

Exercise Sheet 10

Due date: January 31st. You can hand in your solutions at the start of the exercise class.

Hint: Notation is as in the lecture. That is, c is a program, b a Boolean expression, σ a program state, etc.

Task 1: Well-typedness (3 Points)

Check whether the following program is well-typed by constructing a corresponding proof tree.

```
x = 50;
while(x > 0) {
  y = 0.3;
  z = 0.134;
  x = x - 2;
  if(x == 0) {
    y = y * z
  } else {
    z = z * z + x
  };
}
```

Task 2: Type Casts (2 Points)

We introduce new arithmetic expressions to change the type of an arithmetic expression (from integer to real and vice versa). More precisely, if $\Gamma \vdash a : \text{int}$ and $\langle a, \sigma \rangle \rightarrow z$ then $\Gamma \vdash \text{castReal}(a) : \text{real}$ and $\langle \text{castReal}(a), \sigma \rangle \rightarrow z$.

Similarly, if $\Gamma \vdash a : \text{real}$ and $\langle a, \sigma \rangle \rightarrow r$ then $\Gamma \vdash \text{castInt}(a) : \text{int}$ and $\langle \text{castInt}(a), \sigma \rangle \rightarrow \lfloor r \rfloor$.

Extend the rules for typed evaluation of arithmetic expressions (Def. 18.6 from the lecture) by rules to support type casts as specified above.

Task 2: Confinement (5 Points)

Let $c \in \text{Cmd}$ such that $\text{min } SL \vdash c$. Moreover, let $\ell \in SL$ and $\sigma_1, \sigma_2, \sigma'_1, \sigma'_2 \in \Sigma$ such that

1. $\sigma_1 =_{\leq \ell} \sigma_2$,
2. $\langle c_1, \sigma_2 \rangle \rightarrow \sigma'_1$, and
3. $\langle c_2, \sigma_2 \rangle \rightarrow \sigma'_2$.

Show that $\sigma'_1 =_{\leq \ell} \sigma'_2$.