# Semantics and Verification of Software

## Winter Semester 2017/18

## Lecture 1: Introduction

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http://moves.rwth-aachen.de/teaching/ws-1718/sv-sw/

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# Preliminaries

## Staff

- Lectures: Thomas Noll
  - Lehrstuhl für Informatik 2, Room 4211
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- Exercise classes:
  - Benjamin Kaminski (benjamin.kaminski@cs.rwth-aachen.de)
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- Student assistants:
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## Target Audience

- MSc Informatik:
  - Theoretische Informatik
- MSc Software Systems Engineering:
  - Theoretical Foundations of SSE
- In general:
  - interest in formal models for programming languages
  - application of mathematical reasoning methods
- Expected: basic knowledge in
  - essential concepts of imperative programming languages
  - formal languages and automata theory
  - mathematical logic

## Organisation

- Schedule:
  - Lecture Tue 10:15–11:45 9U10 (starting 10 Oct)
  - Lecture Thu 10:15–11:45 AH 6 (starting 12 Oct)
  - Exercise class Wed 12:00–13:30 AH 3 (starting 25 Oct)
- Irregular lecture dates – checkout web page!
- 1st assignment sheet: 18 Oct on web page
  - submission by 25 Oct before exercise class
  - presentation on 25 Oct
- Work on assignments in groups of three
- Examination (6 ECTS credits):
  - oral or written (depending on number of participants)
  - date to be fixed
- Admission requires at least 50% of the points in the exercises
- Written material in English, lecture and exercise classes “on demand”, rest up to you

## Aspects of Programming Languages

Syntax: "How does a program look like?"

- hierarchical composition of programs from structural components

$\Rightarrow$ *Compiler Construction*

Semantics: "What does this program mean?"

- output/behaviour/... in dependence of input/environment/...

$\Rightarrow$ This course

Pragmatics:

- length and understandability of programs
- learnability of programming language
- appropriateness for specific applications, ...

$\Rightarrow$ *Software Engineering*

## Historic development:

- Formal syntax since 1960s (scanners, LL/LR parsers); semantics defined by compiler/interpreter
- Formal semantics since 1970s (operational/denotational/axiomatic)

## Why Semantics?

**Idea:** compiler = ultimate semantics!

- Compiler gives each individual program a semantics
  (= “behaviour” of generated machine code)

**But:**

- Compilers are highly complicated software systems
  - code optimisations
  - memory management
  - interaction with runtime system
  - ...
- Most languages have more than one compiler (with different outputs)
- Most compilers have bugs

$\Rightarrow$ Does not help with formal reasoning about programming language or individual programs

## The Semantics of "Semantics"

Originally: study of meaning of symbols (linguistics)

Semantics of a program: meaning of a concrete program

- mapping input $\rightarrow$ output values
- interaction behaviour (shared variables, communication, ...)
- ...

Semantics of a programming language: mapping of each (syntactically correct) program of a programming language to its meaning

Semantics of software: various techniques for defining the semantics of diverse programming languages

- operational
- denotational
- axiomatic
- ...

## Motivation for Rigorous Formal Treatment I

### Example 1.1

1. How often will the following loop be traversed?

```
for i := 2 to 1 do ...
```

   FORTRAN IV: once
   PASCAL: never

2. What if `p = nil` in the following program?

```
while p <> nil and p^.key < val do ...
```

   Pascal: strict boolean operations ↯
   Modula: non-strict boolean operations ✓

## Motivation for Rigorous Formal Treatment II

- Support for development of
  - new programming languages: missing details, ambiguities and inconsistencies can be recognized
  - compilers: automatic compiler generation from appropriately defined semantics
  - programs: exact understanding of semantics avoids uncertainties in the implementation of algorithms
- Support for correctness proofs of
  - programs: comparison of program semantics with desired behavior (e.g., termination properties, absence of deadlocks, ...)
  - compilers:

$$\begin{array}{ccc} \text{programming language} & \xrightarrow{\text{compiler}} & \text{machine code} \\ \downarrow \text{semantics} & & \downarrow \text{(simple) semantics} \\ \text{meaning} & \overset{?}{=} & \text{meaning} \end{array}$$

  - optimising transformations:

$$\begin{array}{ccc} \text{code} & \xrightarrow{\text{optimisation}} & \text{code} \\ \downarrow \text{semantics} & & \downarrow \text{semantics} \\ \text{meaning} & \overset{?}{=} & \text{meaning} \end{array}$$

## (Complementary) Kinds of Formal Semantics

Operational semantics: describes computation of the program on some (very) abstract machine (G. Plotkin)

- example: $\text{(seq)}\dfrac{\langle c_1, \sigma\rangle \to \sigma' \quad \langle c_2, \sigma'\rangle \to \sigma''}{\langle c_1 ; c_2, \sigma\rangle \to \sigma''}$
- application: implementation of programming languages (compilers, interpreters, ...)

Denotational semantics: mathematical definition of input/output relation of the program by induction on its syntactic structure (D. Scott, C. Strachey)

- example: $\mathfrak{C}[\![.]\!] : \mathit{Cmd} \to (\Sigma \dashrightarrow \Sigma)$
  $\mathfrak{C}[\![c_1 ; c_2]\!] := \mathfrak{C}[\![c_2]\!] \circ \mathfrak{C}[\![c_1]\!]$
- application: program analysis

Axiomatic semantics: formalisation of special properties of programs by logical formulae (assertions/proof rules; R. Floyd, T. Hoare)

- example: $\text{(seq)}\dfrac{\{A\}\, c_1\, \{C\} \quad \{C\}\, c_2\, \{B\}}{\{A\}\, c_1 ; c_2\, \{B\}}$
- application: program verification

## Overview of the Course

1. The imperative model language WHILE
2. Operational semantics of WHILE
3. Denotational semantics of WHILE
4. Equivalence of operational and denotational semantics
5. Axiomatic semantics of WHILE
6. Extensions: procedures and dynamic data structures
7. Applications: compiler correctness etc.

## Literature

(also see the collection “Handapparat Softwaremodellierung und Verifikation” at the CS Library)

- Formal semantics
  - G. Winskel: *The Formal Semantics of Programming Languages*, The MIT Press, 1996
- Compiler correctness
  - H.R. Nielson, F. Nielson: *Semantics with Applications: An Appetizer*, Springer Undergraduate Topics in Computer Science, 2007

## Syntactic Categories

WHILE: simple imperative programming language without procedures or advanced data structures

Syntactic categories:

| Category | Domain | Meta variable |
|---|---|---|
| Numbers | $\mathbb{Z} = \{0, 1, -1, \ldots\}$ | $z$ |
| Truth values | $\mathbb{B} = \{\text{true}, \text{false}\}$ | $t$ |
| Variables | $\mathit{Var} = \{\mathrm{x}, \mathrm{y}, \ldots\}$ | $x$ |
| Arithmetic expressions | *AExp* (next slide) | $a$ |
| Boolean expressions | *BExp* (next slide) | $b$ |
| Commands (statements) | *Cmd* (next slide) | $c$ |

## Syntax of WHILE Programs

### Definition 1.2 (Syntax of WHILE)

The syntax of WHILE Programs is defined by the following context-free grammar:

$a ::= z \mid x \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 * a_2 \in \mathit{AExp}$

$b ::= t \mid a_1 = a_2 \mid a_1 > a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in \mathit{BExp}$

$c ::= \texttt{skip} \mid x := a \mid c_1 \,;\, c_2 \mid \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2\ \texttt{end} \mid \texttt{while}\ b\ \texttt{do}\ c\ \texttt{end} \in \mathit{Cmd}$

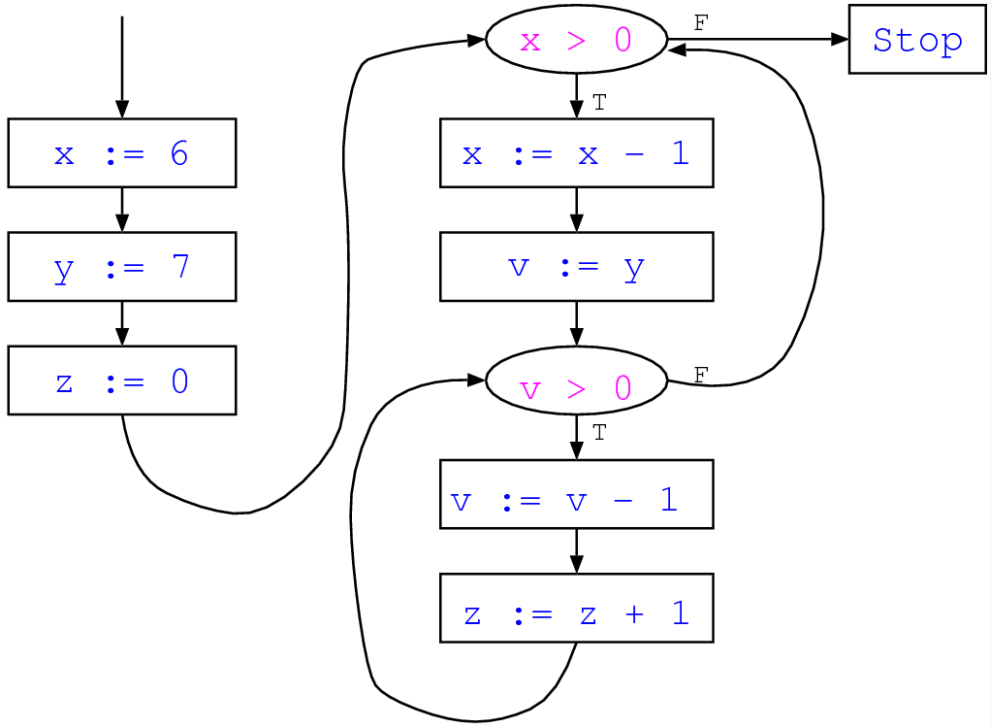
**Remarks:** we assume that

- the syntax of numbers, truth values and variables is predefined (i.e., no “lexical analysis”)
- the syntactic interpretation of ambiguous constructs (expressions) is uniquely determined (by brackets or priorities)

## A WHILE Program and Its Control-Flow Diagram

### Example 1.3

```
x := 6;
y := 7;
z := 0;
while x > 0 do
  x := x - 1;
  v := y;
  while v > 0 do
    v := v - 1;
    z := z + 1
  end
end
```




Effect: `z := x * y = 42`