ are called $LR(0)$-equivalent (notation: $I_1 \sim_0 I_2$) if $\mathrm{lr}_0(I_1) = \mathrm{lr}_0(I_2)$.

$LALR(1)$ Sets

Corollary 11.2

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

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Idea: merge $LR(0)$-equivalent $LR(1)$ sets
(maintaining the lookahead information, but possibly introducing conflicts)

Definition 11.3 ($LALR(1)$ sets)

Let $G \in CFG_\Sigma$.

- An information $I \in LR(1)(G)$ determines the $LALR(1)$ set

$$\bigcup [I]_{\sim_0} = \bigcup \{I' \in LR(1)(G) \mid I' \sim_0 I\}.$$

- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

$LALR(1)$ Sets

Corollary 11.2

For every $G \in CFG_\Sigma$, $|LR(1)(G)/\sim_0| = |LR(0)(G)|$.

Idea: merge $LR(0)$-equivalent $LR(1)$ sets
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- The set of all $LALR(1)$ sets of G is denoted by $LALR(1)(G)$.

Remark: by Corollary 11.2, $|LALR(1)(G)| = |LR(0)(G)|$
(but $LALR(1)$ sets provide additional lookahead information)

$LALR(1)$ Parsing

Sketch of $LALR(1)$ Parsing

- $LALR(1)$ action function

$$\mathrm{act} : LALR(1)(G) \times \Sigma_\varepsilon \to \{\mathrm{red}\, i \mid i \in [p]\} \cup \{\mathrm{shift}, \mathrm{accept}, \mathrm{error}\}$$

  defined in analogy to the $LR(1)$ case (Definition 10.18)
- $G \in CFG_\Sigma$ has the $LALR(1)$ property ($G \in LALR(1)$) if its $LALR(1)$ action function is well defined
- Also $LR(1)$ goto function (Definition 10.20) carries over to the $LALR(1)$ case:

$$\mathrm{goto} : LALR(1)(G) \times X \to LALR(1)(G)$$

- act and goto form the $LALR(1)$ parsing table
- **But:** merging of $LR(1)$ sets can produce new conflicts
- $LALR(1)$ used by `yacc`/`bison` parser generator (later)

Outline of Lecture 11

Ambiguous Grammars

Reminder (Definition 5.5): $G \in CFG_\Sigma$ is called unambiguous if every word $w \in L(G)$ has exactly one syntax tree. Otherwise it is called ambiguous.

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Proof.

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Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.
Hence there exists $w \in L(G)$ with different right derivations. Let αAv be the last common sentence of the two derivations (i.e., $\beta \neq \gamma$):

$$S \Rightarrow_r^* \alpha Av \begin{cases} \Rightarrow_r \alpha\beta v \Rightarrow_r^* w \\ \Rightarrow_r \alpha\gamma v \Rightarrow_r^* w \end{cases}$$

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Proof.

Assume that there exist $k \in \mathbb{N}$ and $G \in LR(k)$ such that G is ambiguous.
Hence there exists $w \in L(G)$ with different right derivations. Let $\alpha A v$ be the last common sentence of the two derivations (i.e., $\beta \neq \gamma$):

$$S \Rightarrow_r^* \alpha A v \begin{cases} \Rightarrow_r \alpha\beta v \Rightarrow_r^* w \\ \Rightarrow_r \alpha\gamma v \Rightarrow_r^* w \end{cases}$$

But since $\mathrm{first}_k(v) = \mathrm{first}_k(v)$ for every $v \in \Sigma^*$, Definition 9.3 yields $\beta = \gamma$. $\lightning$ □

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But since $\mathrm{first}_k(v) = \mathrm{first}_k(v)$ for every $v \in \Sigma^*$, Definition 9.3 yields $\beta = \gamma$. $\lightning$ □

However ambiguity is a natural specification method which generally avoids involved syntactic constructs.

Bottom-Up Parsing of Ambiguous Grammars I

Example 11.5 (Simple arithmetic expressions)

$G : E' \to E \quad (0) \qquad E \to E{+}E \mid E{*}E \mid \mathrm{a} \quad (1, 2, 3)$

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$LR(0)(G)$:

$$
\begin{aligned}
I_0 &:= LR(0)(\varepsilon) : && [E' \to \cdot E] && [E \to \cdot E{+}E] && [E \to \cdot E{*}E] && [E \to \cdot \mathtt{a}] \\
I_1 &:= LR(0)(E) : && [E' \to E\cdot] && [E \to E \cdot {+}E] && [E \to E \cdot {*}E] \\
I_2 &:= LR(0)(\mathtt{a}) : && [E \to \mathtt{a}\cdot] \\
I_3 &:= LR(0)(E{+}) : && [E \to E{+} \cdot E] && [E \to \cdot E{+}E] && [E \to \cdot E{*}E] && [E \to \cdot \mathtt{a}] \\
I_4 &:= LR(0)(E{*}) : && [E \to E{*} \cdot E] && [E \to \cdot E{+}E] && [E \to \cdot E{*}E] && [E \to \cdot \mathtt{a}] \\
I_5 &:= LR(0)(E{+}E) : && [E \to E{+}E\cdot] && [E \to E \cdot {+}E] && [E \to E \cdot {*}E] \\
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Conflicts: I_1: $SLR(1)$-solvable (reduce on ε, shift on $+/*$)

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I_5, I_6: not $SLR(1)$-solvable ($+, * \in \mathrm{fo}(E)$)

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Solution: I_5: $* > + \implies \mathrm{act}(I_5, *) :=$ shift, + left assoc. $\implies \mathrm{act}(I_5, +) :=$ red 1

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I_6: $* > + \implies \mathrm{act}(I_6, +) :=$ red 2, $*$ left assoc. $\implies \mathrm{act}(I_6, *) :=$ red 2

Bottom-Up Parsing of Ambiguous Grammars II

Example 11.6 (“Dangling `else`”)

$G : S' \to S \quad S \to \texttt{i}S\texttt{e}S \mid \texttt{i}S \mid \texttt{a}$

Bottom-Up Parsing of Ambiguous Grammars II

Example 11.6 (“Dangling `else`”)

$G : S' \to S \quad S \to \texttt{i}S\texttt{e}S \mid \texttt{i}S \mid \texttt{a}$

Ambiguity: `iiaea` := (1) `i(iaea)` (common) or (2) `i(ia)ea`

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$I_6 := LR(0)(\mathtt{i}S\mathtt{e}S)$: $[S \to \mathtt{i}S\mathtt{e}S\cdot]$

Conflict in I_4: $\mathtt{e} \in \mathrm{fo}(S) \implies$ not $SLR(1)$-solvable

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$I_4 := LR(0)(\mathtt{i}S) : \quad [S \to \mathtt{i}S \cdot \mathtt{e}S]\,[S \to \mathtt{i}S\cdot]$

$I_5 := LR(0)(\mathtt{i}S\mathtt{e}) : \quad [S \to \mathtt{i}S\mathtt{e} \cdot S]\,[S \to \cdot \mathtt{i}S\mathtt{e}S]\,[S \to \cdot \mathtt{i}S]\,[S \to \cdot \mathtt{a}]$

$I_6 := LR(0)(\mathtt{i}S\mathtt{e}S) : \quad [S \to \mathtt{i}S\mathtt{e}S\cdot]$

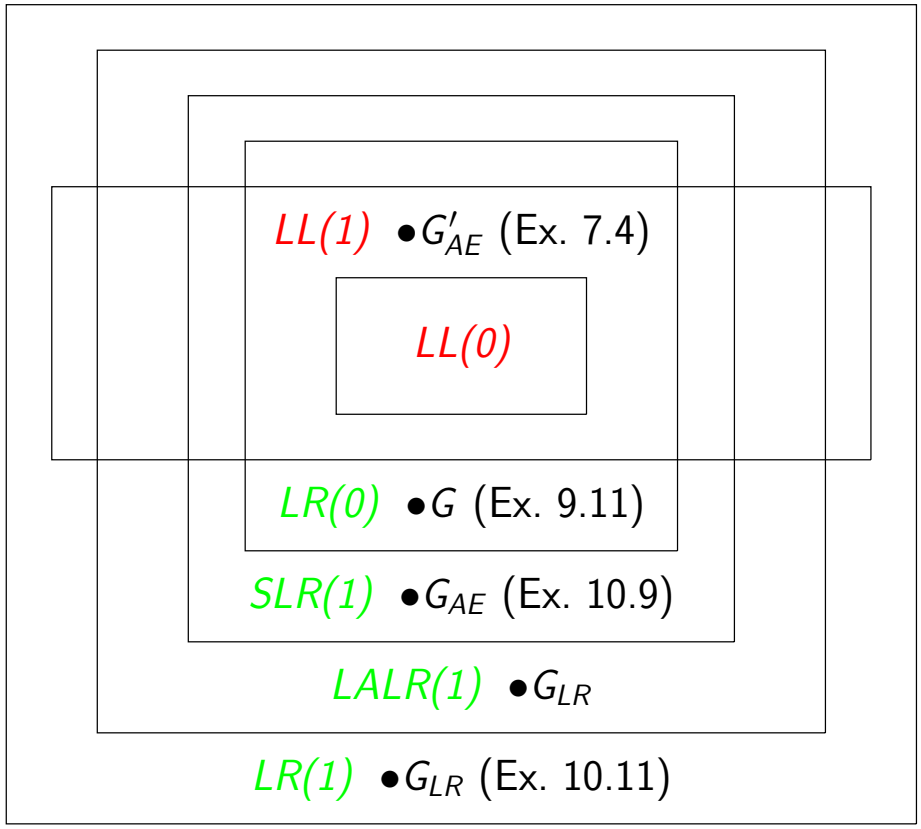
Conflict in I_4: $\mathtt{e} \in \mathrm{fo}(S) \implies$ not $SLR(1)$-solvable

Solution (1): $\mathrm{act}(I_4, \mathtt{e})$:= shift

Outline of Lecture 11

Expressiveness of LL and LR Grammars

Overview of Grammar Classes



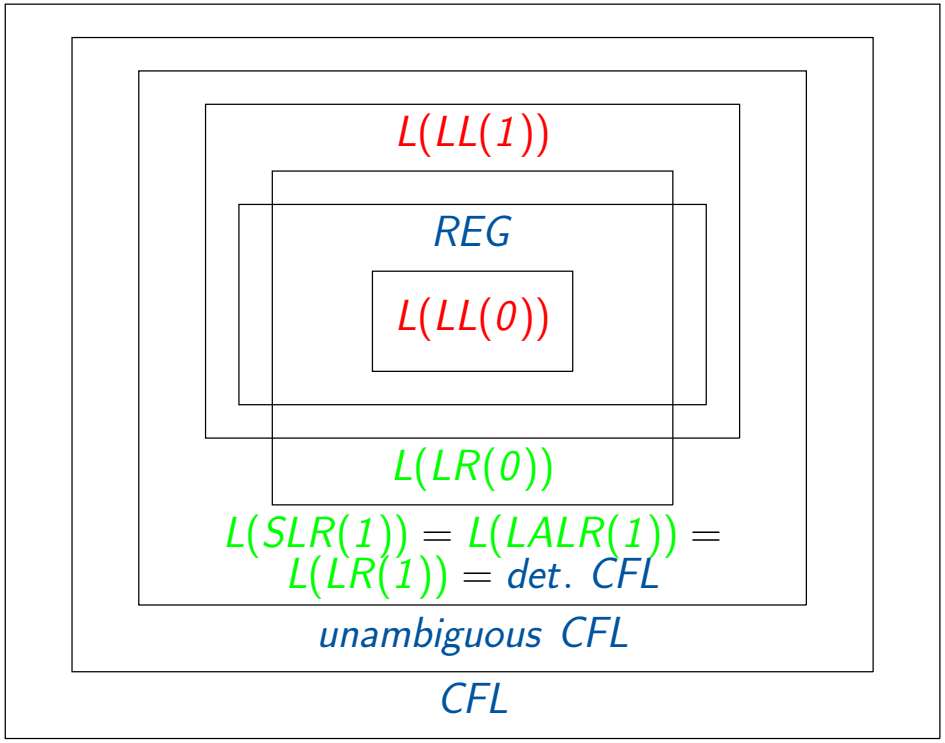
Moreover:

- $LL(k) \subsetneq LL(k+1)$ for every $k \in \mathbb{N}$
- $LR(k) \subsetneq LR(k+1)$ for every $k \in \mathbb{N}$
- $LL(k) \subseteq LR(k)$ for every $k \in \mathbb{N}$

Expressiveness of LL and LR Grammars

Overview of Language Classes

(cf. O. Mayer: *Syntaxanalyse*, BI-Verlag, 1978, p. 409ff)



Moreover:

- $L(LL(k)) \subsetneq L(LL(k+1)) \subsetneq L(LR(1))$ for every $k \in \mathbb{N}$
- $L(LR(k)) = L(LR(1))$ for every $k \geq 1$

Prefix-Free Languages

Definition 11.7

A language $L \subseteq \Sigma^*$ is called prefix-free if $L \cdot \Sigma^+ \cap L = \emptyset$, i.e., if no proper prefix of an element of L is again in L.

Expressiveness of LL and LR Grammars

Why $REG \not\subseteq L(LR(0))$?

Lemma 11.8

$G \in LR(0) \implies L(G)$ *prefix-free*

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Proof.

Assumption: $G \in LR(0)$ and $L(G)$ not prefix-free. Then there ex. $v \in \Sigma^*$ and $w \in \Sigma^+$ such that $v, vw \in L(G)$.

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Proof.

Assumption: $G \in LR(0)$ and $L(G)$ not prefix-free. Then there ex. $v \in \Sigma^*$ and $w \in \Sigma^+$ such that $v, vw \in L(G)$. Thus, the $LR(0)$ parsing automaton for G (Def. 10.2) has an accepting run on v:

$$(v, I_0, \varepsilon) \vdash^* (\varepsilon, I_0 I, z) \vdash (\varepsilon, \varepsilon, z\,0) \quad \text{where } \mathrm{act}(I) = \text{accept}$$

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... and an accepting run on vw (deterministic!):

$$(vw, I_0, \varepsilon) \vdash^* (w, I_0 I, z) \vdash^+ (\varepsilon, I_0 I', zz') \vdash (\varepsilon, \varepsilon, zz'\,0) \quad \text{where } \mathrm{act}(I') = \text{accept}$$

which contradicts $\mathrm{act}(I) = \text{accept}$. ↯ □

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Corollary 11.9

$\{a, aa\} \in REG \setminus L(LR(0))$

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which contradicts $\mathrm{act}(I) = \text{accept}$. ↯ □

Corollary 11.9

$\{a, aa\} \in REG \setminus L(LR(0))$

Conjecture: $L \in REG \setminus L(LR(0)) \implies L(G)$ not prefix-free?

Expressiveness of LL and LR Grammars

Why $REG \subseteq L(LL(1))$?

Lemma 11.10 (cf. Lecture 2)

Every $L \in REG$ can be recognized by a DFA.

Expressiveness of LL and LR Grammars

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Every DFA can be transformed into an equivalent $LL(1)$ grammar.

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Lemma 11.11

Every DFA can be transformed into an equivalent $LL(1)$ grammar.

Proof.

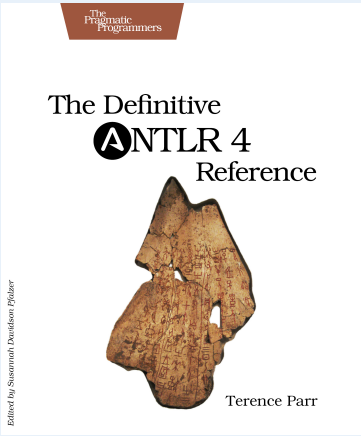
see Exercise 4.2 □

Outline of Lecture 11

Overview of ANTLR

ANother Tool for Language Recognition

- Input: language description using EBNF grammars
- Output: recogniser for the language
- Supports recognisers for three kinds of input:
  - character streams (generation of scanner)
  - token streams (generation of parser)
  - node streams (generation of tree walker)
- Current version: ANTLR 4.7.1
  - generates $LL(*)$ recognisers: flexible choice of lookahead length
  - applies “longest match” principle
  - supports ambiguous grammars by using “first match” principle for rules
  - supports direct left recursion
  - targets Java, C++, C#, Python, JavaScript, Go, Swift
- Details:
  - http://www.antlr.org/
  - T. Parr: *The Definitive ANTLR 4 Reference*, Pragmatic Bookshelf, 2013
  - 3rd Programming Exercise



Example: Infix → Postfix Translator (Simple.g4)

```
grammar Simple;

// productions for syntax analysis
program returns [String s]: e=expr EOF {$s = $e.s;};
expr returns [String s]: t=term r=rest {$s = $t.s + $r.s;};
rest returns [String s]:
  PLUS t=term r=rest                   {$s = $t.s + "+" + $r.s;}
| MINUS t=term r=rest                  {$s = $t.s + "-" + $r.s;}
| /* empty */                          {$s = "";};
term returns [String s]: DIGIT         {$s = $DIGIT.text;};

// productions for lexical analysis
PLUS : '+';
MINUS : '-';
DIGIT : [0-9];
```

Java Code for Using Translator

```
import java.io.*;
import org.antlr.v4.runtime.*;
public class SimpleMain {
  public static void main(final String[] args)
  throws IOException {
    String printSource = null, printSymTab = null,
      printIR = null, printAsm = null;
    SimpleLexer lexer =  /* Create instance of lexer */
      new SimpleLexer(new ANTLRInputStream(args[0]));
    SimpleParser parser =  /* Create instance of parser */
      new SimpleParser(new CommonTokenStream(lexer));
    String postfix = parser.program().s;  /* Run translator */
    System.out.println(postfix);
  }
}
```

An Example Run

1. After installation, invoke ANTLR:

   ```
   $ java -jar /usr/local/lib/antlr-4.7.1-complete.jar Simple.g4
   ```

   (will generate SimpleLexer.java, SimpleParser.java, Simple.tokens, and SimpleLexer.tokens)
2. Use Java compiler:

   ```
   $ javac -cp /usr/local/lib/antlr-4.7.1-complete.jar Simple*.java
   ```
3. Run translator:

   ```
   $ java -cp .:/usr/local/lib/antlr-4.7.1-complete.jar SimpleMain '9-5+2'
   95-2+
   ```

Advantages of ANTLR

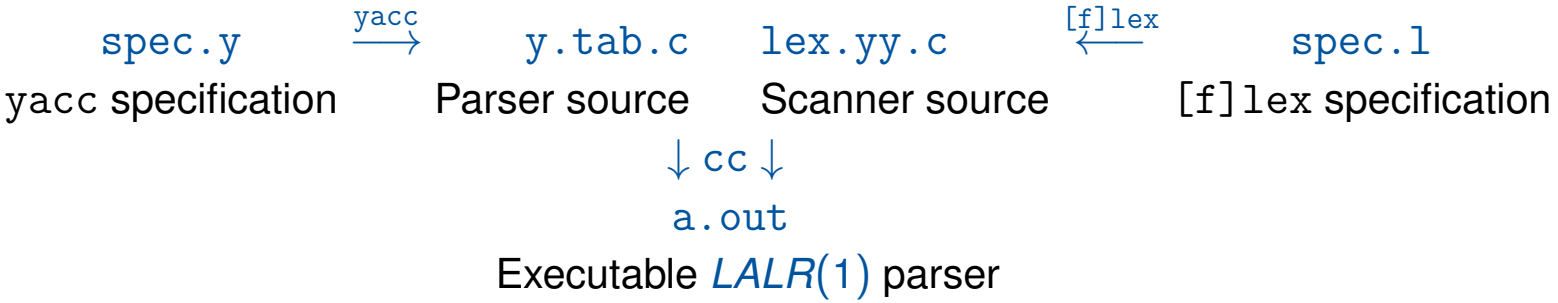
Advantages of ANTLR

- Generated (Java) code is similar to hand-written code
  - possible (and easy) to read and debug generated code
- Syntax for specifying scanners, parsers and tree walkers is identical
- Support for many target programming languages
- ANTLR is well supported and has an active user community

Outline of Lecture 11

The `yacc` and `bison` Tools

Usage of `yacc` ("yet another compiler compiler"):



Generating Bottom-Up Parsers Using `yacc` and `bison`

The `yacc` and `bison` Tools

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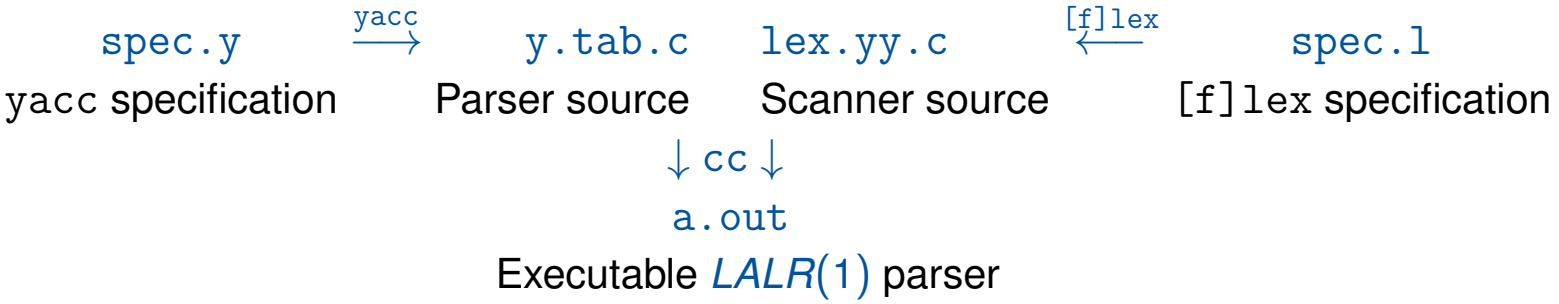

Like for `[f]lex`, a `yacc` specification is of the form

Declarations (optional)
%%
Rules
%%
Auxiliary procedures (optional)

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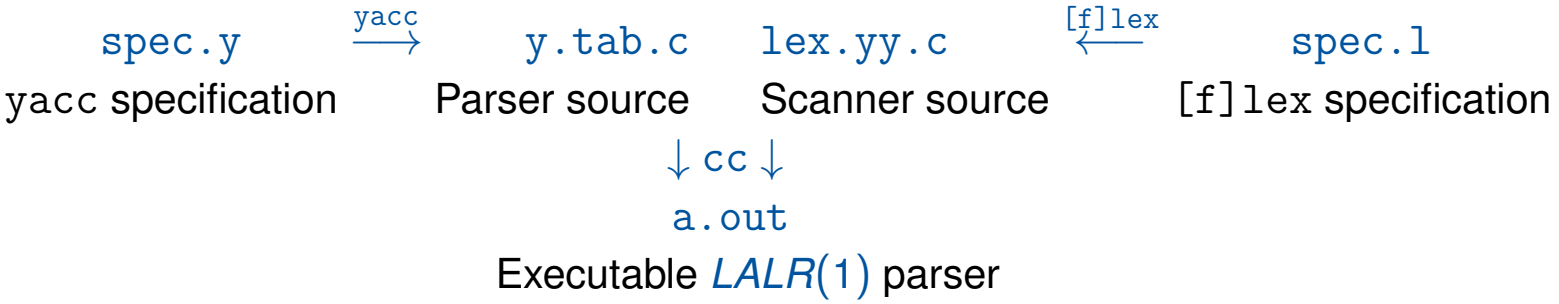

Like for `[f]lex`, a `yacc` specification is of the form

Declarations (optional)
%%
Rules
%%
Auxiliary procedures (optional)

`bison`: upward-compatible GNU implementation of `yacc`
(more flexible w.r.t. file names, ...)

`yacc` Specifications

Declarations:
- Token definitions: `%token` *Tokens*
  - Not every token needs to be declared (`'+'`, `'='`, `...`)
  - Start symbol: `%start` *Symbol* (optional)
  - C code for declarations etc.: `%{` *Code* `%}`

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Rules: context-free productions and semantic actions
- $A \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$ represented as

$$\begin{array}{lll} A & : & \alpha_1 \ \{Action_1\} \\ & \mid & \alpha_2 \ \{Action_2\} \\ & \vdots & \\ & \mid & \alpha_n \ \{Action_n\}\,; \end{array}$$

  - Semantic actions = C statements for computing attribute values
  - `$$` = attribute value of A
  - `$i` = attribute value of i-th symbol on right-hand side
  - Default action: `$$ = $1`

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  - Semantic actions = C statements for computing attribute values
  - `$$` = attribute value of A
  - `$i` = attribute value of i-th symbol on right-hand side
  - Default action: `$$ = $1`

Auxiliary procedures: scanner (if not generated by `[f]lex`), error routines, ...

Example: Simple Desk Calculator I

```
%{/* SLR(1) grammar for arithmetic expressions (Example 10.5) */
  #include <stdio.h>
  #include <ctype.h>
%}
%token DIGIT
%%
line   : expr '\n'          { printf("%d\n", $1); };
expr   : expr '+' term      { $$ = $1 + $3; }
       | term               { $$ = $1; };
term   : term '*' factor    { $$ = $1 * $3; }
       | factor             { $$ = $1; };
factor : '(' expr ')'       { $$ = $2; }
       | DIGIT              { $$ = $1; };
%%
yylex() {
  int c;
  c = getchar();
  if (isdigit(c)) yylval = c - '0'; return DIGIT;
  return c;
}
```

Example: Simple Desk Calculator II

```
$ yacc calc.y
$ cc y.tab.c -ly
$ a.out
2+3
5
$ a.out
2+3*5
17
```

An Ambiguous Grammar I

```
%{/* Ambiguous grammar for arithmetic expressions (Example 11.5) */
  #include <stdio.h>
  #include <ctype.h>
%}
%token DIGIT
%%
line   : expr '\n'          { printf("%d\n", $1); };
expr   : expr '+' expr      { $$ = $1 + $3; }
       | expr '*' expr      { $$ = $1 * $3; }
       | DIGIT              { $$ = $1; };
%%
yylex() {
  int c;
  c = getchar();
  if (isdigit(c)) {yylval = c - '0'; return DIGIT;}
  return c;
}
```

An Ambiguous Grammar II

Invoking yacc with the option -v produces a report y.output:

```
State 8
    2 expr: expr . '+' expr
    2     | expr '+' expr .
    3     | expr . '*' expr

    '+'  shift and goto state 6
    '*'  shift and goto state 7
    '+'       [reduce with rule 2 (expr)]
    '*'       [reduce with rule 2 (expr)]

State 9
    2 expr: expr . '+' expr
    3     | expr . '*' expr
    3     | expr '*' expr .

    '+'  shift and goto state 6
    '*'  shift and goto state 7
    '+'       [reduce with rule 3 (expr)]
    '*'       [reduce with rule 3 (expr)]
```

Conflict Handling in yacc

Default conflict resolving strategy in yacc:
reduce/reduce: choose first conflicting production in specification

Conflict Handling in yacc

Default conflict resolving strategy in yacc:

reduce/reduce: choose first conflicting production in specification

shift/reduce: prefer shift

- resolves dangling-else ambiguity (Example 11.6) correctly
- also adequate for strong following weak operator ($*$ after $+$; Example 11.5) and for right-associative operators
- not appropriate for weak following strong operator and for left-associative operators ($\Longrightarrow$ reduce; see Example 11.5)

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Default conflict resolving strategy in yacc:

reduce/reduce: choose first conflicting production in specification

shift/reduce: prefer shift

- resolves dangling-else ambiguity (Example 11.6) correctly
- also adequate for strong following weak operator (∗ after +; Example 11.5) and for right-associative operators
- not appropriate for weak following strong operator and for left-associative operators ($\Longrightarrow$ reduce; see Example 11.5)

For ambiguous grammar:

```
$ yacc ambig.y
conflicts: 4 shift/reduce
$ cc y.tab.c -ly
$ a.out
2+3*5
17
$ a.out
2*3+5
16
```

Precedences and Associativities in `yacc` I

General mechanism for resolving conflicts:

$$\%[\texttt{left}|\texttt{right}]\ \mathit{Operators}_1$$
$$\vdots$$
$$\%[\texttt{left}|\texttt{right}]\ \mathit{Operators}_n$$

- operators in one line have given associativity and same precedence
- precedence increases over lines

Precedences and Associativities in `yacc` I

General mechanism for resolving conflicts:

$$\%[\texttt{left}|\texttt{right}]\ \mathit{Operators}_1$$
$$\vdots$$
$$\%[\texttt{left}|\texttt{right}]\ \mathit{Operators}_n$$

- operators in one line have given associativity and same precedence
- precedence increases over lines

Example 11.12

```
%left '+' '-'
%left '*' '/'
%right '^'
```

`^` (right associative) binds stronger than `*` and `/` (left associative), which in turn bind stronger than `+` and `-` (left associative)

Precedences and Associativities in yacc II

```
%{/* Ambiguous grammar for arithmetic expressions
     with precedences and associativities */
  #include <stdio.h>
  #include <ctype.h>
%}
%token DIGIT
%left '+'
%left '*'
%%
line   : expr '\n'  { printf("%d\n", $1); };
expr   : expr '+' expr    { $$ = $1 + $3; }
       | expr '*' expr    { $$ = $1 * $3; }
       | DIGIT            { $$ = $1; };
%%
yylex() {
  int c;
  c = getchar();
  if (isdigit(c)) {yylval = c - '0'; return DIGIT;}
  return c;
}
```

Precedences and Associativities in yacc III

```
$ yacc ambig-prio.y
$ cc y.tab.c -ly
$ a.out
2*3+5
11
$ a.out
2+3*5
17
```