



# Compiler Construction

Lecture 10: Syntax Analysis VI ( $SLR(1)$  and  $LR(1)$  Parsing)

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Thomas Noll

Software Modeling and Verification Group

RWTH Aachen University

<https://moves.rwth-aachen.de/teaching/ws-1819/cc/>

# Recap: $LR(0)$ Parsing

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## Outline of Lecture 10

Recap:  $LR(0)$  Parsing

The  $LR(0)$  Parsing Automaton

$SLR(1)$  Parsing

Examples of  $SLR(1)$  Conflicts

$LR(1)$  Parsing

# Recap: LR(0) Parsing

## LR(0) Items and Sets

### Definition (LR(0) items and sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated by  $S' \rightarrow S$  and  $S' \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha \beta_1 \beta_2 w$  (i.e.,  $A \rightarrow \beta_1 \beta_2 \in P$ ).

- $[A \rightarrow \beta_1 \cdot \beta_2]$  is called an **LR(0) item** for  $\alpha \beta_1$ .
- Given  $\gamma \in X^*$ ,  $LR(0)(\gamma)$  denotes the set of all **LR(0)** items for  $\gamma$ , called the **LR(0) set** (or: **LR(0) information**) of  $\gamma$ .
- $LR(0)(G) := \{LR(0)(\gamma) \mid \gamma \in X^*\}$ .

### Corollary

1. For every  $\gamma \in X^*$ ,  $LR(0)(\gamma)$  is finite.
2.  $LR(0)(G)$  is finite.
3. The item  $[A \rightarrow \beta \cdot] \in LR(0)(\gamma)$  indicates the possible **reduction**  $(w, \alpha \beta, z) \vdash (w, \alpha A, zi)$  where  $\pi_i = A \rightarrow \beta$  and  $\gamma = \alpha \beta$ .
4. The item  $[A \rightarrow \beta_1 \cdot Y \beta_2] \in LR(0)(\gamma)$  indicates an **incomplete handle**  $\beta_1$  (to be completed by shift operations or  $\varepsilon$ -reductions).

# Recap: $LR(0)$ Parsing

## $LR(0)$ Conflicts

### Definition ( $LR(0)$ conflicts)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  and  $I \in LR(0)(G)$ .

- $I$  has a **shift/reduce conflict** if there exist  $A \rightarrow \alpha_1 a \alpha_2, B \rightarrow \beta \in P$  such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2], [B \rightarrow \beta \cdot] \in I.$$

- $I$  has a **reduce/reduce conflict** if there exist  $A \rightarrow \alpha, B \rightarrow \beta \in P$  with  $A \neq B$  or  $\alpha \neq \beta$  such that

$$[A \rightarrow \alpha \cdot], [B \rightarrow \beta \cdot] \in I.$$

### Lemma

$G \in LR(0)$  iff no  $I \in LR(0)(G)$  contains conflicting items.

### Proof.

omitted □

# Recap: LR(0) Parsing

## Computing LR(0) Sets

### Theorem (Computing LR(0) sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated by  $S' \rightarrow S$  and reduced.

1.  $LR(0)(\varepsilon)$  is the least set such that

- $[S' \rightarrow \cdot S] \in LR(0)(\varepsilon)$  and
- if  $[A \rightarrow \cdot B\gamma] \in LR(0)(\varepsilon)$  and  $B \rightarrow \beta \in P$ ,  
then  $[B \rightarrow \cdot \beta] \in LR(0)(\varepsilon)$ .

2.  $LR(0)(\alpha Y)$  ( $\alpha \in X^*$ ,  $Y \in X$ ) is the least set such that

- if  $[A \rightarrow \gamma_1 \cdot Y\gamma_2] \in LR(0)(\alpha)$ ,  
then  $[A \rightarrow \gamma_1 Y \cdot \gamma_2] \in LR(0)(\alpha Y)$  and
- if  $[A \rightarrow \gamma_1 \cdot B\gamma_2] \in LR(0)(\alpha Y)$  and  $B \rightarrow \beta \in P$ ,  
then  $[B \rightarrow \cdot \beta] \in LR(0)(\alpha Y)$ .

## Recap: $LR(0)$ Parsing

### The goto Function

**Observation:** if  $G \in LR(0)$ , then  $LR(0)(\gamma)$  yields **deterministic shift/reduce decision** for  $NBA(G)$  in a configuration with pushdown  $\gamma$

$\implies$  **new pushdown alphabet:**  $LR(0)(G)$  in place of  $X$

Moreover  $LR(0)(\gamma Y)$  is determined by  $LR(0)(\gamma)$  and  $Y$  but **independent from  $\gamma$**  in the following sense:

$$LR(0)(\gamma) = LR(0)(\gamma') \implies LR(0)(\gamma Y) = LR(0)(\gamma' Y)$$

### Definition ( $LR(0)$ goto function)

The function  $\text{goto} : LR(0)(G) \times X \rightarrow LR(0)(G)$  is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(0)(\gamma) \text{ and } I' = LR(0)(\gamma Y).$$

## Recap: $LR(0)$ Parsing

### The $LR(0)$ Action Function

The parsing automaton will be defined using another table, the **action function**, which determines the shift/reduce decision (reminder:  $\pi_0 = S' \rightarrow S$ ).

#### Definition ( $LR(0)$ action function)

The  **$LR(0)$  action function**  $\text{act} : LR(0)(G) \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$  is defined by

$$\text{act}(I) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot a\alpha_2] \in I \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \\ \text{error} & \text{if } I = \emptyset \end{cases}$$

#### Corollary

For every  $G \in CFG_{\Sigma}$ ,  $G \in LR(0)$  iff  $\text{act}$  is well defined.

Together,  $\text{act}$  and  $\text{goto}$  form the  **$LR(0)$  parsing table** of  $G$ .

# The $LR(0)$ Parsing Automaton

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# The $LR(0)$ Parsing Automaton

## The $LR(0)$ Parsing Table

### Example 10.1 (cf. Example 9.15)

$G: S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
 $B \rightarrow aB \mid b \quad (3, 4)$   
 $C \rightarrow aC \mid c \quad (5, 6)$

$l_0 := LR(0)(\varepsilon) : [S' \rightarrow \cdot S]$   
 $[S \rightarrow \cdot B] \quad [S \rightarrow \cdot C]$   
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C]$   
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

$l_9 := \emptyset$

# The LR(0) Parsing Automaton

## The LR(0) Parsing Table

### Example 10.1 (cf. Example 9.15)

$G: S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
 $B \rightarrow aB \mid b \quad (3, 4)$   
 $C \rightarrow aC \mid c \quad (5, 6)$

| $LR(0)(G)$ | act    | goto  |       |       |       |       |       |
|------------|--------|-------|-------|-------|-------|-------|-------|
|            |        | $S$   | $B$   | $C$   | $a$   | $b$   | $c$   |
| $l_0$      | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$      | accept |       |       |       |       |       |       |
| $l_2$      | red 1  |       |       |       |       |       |       |
| $l_3$      | red 2  |       |       |       |       |       |       |
| $l_4$      | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$      | red 4  |       |       |       |       |       |       |
| $l_6$      | red 6  |       |       |       |       |       |       |
| $l_7$      | red 3  |       |       |       |       |       |       |
| $l_8$      | red 5  |       |       |       |       |       |       |
| $l_9$      | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

$l_0 := LR(0)(\epsilon) : [S' \rightarrow \cdot S]$   
 $[S \rightarrow \cdot B] \quad [S \rightarrow \cdot C]$   
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_1 := LR(0)(S) : [S' \rightarrow S \cdot]$

$l_2 := LR(0)(B) : [S \rightarrow B \cdot]$

$l_3 := LR(0)(C) : [S \rightarrow C \cdot]$

$l_4 := LR(0)(a) : [B \rightarrow a \cdot B] \quad [C \rightarrow a \cdot C]$   
 $[B \rightarrow \cdot aB] \quad [B \rightarrow \cdot b]$   
 $[C \rightarrow \cdot aC] \quad [C \rightarrow \cdot c]$

$l_5 := LR(0)(b) : [B \rightarrow b \cdot]$

$l_6 := LR(0)(c) : [C \rightarrow c \cdot]$

$l_7 := LR(0)(aB) : [B \rightarrow aB \cdot]$

$l_8 := LR(0)(aC) : [C \rightarrow aC \cdot]$

$l_9 := \emptyset$

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton I

### Definition 10.2 (LR(0) parsing automaton)

Let  $G = \langle N, \Sigma, P, S \rangle \in LR(0)$ . The (deterministic) LR(0) parsing automaton of  $G$  is defined by the following components:

- Input alphabet  $\Sigma$
- Pushdown alphabet  $\Gamma := LR(0)(G)$
- Output alphabet  $\Delta := [\rho] \cup \{0, \text{error}\}$
- Configurations  $\Sigma^* \times \Gamma^* \times \Delta^*$
- Initial configuration  $(w, l_0, \varepsilon)$  where  $l_0 := LR(0)(\varepsilon)$
- Final configurations  $\{\varepsilon\} \times \{\varepsilon\} \times \Delta^*$
- Transitions:
  - shift:  $(aw, \alpha l, z) \vdash (w, \alpha l', z)$  if  $\text{act}(l) = \text{shift}$  and  $\text{goto}(l, a) = l'$
  - reduce:  $(w, \alpha l_1 \dots l_n, z) \vdash (w, \alpha l', zi)$  if  $\text{act}(l_n) = \text{red } i$ ,  $\pi_i = A \rightarrow Y_1 \dots Y_n$ ,  $\text{goto}(l, A) = l'$
  - accept:  $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$  if  $\text{act}(l) = \text{accept}$
  - error:  $(w, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$  if  $\text{act}(l) = \text{error}$

# The $LR(0)$ Parsing Automaton

## The $LR(0)$ Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G : S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
 $B \rightarrow aB \mid b \quad (3, 4)$   
 $C \rightarrow aC \mid c \quad (5, 6)$

| $LR(0)(G)$ | act    | goto  |       |       |       |       |       |
|------------|--------|-------|-------|-------|-------|-------|-------|
|            |        | $S$   | $B$   | $C$   | $a$   | $b$   | $c$   |
| $l_0$      | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$      | accept |       |       |       |       |       |       |
| $l_2$      | red 1  |       |       |       |       |       |       |
| $l_3$      | red 2  |       |       |       |       |       |       |
| $l_4$      | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$      | red 4  |       |       |       |       |       |       |
| $l_6$      | red 6  |       |       |       |       |       |       |
| $l_7$      | red 3  |       |       |       |       |       |       |
| $l_8$      | red 5  |       |       |       |       |       |       |
| $l_9$      | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G: S' \rightarrow S \quad (0)$   
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LR(0) parsing of *aac*:  
 (*aac*,  $l_0$ ,  $\epsilon$ )

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
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| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
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| $l_9$    | error  |       |       |       |       |       |       |

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# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:  
 $(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
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# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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LR(0) parsing of *aac*:

$(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$   
 $\vdash (c, l_0 l_4 l_4, \epsilon)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
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# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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LR(0) parsing of *aac*:

$(aac, l_0, \varepsilon)$   
 $\vdash (ac, l_0 l_4, \varepsilon)$   
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |  |
|----------|--------|-------|-------|-------|-------|-------|-------|--|
|          |        | S     | B     | C     | a     | b     | c     |  |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |  |
| $l_1$    | accept |       |       |       |       |       |       |  |
| $l_2$    | red 1  |       |       |       |       |       |       |  |
| $l_3$    | red 2  |       |       |       |       |       |       |  |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |  |
| $l_5$    | red 4  |       |       |       |       |       |       |  |
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# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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LR(0) parsing of *aac*:

$(aac, l_0, \varepsilon)$   
 $\vdash (ac, l_0 l_4, \varepsilon)$   
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
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# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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LR(0) parsing of *aac*:

$(aac, l_0, \varepsilon)$   
 $\vdash (ac, l_0 l_4, \varepsilon)$   
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$   
 $(*)$   
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
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| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

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$G: S' \rightarrow S \quad (0)$   
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LR(0) parsing of *aac*:

$(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$   
 $\vdash (c, l_0 l_4 l_4, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_8, 6)$   
 $(*)$   
 $\vdash (\epsilon, l_0 l_4 l_8, 65)$   
 $\vdash (\epsilon, l_0 l_3, 655)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
| $l_8$    | red 5  |       |       |       |       |       |       |
| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G: S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
 $B \rightarrow aB \mid b \quad (3, 4)$   
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
| $l_8$    | red 5  |       |       |       |       |       |       |
| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

$(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$   
 $\vdash (c, l_0 l_4 l_4, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_8, 6)$   
 (\*)  
 $\vdash (\epsilon, l_0 l_4 l_8, 65)$   
 $\vdash (\epsilon, l_0 l_3, 655)$   
 $\vdash (\epsilon, l_0 l_1, 6552)$

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G: S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
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| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
| $l_8$    | red 5  |       |       |       |       |       |       |
| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

$(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$   
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 $\vdash (\epsilon, l_0 l_1, 6552)$   
 $\vdash (\epsilon, \epsilon, 65520)$

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G: S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
 $B \rightarrow aB \mid b \quad (3, 4)$   
 $C \rightarrow aC \mid c \quad (5, 6)$

LR(0) parsing of *aac*:

$(aac, l_0, \varepsilon)$   
 $\vdash (ac, l_0 l_4, \varepsilon)$   
 $\vdash (c, l_0 l_4 l_4, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_6, \varepsilon)$   
 $\vdash (\varepsilon, l_0 l_4 l_4 l_8, 6)$   
 $(*)$   
 $\vdash (\varepsilon, l_0 l_4 l_8, 65)$   
 $\vdash (\varepsilon, l_0 l_3, 655)$   
 $\vdash (\varepsilon, l_0 l_1, 6552)$   
 $\vdash (\varepsilon, \varepsilon, 65520)$

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
| $l_8$    | red 5  |       |       |       |       |       |       |
| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

Check by rightmost derivation (on the board)

# The LR(0) Parsing Automaton

## The LR(0) Parsing Automaton II

### Example 10.3 (cf. Example 10.1)

$G : S' \rightarrow S \quad (0)$   
 $S \rightarrow B \mid C \quad (1, 2)$   
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LR(0) parsing of *aac*:

| LR(0)(G) | act    | goto  |       |       |       |       |       |
|----------|--------|-------|-------|-------|-------|-------|-------|
|          |        | S     | B     | C     | a     | b     | c     |
| $l_0$    | shift  | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ |
| $l_1$    | accept |       |       |       |       |       |       |
| $l_2$    | red 1  |       |       |       |       |       |       |
| $l_3$    | red 2  |       |       |       |       |       |       |
| $l_4$    | shift  |       | $l_7$ | $l_8$ | $l_4$ | $l_5$ | $l_6$ |
| $l_5$    | red 4  |       |       |       |       |       |       |
| $l_6$    | red 6  |       |       |       |       |       |       |
| $l_7$    | red 3  |       |       |       |       |       |       |
| $l_8$    | red 5  |       |       |       |       |       |       |
| $l_9$    | error  |       |       |       |       |       |       |

(empty =  $l_9$ )

$(aac, l_0, \epsilon)$   
 $\vdash (ac, l_0 l_4, \epsilon)$   
 $\vdash (c, l_0 l_4 l_4, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_6, \epsilon)$   
 $\vdash (\epsilon, l_0 l_4 l_4 l_8, 6)$   
 (\*)  
 $\vdash (\epsilon, l_0 l_4 l_8, 65)$   
 $\vdash (\epsilon, l_0 l_3, 655)$   
 $\vdash (\epsilon, l_0 l_1, 6552)$   
 $\vdash (\epsilon, \epsilon, 65520)$

Check by rightmost derivation (on the board)

**Remark:** in the corresponding computation of  $NBA(G)$ , (\*) is nondeterministic (handle  $C$  vs.  $aC$ )

# The $LR(0)$ Parsing Automaton

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## The $LR(0)$ Parsing Automaton III

### Theorem 10.4 (Correctness of $LR(0)$ Parsing Automaton)

If  $G \in LR(0)$ , then the  $LR(0)$  parsing automaton of  $G$  is deterministic, and for every  $w \in \Sigma^*$  and  $z \in \{0, \dots, p\}^*$ :

$$(w, l_0, \varepsilon) \vdash^* (\varepsilon, \varepsilon, z) \quad \text{iff} \quad \overleftarrow{z} \text{ is a rightmost analysis of } w$$

Proof.

omitted □



# *SLR*(1) Parsing

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## Outline of Lecture 10

Recap: *LR*(0) Parsing

The *LR*(0) Parsing Automaton

*SLR*(1) Parsing

Examples of *SLR*(1) Conflicts

*LR*(1) Parsing

# SLR(1) Parsing

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## Removing Conflicts in $LR(0)$ Parsing

In practice: often  $G \notin LR(0)$

### Example 10.5

$$G_{AE} : \begin{array}{ll} E' \rightarrow E & E \rightarrow E+T \mid T \\ T \rightarrow T*F \mid F & F \rightarrow (E) \mid a \mid b \end{array}$$

# SLR(1) Parsing

## Removing Conflicts in LR(0) Parsing

In practice: often  $G \notin LR(0)$

### Example 10.5

$$G_{AE} : E' \rightarrow E \qquad E \rightarrow E+T \mid T$$
$$T \rightarrow T*F \mid F \qquad F \rightarrow (E) \mid a \mid b$$

LR(0)( $G_{AE}$ ) with conflicts:

$$I_0 : [E' \rightarrow \cdot E] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T]$$
$$[T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F] \quad [F \rightarrow \cdot (E)]$$
$$[F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$
$$I_4 : [F \rightarrow (\cdot E)] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T]$$
$$[T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F] \quad [F \rightarrow \cdot (E)]$$
$$[F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$
$$I_8 : [T \rightarrow T* \cdot F] \quad [F \rightarrow \cdot (E)]$$
$$[F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$
$$I_{11} : [T \rightarrow T*F \cdot]$$
$$I_1 : [E' \rightarrow E \cdot] \quad [E \rightarrow E \cdot +T]$$
$$I_2 : [E \rightarrow T \cdot] \quad [T \rightarrow T \cdot *F]$$
$$I_3 : [T \rightarrow F \cdot]$$
$$I_5 : [F \rightarrow a \cdot]$$
$$I_6 : [F \rightarrow b \cdot]$$
$$I_7 : [E \rightarrow E+ \cdot T] \quad [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F]$$
$$[F \rightarrow \cdot (E)] \quad [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$$
$$I_9 : [F \rightarrow (E \cdot)] \quad [E \rightarrow E \cdot +T]$$
$$I_{10} : [E \rightarrow E+T \cdot] \quad [T \rightarrow T \cdot *F]$$
$$I_{12} : [F \rightarrow (E) \cdot]$$

## Adding Lookahead I

**Goal:** resolving conflicts by considering next input symbol

# SLR(1) Parsing

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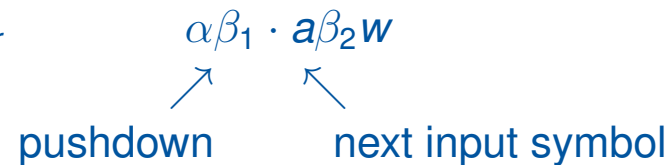
## Adding Lookahead I

**Goal:** resolving conflicts by considering next input symbol

### Observations:

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$   
 $\implies \exists \alpha \in X^*, w \in \Sigma^* :$

$$S' \Rightarrow_r^* \alpha A w \Rightarrow_r$$



**Thus:** shift only on lookahead  $a$

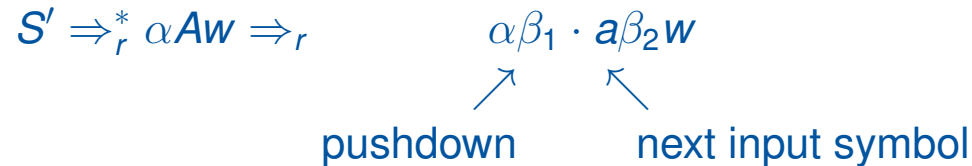
# SLR(1) Parsing

## Adding Lookahead I

**Goal:** resolving conflicts by considering next input symbol

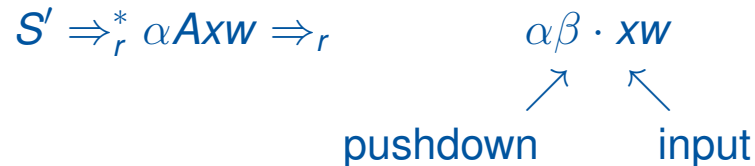
**Observations:**

- $[A \rightarrow \beta_1 \cdot a\beta_2] \in LR(0)(\alpha\beta_1)$   
 $\implies \exists \alpha \in X^*, w \in \Sigma^* :$



**Thus:** shift only on lookahead  $a$

- $[A \rightarrow \beta \cdot] \in LR(0)(\alpha\beta)$   
 $\implies \exists \alpha \in X^*, x \in \Sigma_\varepsilon, w \in \Sigma^* (x = \varepsilon \text{ only if } w = \varepsilon):$



$\implies x \in \text{fo}(A) \subseteq \Sigma_\varepsilon$

**Thus:** reduce with  $A \rightarrow \beta$  only if lookahead  $x \in \text{fo}(A)$

## Adding Lookahead II

### Example 10.6 (cf. Example 10.5)

$$\begin{array}{ll} G_{AE} : E' \rightarrow E & (0) \\ E \rightarrow E+T \mid T & (1, 2) \\ T \rightarrow T*F \mid F & (3, 4) \\ F \rightarrow (E) \mid a \mid b & (5, 6, 7) \end{array}$$

| $A \in N$ | $\text{fo}(A)$          |
|-----------|-------------------------|
| $E'$      | $\{\varepsilon\}$       |
| $E$       | $\{+, ), \varepsilon\}$ |

## Adding Lookahead II

### Example 10.6 (cf. Example 10.5)

$$\begin{aligned} G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7) \end{aligned}$$

| $A \in N$ | $\text{fo}(A)$          |
|-----------|-------------------------|
| $E'$      | $\{\varepsilon\}$       |
| $E$       | $\{+, ), \varepsilon\}$ |

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot + T]\}$ :
  - accept on lookahead  $\varepsilon$
  - shift on lookahead  $+$



## Adding Lookahead II

### Example 10.6 (cf. Example 10.5)

$$\begin{aligned}G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7)\end{aligned}$$

| $A \in N$ | $\text{fo}(A)$          |
|-----------|-------------------------|
| $E'$      | $\{\varepsilon\}$       |
| $E$       | $\{+, ), \varepsilon\}$ |

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$ :
  - accept on lookahead  $\varepsilon$
  - shift on lookahead  $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$ :
  - red 2 on lookahead  $+/)/\varepsilon$
  - shift on lookahead  $*$

## Adding Lookahead II

### Example 10.6 (cf. Example 10.5)

$$\begin{aligned} G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7) \end{aligned}$$

| $A \in N$ | $\text{fo}(A)$          |
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- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$ :
  - accept on lookahead  $\varepsilon$
  - shift on lookahead  $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$ :
  - red 2 on lookahead  $+/)/\varepsilon$
  - shift on lookahead  $*$
- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$ :
  - red 1 on lookahead  $+/)/\varepsilon$
  - shift on lookahead  $*$

# SLR(1) Parsing

## Adding Lookahead II

### Example 10.6 (cf. Example 10.5)

$$\begin{aligned}G_{AE} : E' &\rightarrow E && (0) \\ E &\rightarrow E+T \mid T && (1, 2) \\ T &\rightarrow T*F \mid F && (3, 4) \\ F &\rightarrow (E) \mid a \mid b && (5, 6, 7)\end{aligned}$$

| $A \in N$ | $\text{fo}(A)$          |
|-----------|-------------------------|
| $E'$      | $\{\varepsilon\}$       |
| $E$       | $\{+, ), \varepsilon\}$ |

- $I_1 = \{[E' \rightarrow E\cdot], [E \rightarrow E\cdot +T]\}$ :
  - accept on lookahead  $\varepsilon$
  - shift on lookahead  $+$
- $I_2 = \{[E \rightarrow T\cdot], [T \rightarrow T\cdot *F]\}$ :
  - red 2 on lookahead  $+/)/\varepsilon$
  - shift on lookahead  $*$
- $I_{10} = \{[E \rightarrow E+T\cdot], [T \rightarrow T\cdot *F]\}$ :
  - red 1 on lookahead  $+/)/\varepsilon$
  - shift on lookahead  $*$

$\implies$  **SLR(1) parsing** (Simple LR(1))

## The SLR(1) Action Function

Definition 10.7 (SLR(1) action function)

The SLR(1) action function

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, \mathbf{x}) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \text{ and } \mathbf{x} \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot \mathbf{x} \alpha_2] \in I \text{ and } \mathbf{x} \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } \mathbf{x} = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

# SLR(1) Parsing

## The SLR(1) Action Function

Definition 10.7 (SLR(1) action function)

The **SLR(1) action function**

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

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Definition 10.8 (SLR(1) grammar)

A grammar  $G \in CFG_\Sigma$  has the **SLR(1) property** (notation:  $G \in SLR(1)$ ) if its **SLR(1)** action function is well defined.

# SLR(1) Parsing

## The SLR(1) Action Function

Definition 10.7 (SLR(1) action function)

The **SLR(1) action function**

$$\text{act} : LR(0)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, \mathbf{x}) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha, [A \rightarrow \alpha \cdot] \in I, \text{ and } \mathbf{x} \in \text{fo}(A) \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot \mathbf{x} \alpha_2] \in I \text{ and } \mathbf{x} \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot] \in I \text{ and } \mathbf{x} = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

Definition 10.8 (SLR(1) grammar)

A grammar  $G \in CFG_\Sigma$  has the **SLR(1) property** (notation:  $G \in SLR(1)$ ) if its **SLR(1) action function** is well defined.

**act** and the **LR(0) goto** function (Definition 9.14) form the **SLR(1) parsing table** of  $G$ .

# SLR(1) Parsing

## The SLR(1) Parsing Table

### Example 10.9 (cf. Example 10.5)

$l_0$  :  $[E' \rightarrow \cdot E]$   $[E \rightarrow \cdot E+T]$   $[E \rightarrow \cdot T]$   $l_1$  :  $[E' \rightarrow E \cdot]$   $[E \rightarrow E \cdot +T]$   
 $[T \rightarrow \cdot T*F]$   $[T \rightarrow \cdot F]$   $[F \rightarrow \cdot (E)]$   $l_2$  :  $[E \rightarrow T \cdot]$   $[T \rightarrow T \cdot *F]$   
 $[F \rightarrow \cdot a]$   $[F \rightarrow \cdot b]$   $l_3$  :  $[T \rightarrow F \cdot]$   
 $l_4$  :  $[F \rightarrow (\cdot E)]$   $[E \rightarrow \cdot E+T]$   $[E \rightarrow \cdot T]$   $l_5$  :  $[F \rightarrow a \cdot]$   
 $[T \rightarrow \cdot T*F]$   $[T \rightarrow \cdot F]$   $[F \rightarrow \cdot (E)]$   $l_6$  :  $[F \rightarrow b \cdot]$   
 $[F \rightarrow \cdot a]$   $[F \rightarrow \cdot b]$   $l_7$  :  $[E \rightarrow E+ \cdot T]$   $[T \rightarrow \cdot T*F]$   $[T \rightarrow \cdot F]$   
 $l_8$  :  $[T \rightarrow T* \cdot F]$   $[F \rightarrow \cdot (E)]$   $[F \rightarrow \cdot a]$   $[F \rightarrow \cdot b]$   
 $[F \rightarrow \cdot a]$   $[F \rightarrow \cdot b]$   $l_9$  :  $[F \rightarrow (E \cdot)]$   $[E \rightarrow E \cdot +T]$   
 $l_{11}$  :  $[T \rightarrow T*F \cdot]$   $l_{10}$  :  $[E \rightarrow E+T \cdot]$   $[T \rightarrow T \cdot *F]$   
 $l_{12}$  :  $[F \rightarrow (E) \cdot]$

| $A \in N$ | $fo(A)$                     |
|-----------|-----------------------------|
| $E'$      | $\{\epsilon\}$              |
| $E$       | $\{+, \cdot, \epsilon\}$    |
| $T$       | $\{+, *, \cdot, \epsilon\}$ |
| $F$       | $\{+, *, \cdot, \epsilon\}$ |

# SLR(1) Parsing

## The SLR(1) Parsing Table

### Example 10.9 (cf. Example 10.5)

$l_0 : [E' \rightarrow \cdot E] \quad [E \rightarrow \cdot E+T] \quad [E \rightarrow \cdot T] \quad [F \rightarrow \cdot (E)]$   
 $l_1 : [E' \rightarrow E \cdot] \quad [E \rightarrow E \cdot +T]$   
 $l_2 : [E \rightarrow T \cdot] \quad [T \rightarrow T \cdot *F]$   
 $l_3 : [T \rightarrow F \cdot]$   
 $l_4 : [F \rightarrow a \cdot]$   
 $l_5 : [F \rightarrow b \cdot]$   
 $l_6 : [F \rightarrow b \cdot]$   
 $l_7 : [E \rightarrow E+ \cdot T] \quad [T \rightarrow \cdot T*F] \quad [T \rightarrow \cdot F]$   
 $l_8 : [T \rightarrow T* \cdot F] \quad [F \rightarrow \cdot (E)] \quad [F \rightarrow \cdot a] \quad [F \rightarrow \cdot b]$   
 $l_9 : [F \rightarrow (E \cdot)] \quad [E \rightarrow E \cdot +T]$   
 $l_{10} : [E \rightarrow E+T \cdot] \quad [T \rightarrow T \cdot *F]$   
 $l_{11} : [T \rightarrow T*F \cdot]$   
 $l_{12} : [F \rightarrow (E) \cdot]$

| $A \in N$ | $fo(A)$                     |
|-----------|-----------------------------|
| $E'$      | $\{\epsilon\}$              |
| $E$       | $\{+, \cdot, \epsilon\}$    |
| $T$       | $\{+, *, \cdot, \epsilon\}$ |
| $F$       | $\{+, *, \cdot, \epsilon\}$ |

| $LR(0)(G_{AE})$ | act   |       |       |       |       |       |            | goto  |          |          |       |       |       |          |       |       |
|-----------------|-------|-------|-------|-------|-------|-------|------------|-------|----------|----------|-------|-------|-------|----------|-------|-------|
|                 | +     | *     | (     | )     | a     | b     | $\epsilon$ | E     | T        | F        | +     | *     | (     | )        | a     | b     |
| $l_0$           |       |       | shift |       | shift | shift |            | $l_1$ | $l_2$    | $l_3$    |       |       | $l_4$ |          | $l_5$ | $l_6$ |
| $l_1$           | shift |       |       |       |       |       | accept     |       |          |          |       | $l_7$ |       |          |       |       |
| $l_2$           | red 2 | shift |       | red 2 |       |       | red 2      |       |          |          |       |       | $l_8$ |          |       |       |
| $l_3$           | red 4 | red 4 |       | red 4 |       |       | red 4      |       |          |          |       |       |       |          |       |       |
| $l_4$           |       |       | shift |       | shift | shift |            | $l_9$ | $l_2$    | $l_3$    |       |       | $l_4$ |          | $l_5$ | $l_6$ |
| $l_5$           | red 6 | red 6 |       | red 6 |       |       | red 6      |       |          |          |       |       |       |          |       |       |
| $l_6$           | red 7 | red 7 |       | red 7 |       |       | red 7      |       |          |          |       |       |       |          |       |       |
| $l_7$           |       |       | shift |       | shift | shift |            |       | $l_{10}$ | $l_3$    |       |       | $l_4$ |          | $l_5$ | $l_6$ |
| $l_8$           |       |       | shift |       | shift | shift |            |       |          | $l_{11}$ |       |       | $l_4$ |          | $l_5$ | $l_6$ |
| $l_9$           | shift |       |       | shift |       |       |            |       |          |          | $l_7$ |       |       | $l_{12}$ |       |       |
| $l_{10}$        | red 1 | shift |       | red 1 |       |       | red 1      |       |          |          |       | $l_8$ |       |          |       |       |
| $l_{11}$        | red 3 | red 3 |       | red 3 |       |       | red 3      |       |          |          |       |       |       |          |       |       |
| $l_{12}$        | red 5 | red 5 |       | red 5 |       |       | red 5      |       |          |          |       |       |       |          |       |       |



## The SLR(1) Parsing Automaton

### Definition 10.10 (SLR(1) parsing automaton)

The **SLR(1) parsing automaton** is defined as in the **LR(0)** case (see Definition 10.2), except for the **transition relation**:

shift:  $(aw, \alpha l, z) \vdash (w, \alpha l', z)$  if  $\text{act}(l, a) = \text{shift}$  and  $\text{goto}(l, a) = l'$

reduce<sub>a</sub>:  $(aw, \alpha ll_1 \dots l_n, z) \vdash (aw, \alpha l', zi)$  if  $\text{act}(l_n, a) = \text{red } i$ ,  $\pi_i = A \rightarrow Y_1 \dots Y_n$ , and  $\text{goto}(l, A) = l'$

reduce<sub>ε</sub>:  $(\varepsilon, \alpha ll_1 \dots l_n, z) \vdash (\varepsilon, \alpha l', zi)$  if  $\text{act}(l_n, \varepsilon) = \text{red } i$ ,  $\pi_i = A \rightarrow Y_1 \dots Y_n$ , and  $\text{goto}(l, A) = l'$

accept:  $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$  if  $\text{act}(l, \varepsilon) = \text{accept}$

error<sub>a</sub>:  $(aw, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$  if  $\text{act}(l, a) = \text{error}$

error<sub>ε</sub>:  $(\varepsilon, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$  if  $\text{act}(l, \varepsilon) = \text{error}$

# Examples of $SLR(1)$ Conflicts

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## Outline of Lecture 10

Recap:  $LR(0)$  Parsing

The  $LR(0)$  Parsing Automaton

$SLR(1)$  Parsing

Examples of  $SLR(1)$  Conflicts

$LR(1)$  Parsing

# Examples of $SLR(1)$ Conflicts

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## $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using fo sets

### Example 10.11

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

# Examples of $SLR(1)$ Conflicts

## $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using fo sets

### Example 10.11

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$LR(0)(G_{LR}) : I_0 := LR(0)(\varepsilon) : \quad [S' \rightarrow \cdot S] \quad [S \rightarrow \cdot L=R] \quad [S \rightarrow \cdot R]$   
 $\quad [L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a] \quad [R \rightarrow \cdot L]$

$I_1 := LR(0)(S) : \quad [S' \rightarrow S \cdot]$

$I_2 := LR(0)(L) : \quad [S \rightarrow L \cdot =R] \quad [R \rightarrow L \cdot]$

$I_3 := LR(0)(R) : \quad [S \rightarrow R \cdot]$

$I_4 := LR(0)(*) : \quad [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \quad [L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$

$I_5 := LR(0)(a) : \quad [L \rightarrow a \cdot]$

$I_6 := LR(0)(L=) : \quad [S \rightarrow L= \cdot R] \quad [R \rightarrow \cdot L] \quad [L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$

$I_7 := LR(0)(*R) : \quad [L \rightarrow *R \cdot]$

$I_8 := LR(0)(*L) : \quad [R \rightarrow L \cdot]$

$I_9 := LR(0)(L=R) : \quad [S \rightarrow L=R \cdot]$

# Examples of $SLR(1)$ Conflicts

## $SLR(1)$ Conflicts

**Problem:** not all conflicts can be resolved using  $fo$  sets

### Example 10.11

$G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

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$I_3 := LR(0)(R) : \quad [S \rightarrow R \cdot]$

$I_4 := LR(0)(*) : \quad [L \rightarrow * \cdot R] \quad [R \rightarrow \cdot L] \quad [L \rightarrow \cdot *R] \quad [L \rightarrow \cdot a]$

$I_5 := LR(0)(a) : \quad [L \rightarrow a \cdot]$

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$I_7 := LR(0)(*R) : \quad [L \rightarrow *R \cdot]$

$I_8 := LR(0)(*L) : \quad [R \rightarrow L \cdot]$

$I_9 := LR(0)(L=R) : \quad [S \rightarrow L=R \cdot]$

But: conflict in  $I_2$  not  $SLR(1)$ -solvable since  $= \in fo(R)$

# *LR*(1) Parsing

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## Outline of Lecture 10

Recap: *LR*(0) Parsing

The *LR*(0) Parsing Automaton

*SLR*(1) Parsing

Examples of *SLR*(1) Conflicts

*LR*(1) Parsing

# LR(1) Parsing

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## LR(1) Items and Sets I

**Observation:** not every element of  $\text{fo}(A)$  can follow every occurrence of  $A$   
 $\implies$  refinement of  $LR(0)$  items by adding possible lookahead symbols

# LR(1) Parsing

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### Definition 10.12 ( $LR(1)$ items and sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in \text{CFG}_\Sigma$  be start separated by  $S' \rightarrow S$ .

- If  $S' \Rightarrow_r^* \alpha A a w \Rightarrow_r \alpha \beta_1 \beta_2 a w$ , then  $[A \rightarrow \beta_1 \cdot \beta_2, a]$  is called an  $LR(1)$  item for  $\alpha \beta_1$ .



# LR(1) Parsing

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- If  $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$ , then  $[A \rightarrow \beta_1 \cdot \beta_2, \varepsilon]$  is called an **LR(1) item** for  $\alpha \beta_1$ .

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- If  $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$ , then  $[A \rightarrow \beta_1 \cdot \beta_2, \varepsilon]$  is called an  $LR(1)$  item for  $\alpha \beta_1$ .
- Given  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  denotes the set of all  $LR(1)$  items for  $\gamma$ , called the  $LR(1)$  set (or:  $LR(1)$  information) of  $\gamma$ .

# LR(1) Parsing

## LR(1) Items and Sets I

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- If  $S' \Rightarrow_r^* \alpha A \Rightarrow_r \alpha \beta_1 \beta_2$ , then  $[A \rightarrow \beta_1 \cdot \beta_2, \varepsilon]$  is called an **LR(1) item** for  $\alpha \beta_1$ .
- Given  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  denotes the set of all **LR(1) items** for  $\gamma$ , called the **LR(1) set** (or: **LR(1) information**) of  $\gamma$ .
- $LR(1)(G) := \{LR(1)(\gamma) \mid \gamma \in X^*\}$ .

## LR(1) Items and Sets II

### Corollary 10.13

1. For every  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  is finite.

## LR(1) Items and Sets II

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1. For every  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  is finite.
2.  $LR(1)(G)$  is finite.
3. For every  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  “contains”  $LR(0)(\gamma)$ , i.e.,

$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

## LR(1) Items and Sets II

### Corollary 10.13

1. For every  $\gamma \in X^*$ ,  $LR(1)(\gamma)$  is finite.
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$$\{[A \rightarrow \beta_1 \cdot \beta_2] \mid [A \rightarrow \beta_1 \cdot \beta_2, x] \in LR(1)(\gamma)\} = LR(0)(\gamma).$$

4.  $[A \rightarrow \beta_1 \cdot \beta_2, x] \in I \in LR(1)(G) \implies x \in \text{fo}(A)$

# LR(1) Parsing

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## LR(1) Conflicts

### Definition 10.14 (LR(1) conflicts)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  and  $I \in LR(1)(G)$ .

- $I$  has a **shift/reduce conflict** if there exist  $A \rightarrow \alpha_1 a \alpha_2$ ,  $B \rightarrow \beta \in P$  and  $x \in \Sigma_{\epsilon}$  such that

$$[A \rightarrow \alpha_1 \cdot a \alpha_2, x], [B \rightarrow \beta \cdot, a] \in I.$$



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- $I$  has a **reduce/reduce conflict** if there exist  $x \in \Sigma_{\epsilon}$  and  $A \rightarrow \alpha$ ,  $B \rightarrow \beta \in P$  with  $A \neq B$  or  $\alpha \neq \beta$  such that

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# LR(1) Parsing

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$$[A \rightarrow \alpha \cdot, x], [B \rightarrow \beta \cdot, x] \in I.$$

### Lemma 10.15

$G \in LR(1)$  iff no  $I \in LR(1)(G)$  contains conflicting items.

## Computing LR(1) Sets I

The computation of LR(0) sets (cf. Theorem 9.10) can be extended to cover right contexts:

### Theorem 10.16 (Computing LR(1) sets)

Let  $G = \langle N, \Sigma, P, S \rangle \in CFG_{\Sigma}$  be start separated by  $S' \rightarrow S$  and reduced.

1.  $LR(1)(\varepsilon)$  is the least set such that

–  $[S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$  and

– if  $[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon)$ ,  $B \rightarrow \beta \in P$ , and  $y \in \text{fi}(\gamma x)$ , then  $[B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$ .

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– if  $[A \rightarrow \cdot B\gamma, \mathbf{x}] \in LR(1)(\varepsilon)$ ,  $B \rightarrow \beta \in P$ , and  $\mathbf{y} \in \text{fi}(\gamma\mathbf{x})$ , then  $[B \rightarrow \cdot \beta, \mathbf{y}] \in LR(1)(\varepsilon)$ .

2.  $LR(1)(\alpha Y)$  ( $\alpha \in X^*$ ,  $Y \in X$ ) is the least set such that

– if  $[A \rightarrow \gamma_1 \cdot Y\gamma_2, \mathbf{x}] \in LR(1)(\alpha)$ , then  $[A \rightarrow \gamma_1 Y \cdot \gamma_2, \mathbf{x}] \in LR(1)(\alpha Y)$  and

– if  $[A \rightarrow \gamma_1 \cdot B\gamma_2, \mathbf{x}] \in LR(1)(\alpha Y)$ ,  $B \rightarrow \beta \in P$ , and  $\mathbf{y} \in \text{fi}(\gamma_2\mathbf{x})$ , then  $[B \rightarrow \cdot \beta, \mathbf{y}] \in LR(1)(\alpha Y)$ .

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

LR(1)( $G_{LR}$ ) for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

## Computing $LR(1)$ Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[S' \rightarrow \cdot S, \varepsilon] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : \quad [S' \rightarrow \cdot S, \varepsilon]$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) : \quad [S' \rightarrow \cdot S, \varepsilon] \quad [S \rightarrow \cdot L=R, \varepsilon] \quad [S \rightarrow \cdot R, \varepsilon]$

## Computing $LR(1)$ Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

|   |  |  |                               |
|---|--|--|-------------------------------|
| $[S' \rightarrow \cdot S, \varepsilon]$ | $[S \rightarrow \cdot L=R, \varepsilon]$ | $[S \rightarrow \cdot R, \varepsilon]$ | $[L \rightarrow \cdot *R, =]$ |
| $[L \rightarrow \cdot a, =]$            |  |  |                               |



## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

|   |  |  |                               |
|---|--|--|-------------------------------|
| $[S' \rightarrow \cdot S, \varepsilon]$ | $[S \rightarrow \cdot L=R, \varepsilon]$ | $[S \rightarrow \cdot R, \varepsilon]$ | $[L \rightarrow \cdot *R, =]$ |
| $[L \rightarrow \cdot a, =]$            | $[R \rightarrow \cdot L, \varepsilon]$   |  |                               |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \cdot B\gamma, x] \in LR(1)(\varepsilon), B \rightarrow \beta \in P, y \in \text{fi}(\gamma x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\varepsilon)$

$I'_0 := LR(1)(\varepsilon) :$

|   |  |   |  |
|---|--|---|--|
| $[S' \rightarrow \cdot S, \varepsilon]$ | $[S \rightarrow \cdot L=R, \varepsilon]$ | $[S \rightarrow \cdot R, \varepsilon]$  | $[L \rightarrow \cdot *R, =]$          |
| $[L \rightarrow \cdot a, =]$            | $[R \rightarrow \cdot L, \varepsilon]$   | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot a, \varepsilon]$ |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$

|                                      |                                       |                                      |                                     |
|--------------------------------------|---------------------------------------|--------------------------------------|-------------------------------------|
| $[S' \rightarrow \cdot S, \epsilon]$ | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
| $[L \rightarrow \cdot a, =]$         | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |

$I'_1 := LR(1)(S) :$

|                                      |  |  |  |
|--------------------------------------|--|--|--|
| $[S' \rightarrow S \cdot, \epsilon]$ |  |  |  |
|--------------------------------------|--|--|--|

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$        $[S' \rightarrow \cdot S, \epsilon]$        $[S \rightarrow \cdot L=R, \epsilon]$        $[S \rightarrow \cdot R, \epsilon]$        $[L \rightarrow \cdot *R, =]$   
                                  $[L \rightarrow \cdot a, =]$        $[R \rightarrow \cdot L, \epsilon]$        $[L \rightarrow \cdot *R, \epsilon]$        $[L \rightarrow \cdot a, \epsilon]$

$I'_1 := LR(1)(S) :$        $[S' \rightarrow S \cdot, \epsilon]$

$I'_2 := LR(1)(L) :$        $[S \rightarrow L \cdot =R, \epsilon]$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\varepsilon) :$

|   |  |   |  |
|---|--|---|--|
| $[S' \rightarrow \cdot S, \varepsilon]$ | $[S \rightarrow \cdot L=R, \varepsilon]$ | $[S \rightarrow \cdot R, \varepsilon]$  | $[L \rightarrow \cdot *R, =]$          |
| $[L \rightarrow \cdot a, =]$            | $[R \rightarrow \cdot L, \varepsilon]$   | $[L \rightarrow \cdot *R, \varepsilon]$ | $[L \rightarrow \cdot a, \varepsilon]$ |

$I'_1 := LR(1)(S) :$

|   |
|---|
| $[S' \rightarrow S \cdot, \varepsilon]$ |
|---|

$I'_2 := LR(1)(L) :$

|   |  |
|---|--|
| $[S \rightarrow L \cdot =R, \varepsilon]$ | $[R \rightarrow L \cdot, \varepsilon]$ |
|---|--|

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### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$I'_0 := LR(1)(\epsilon) :$        $[S' \rightarrow \cdot S, \epsilon]$        $[S \rightarrow \cdot L=R, \epsilon]$        $[S \rightarrow \cdot R, \epsilon]$        $[L \rightarrow \cdot *R, =]$   
                                  $[L \rightarrow \cdot a, =]$        $[R \rightarrow \cdot L, \epsilon]$        $[L \rightarrow \cdot *R, \epsilon]$        $[L \rightarrow \cdot a, \epsilon]$

$I'_1 := LR(1)(S) :$        $[S' \rightarrow S \cdot, \epsilon]$

$I'_2 := LR(1)(L) :$        $[S \rightarrow L \cdot =R, \epsilon]$        $[R \rightarrow L \cdot, \epsilon]$

$I'_3 := LR(1)(R) :$        $[S \rightarrow R \cdot, \epsilon]$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$$\begin{aligned} I'_0 &:= LR(1)(\epsilon) : & [S' \rightarrow \cdot S, \epsilon] & \quad [S \rightarrow \cdot L=R, \epsilon] & \quad [S \rightarrow \cdot R, \epsilon] & \quad [L \rightarrow \cdot *R, =] \\ & & [L \rightarrow \cdot a, =] & \quad [R \rightarrow \cdot L, \epsilon] & \quad [L \rightarrow \cdot *R, \epsilon] & \quad [L \rightarrow \cdot a, \epsilon] \\ I'_1 &:= LR(1)(S) : & [S' \rightarrow S \cdot, \epsilon] & & & \\ I'_2 &:= LR(1)(L) : & [S \rightarrow L \cdot =R, \epsilon] & \quad [R \rightarrow L \cdot, \epsilon] & & \\ I'_3 &:= LR(1)(R) : & [S \rightarrow R \cdot, \epsilon] & & & \\ I'_4 &:= LR(1)(*) : & [L \rightarrow * \cdot R, =] & \quad [L \rightarrow * \cdot R, \epsilon] & & \end{aligned}$$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$$\begin{aligned} I'_0 &:= LR(1)(\varepsilon) : & [S' \rightarrow \cdot S, \varepsilon] & \quad [S \rightarrow \cdot L=R, \varepsilon] & \quad [S \rightarrow \cdot R, \varepsilon] & \quad [L \rightarrow \cdot *R, =] \\ & & [L \rightarrow \cdot a, =] & \quad [R \rightarrow \cdot L, \varepsilon] & \quad [L \rightarrow \cdot *R, \varepsilon] & \quad [L \rightarrow \cdot a, \varepsilon] \\ I'_1 &:= LR(1)(S) : & [S' \rightarrow S \cdot, \varepsilon] & & & \\ I'_2 &:= LR(1)(L) : & [S \rightarrow L \cdot =R, \varepsilon] & \quad [R \rightarrow L \cdot, \varepsilon] & & \\ I'_3 &:= LR(1)(R) : & [S \rightarrow R \cdot, \varepsilon] & & & \\ I'_4 &:= LR(1)(*) : & [L \rightarrow * \cdot R, =] & \quad [L \rightarrow * \cdot R, \varepsilon] & \quad [R \rightarrow \cdot L, =] & \quad [R \rightarrow \cdot L, \varepsilon] \end{aligned}$$



## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$$\begin{aligned} I'_0 &:= LR(1)(\varepsilon) : & [S' \rightarrow \cdot S, \varepsilon] & & [S \rightarrow \cdot L=R, \varepsilon] & & [S \rightarrow \cdot R, \varepsilon] & & [L \rightarrow \cdot *R, =] \\ & & [L \rightarrow \cdot a, =] & & [R \rightarrow \cdot L, \varepsilon] & & [L \rightarrow \cdot *R, \varepsilon] & & [L \rightarrow \cdot a, \varepsilon] \\ I'_1 &:= LR(1)(S) : & [S' \rightarrow S \cdot, \varepsilon] & & & & & & \\ I'_2 &:= LR(1)(L) : & [S \rightarrow L \cdot =R, \varepsilon] & & [R \rightarrow L \cdot, \varepsilon] & & & & \\ I'_3 &:= LR(1)(R) : & [S \rightarrow R \cdot, \varepsilon] & & & & & & \\ I'_4 &:= LR(1)(*) : & [L \rightarrow * \cdot R, =] & & [L \rightarrow * \cdot R, \varepsilon] & & [R \rightarrow \cdot L, =] & & [R \rightarrow \cdot L, \varepsilon] \\ & & [L \rightarrow \cdot *R, =] & & [L \rightarrow \cdot a, =] & & [L \rightarrow \cdot *R, \varepsilon] & & [L \rightarrow \cdot a, \varepsilon] \end{aligned}$$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

$$\begin{aligned} I'_0 &:= LR(1)(\epsilon) : & [S' \rightarrow \cdot S, \epsilon] & & [S \rightarrow \cdot L=R, \epsilon] & & [S \rightarrow \cdot R, \epsilon] & & [L \rightarrow \cdot *R, =] \\ & & [L \rightarrow \cdot a, =] & & [R \rightarrow \cdot L, \epsilon] & & [L \rightarrow \cdot *R, \epsilon] & & [L \rightarrow \cdot a, \epsilon] \\ I'_1 &:= LR(1)(S) : & [S' \rightarrow S \cdot, \epsilon] & & & & & & \\ I'_2 &:= LR(1)(L) : & [S \rightarrow L \cdot =R, \epsilon] & & [R \rightarrow L \cdot, \epsilon] & & & & \\ I'_3 &:= LR(1)(R) : & [S \rightarrow R \cdot, \epsilon] & & & & & & \\ I'_4 &:= LR(1)(*) : & [L \rightarrow * \cdot R, =] & & [L \rightarrow * \cdot R, \epsilon] & & [R \rightarrow \cdot L, =] & & [R \rightarrow \cdot L, \epsilon] \\ & & [L \rightarrow \cdot *R, =] & & [L \rightarrow \cdot a, =] & & [L \rightarrow \cdot *R, \epsilon] & & [L \rightarrow \cdot a, \epsilon] \\ I'_5 &:= LR(1)(a) : & [L \rightarrow a \cdot, =] & & [L \rightarrow a \cdot, \epsilon] & & & & \\ I'_6 &:= LR(1)(L=) : & [S \rightarrow L= \cdot R, \epsilon] & & & & & & \end{aligned}$$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

$$\begin{aligned} I'_0 &:= LR(1)(\epsilon) : & [S' \rightarrow \cdot S, \epsilon] & [S \rightarrow \cdot L=R, \epsilon] & [S \rightarrow \cdot R, \epsilon] & [L \rightarrow \cdot *R, =] \\ & & [L \rightarrow \cdot a, =] & [R \rightarrow \cdot L, \epsilon] & [L \rightarrow \cdot *R, \epsilon] & [L \rightarrow \cdot a, \epsilon] \\ I'_1 &:= LR(1)(S) : & [S' \rightarrow S \cdot, \epsilon] & & & \\ I'_2 &:= LR(1)(L) : & [S \rightarrow L \cdot =R, \epsilon] & [R \rightarrow L \cdot, \epsilon] & & \\ I'_3 &:= LR(1)(R) : & [S \rightarrow R \cdot, \epsilon] & & & \\ I'_4 &:= LR(1)(*) : & [L \rightarrow * \cdot R, =] & [L \rightarrow * \cdot R, \epsilon] & [R \rightarrow \cdot L, =] & [R \rightarrow \cdot L, \epsilon] \\ & & [L \rightarrow \cdot *R, =] & [L \rightarrow \cdot a, =] & [L \rightarrow \cdot *R, \epsilon] & [L \rightarrow \cdot a, \epsilon] \\ I'_5 &:= LR(1)(a) : & [L \rightarrow a \cdot, =] & [L \rightarrow a \cdot, \epsilon] & & \\ I'_6 &:= LR(1)(L=) : & [S \rightarrow L= \cdot R, \epsilon] & [R \rightarrow \cdot L, \epsilon] & & \end{aligned}$$

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B \gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |



## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L=R \cdot, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  |                                       |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot\beta, y] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot B\gamma_2, x] \in LR(1)(\alpha Y), B \rightarrow \beta \in P, y \in \text{fi}(\gamma_2 x) \implies [B \rightarrow \cdot \beta, y] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

LR(1)( $G_{LR}$ ) for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_{12} := LR(1)(L=a) :$   | $[L \rightarrow a \cdot, \epsilon]$    |                                       |                                      |                                     |

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

$[A \rightarrow \gamma_1 \cdot Y \gamma_2, x] \in LR(1)(\alpha) \implies [A \rightarrow \gamma_1 Y \cdot \gamma_2, x] \in LR(1)(\alpha Y)$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_{12} := LR(1)(L=a) :$   | $[L \rightarrow a \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{13} := LR(1)(L=*R) :$  | $[L \rightarrow *R \cdot, \epsilon]$   |                                       |                                      |                                     |

# LR(1) Parsing

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

LR(1)( $G_{LR}$ ) for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_{12} := LR(1)(L=a) :$   | $[L \rightarrow a \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{13} := LR(1)(L=*R) :$  | $[L \rightarrow *R \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_{14} := \emptyset$      |  |                                       |                                      |                                     |

# LR(1) Parsing

## Computing LR(1) Sets II

### Example 10.17 (cf. Example 10.11)

$LR(1)(G_{LR})$  for  $G_{LR} : S' \rightarrow S \quad S \rightarrow L=R \mid R \quad L \rightarrow *R \mid a \quad R \rightarrow L$

|                             |  |                                       |                                      |                                      |
|-----------------------------|--|---------------------------------------|--------------------------------------|--------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, \epsilon]$ |
|                             | $[L \rightarrow \cdot a, \epsilon]$    | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$  |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                      |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                      |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                      |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, \epsilon]$  | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$  |
|                             | $[L \rightarrow \cdot *R, \epsilon]$   | $[L \rightarrow \cdot a, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$  |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, \epsilon]$    | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                      |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$  |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, \epsilon]$   | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                      |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, \epsilon]$    | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                      |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                      |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                      |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$  |
| $I'_{12} := LR(1)(L=a) :$   | $[L \rightarrow a \cdot, \epsilon]$    |                                       |                                      |                                      |
| $I'_{13} := LR(1)(L=*R) :$  | $[L \rightarrow *R \cdot, \epsilon]$   |                                       |                                      |                                      |
| $I'_{14} := \emptyset$      |  |                                       |                                      |                                      |

In  $I'_2$ : shift on = / reduce on  $\epsilon \implies G_{LR} \in LR(1)$



## The LR(1) Action Function

Definition 10.18 (LR(1) action function)

The LR(1) action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

## The LR(1) Action Function

Definition 10.18 (LR(1) action function)

The LR(1) action function

$$\text{act} : LR(1)(G) \times \Sigma_\varepsilon \rightarrow \{\text{red } i \mid i \in [p]\} \cup \{\text{shift, accept, error}\}$$

is defined by

$$\text{act}(I, x) := \begin{cases} \text{red } i & \text{if } i \neq 0, \pi_i = A \rightarrow \alpha \text{ and } [A \rightarrow \alpha \cdot, x] \in I \\ \text{shift} & \text{if } [A \rightarrow \alpha_1 \cdot x \alpha_2, y] \in I \text{ and } x \in \Sigma \\ \text{accept} & \text{if } [S' \rightarrow S \cdot, \varepsilon] \in I \text{ and } x = \varepsilon \\ \text{error} & \text{otherwise} \end{cases}$$

## Corollary 10.19

For every  $G \in CFG_\Sigma$ ,  $G \in LR(1)$  iff its LR(1) action function is well defined.

## The LR(1) goto Function

The goto function is defined in analogy to the LR(0) case (Definition 9.14).

### Definition 10.20 (LR(1) goto function)

The function  $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$  is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

## The LR(1) goto Function

The goto function is defined in analogy to the LR(0) case (Definition 9.14).

### Definition 10.20 (LR(1) goto function)

The function  $\text{goto} : LR(1)(G) \times X \rightarrow LR(1)(G)$  is determined by

$$\text{goto}(I, Y) = I' \quad \text{iff} \quad \text{there exists } \gamma \in X^* \text{ such that} \\ I = LR(1)(\gamma) \text{ and } I' = LR(1)(\gamma Y).$$

Again, act and goto form the LR(1) parsing table of  $G$ .

# LR(1) Parsing

## The LR(1) Parsing Table

### Example 10.21 (cf. Example 10.17)

|                             |  |                                       |                                      |                                     |
|-----------------------------|--|---------------------------------------|--------------------------------------|-------------------------------------|
| $I'_0 := LR(1)(\epsilon) :$ | $[S' \rightarrow \cdot S, \epsilon]$   | $[S \rightarrow \cdot L=R, \epsilon]$ | $[S \rightarrow \cdot R, \epsilon]$  | $[L \rightarrow \cdot *R, =]$       |
|                             | $[L \rightarrow \cdot a, =]$           | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_1 := LR(1)(S) :$        | $[S' \rightarrow S \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_2 := LR(1)(L) :$        | $[S \rightarrow L \cdot =R, \epsilon]$ | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_3 := LR(1)(R) :$        | $[S \rightarrow R \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_4 := LR(1)(*) :$        | $[L \rightarrow * \cdot R, =]$         | $[L \rightarrow * \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, =]$         | $[R \rightarrow \cdot L, \epsilon]$ |
|                             | $[L \rightarrow \cdot *R, =]$          | $[L \rightarrow \cdot a, =]$          | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_5 := LR(1)(a) :$        | $[L \rightarrow a \cdot, =]$           | $[L \rightarrow a \cdot, \epsilon]$   |                                      |                                     |
| $I'_6 := LR(1)(L=) :$       | $[S \rightarrow L= \cdot R, \epsilon]$ | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_7 := LR(1)(*R) :$       | $[L \rightarrow *R \cdot, =]$          | $[L \rightarrow *R \cdot, \epsilon]$  |                                      |                                     |
| $I'_8 := LR(1)(*L) :$       | $[R \rightarrow L \cdot, =]$           | $[R \rightarrow L \cdot, \epsilon]$   |                                      |                                     |
| $I'_9 := LR(1)(L=R) :$      | $[S \rightarrow L=R \cdot, \epsilon]$  |                                       |                                      |                                     |
| $I'_{10} := LR(1)(L=L) :$   | $[R \rightarrow L \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{11} := LR(1)(L=*) :$   | $[L \rightarrow * \cdot R, \epsilon]$  | $[R \rightarrow \cdot L, \epsilon]$   | $[L \rightarrow \cdot *R, \epsilon]$ | $[L \rightarrow \cdot a, \epsilon]$ |
| $I'_{12} := LR(1)(L=a) :$   | $[L \rightarrow a \cdot, \epsilon]$    |                                       |                                      |                                     |
| $I'_{13} := LR(1)(L=*R) :$  | $[L \rightarrow *R \cdot, \epsilon]$   |                                       |                                      |                                     |
| $I'_{14} := \emptyset$      |  |                                       |                                      |                                     |

|           | act   |       |       |            | goto      |           |           |           |        |   |
|-----------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|           | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$    | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$    |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$    |       | shift |       | red 5      |           |           |           | $I'_6$    |        |   |
| $I'_3$    |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$    | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$    |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$    | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$    |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$    |       | red 5 |       | red 5      |           |           |           |           |        |   |
| $I'_9$    |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$ |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$ | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$ |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$ |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

## The LR(1) Parsing Automaton I

### Definition 10.22 (LR(1) parsing automaton)

The **LR(1) parsing automaton** is defined as in the **LR(0)** case (see Definition 10.2), except for the **transition relation**:

**shift**:  $(aw, \alpha l, z) \vdash (w, \alpha l', z)$  if  $\text{act}(l, a) = \text{shift}$  and  $\text{goto}(l, a) = l'$

**reduce<sub>a</sub>**:  $(aw, \alpha ll_1 \dots l_n, z) \vdash (aw, \alpha l', zi)$  if  $\text{act}(l_n, a) = \text{red } i$ ,  $\pi_i = A \rightarrow Y_1 \dots Y_n$ , and  $\text{goto}(l, A) = l'$

**reduce<sub>ε</sub>**:  $(\varepsilon, \alpha ll_1 \dots l_n, z) \vdash (\varepsilon, \alpha l', zi)$  if  $\text{act}(l_n, \varepsilon) = \text{red } i$ ,  $\pi_i = A \rightarrow Y_1 \dots Y_n$ , and  $\text{goto}(l, A) = l'$

**accept**:  $(\varepsilon, l_0 l, z) \vdash (\varepsilon, \varepsilon, z 0)$  if  $\text{act}(l, \varepsilon) = \text{accept}$

**error<sub>a</sub>**:  $(aw, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$  if  $\text{act}(l, a) = \text{error}$

**error<sub>ε</sub>**:  $(\varepsilon, \alpha l, z) \vdash (\varepsilon, \varepsilon, z \text{error})$  if  $\text{act}(l, \varepsilon) = \text{error}$

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |               | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|---------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\varepsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |               | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept        |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5         |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2         |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |               | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4         |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |               | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3         |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |               |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1         |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5         |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |               | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4         |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3         |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of  $a=*a$ :

( $a=*a, I'_0, \epsilon$ )



# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

$(a=*a, I'_0, \epsilon)$   
 $\vdash (= *a, I'_0 I'_5, \epsilon)$

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (= \*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (= \*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of  $a=*a$ :

$(a=*a, I'_0, \epsilon)$   
 $\vdash (= * a, I'_0 I'_5, \epsilon)$   
 $\vdash (= * a, I'_0 I'_2, 4)$   
 $\vdash (* a, I'_0 I'_2 I'_6, 4)$   
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$   
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of  $a=*a$ :

$(a=*a, I'_0, \epsilon)$   
 $\vdash (=*a, I'_0 I'_5, \epsilon)$   
 $\vdash (=*a, I'_0 I'_2, 4)$   
 $\vdash (*a, I'_0 I'_2 I'_6, 4)$   
 $\vdash (a, I'_0 I'_2 I'_6 I'_{11}, 4)$   
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{12}, 4)$   
 $\vdash (\epsilon, I'_0 I'_2 I'_6 I'_{11} I'_{10}, 44)$

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1, 2) \quad L \rightarrow *R \mid a (3, 4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)



# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_1$ , 445351)

# LR(1) Parsing

## The LR(1) Parsing Automaton II

### Example 10.23 (cf. Example 10.17)

$G_{LR} : S' \rightarrow S (0) \quad S \rightarrow L=R \mid R (1,2) \quad L \rightarrow *R \mid a (3,4) \quad R \rightarrow L (5)$

| $LR(1)(G_{LR})$ | act   |       |       |            | goto      |           |           |           |        |   |
|-----------------|-------|-------|-------|------------|-----------|-----------|-----------|-----------|--------|---|
|                 | *     | =     | a     | $\epsilon$ | S         | L         | R         | *         | =      | a |
| $I'_0$          | shift |       | shift |            | $I'_1$    | $I'_2$    | $I'_3$    | $I'_4$    | $I'_5$ |   |
| $I'_1$          |       |       |       | accept     |           |           |           |           |        |   |
| $I'_2$          |       | shift |       | red 5      |           |           |           |           | $I'_6$ |   |
| $I'_3$          |       |       |       | red 2      |           |           |           |           |        |   |
| $I'_4$          | shift |       | shift |            | $I'_8$    | $I'_7$    | $I'_4$    | $I'_5$    |        |   |
| $I'_5$          |       | red 4 |       | red 4      |           |           |           |           |        |   |
| $I'_6$          | shift |       | shift |            | $I'_{10}$ | $I'_9$    | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_7$          |       | red 3 |       | red 3      |           |           |           |           |        |   |
| $I'_8$          |       | red 5 |       |            |           |           |           |           |        |   |
| $I'_9$          |       |       |       | red 1      |           |           |           |           |        |   |
| $I'_{10}$       |       |       |       | red 5      |           |           |           |           |        |   |
| $I'_{11}$       | shift |       | shift |            | $I'_{10}$ | $I'_{13}$ | $I'_{11}$ | $I'_{12}$ |        |   |
| $I'_{12}$       |       |       |       | red 4      |           |           |           |           |        |   |
| $I'_{13}$       |       |       |       | red 3      |           |           |           |           |        |   |

(empty = error/ $\emptyset$ )

LR(1) parsing of a=\*a:

(a=\*a,  $I'_0$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_5$ ,  $\epsilon$ )  
 $\vdash$  (=\*a,  $I'_0 I'_2$ , 4)  
 $\vdash$  (\*a,  $I'_0 I'_2 I'_6$ , 4)  
 $\vdash$  (a,  $I'_0 I'_2 I'_6 I'_{11}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{12}$ , 4)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{10}$ , 44)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{11} I'_{13}$ , 445)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_{10}$ , 4453)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_2 I'_6 I'_9$ , 44535)  
 $\vdash$  ( $\epsilon$ ,  $I'_0 I'_1$ , 445351)  
 $\vdash$  ( $\epsilon$ ,  $\epsilon$ , 4453510)