Exercise Sheet 9

Due date: January 17th. You can hand in your solutions at the start of the exercise class.

Hint: Notation is as in the lecture. That is, c is a program, b a Boolean expression, σ a program state, etc.

Task 1: Equivalence of Statements in `ParWHILE` (2 Points)

Recall that two statements c_1, c_2 are *equivalent*, written $c_1 \approx c_2$, if and only if

$$\forall \sigma, \sigma' \in \Sigma \ : \ \langle c_1, \sigma \rangle \to^* \sigma' \quad \Leftrightarrow \quad \langle c_2, \sigma \rangle \to^* \sigma'.$$

Previously, we occasionally made use of the fact that program statements can be replaced by equivalent ones without changing the programs behavior. That is, for all `WHILE` programs P containing a single occurrence of a statement c, $c_1 \approx c_2$ implies $P[c \mapsto c_1] \approx P[c \mapsto c_2]$. Here, $P[c \mapsto c']$ denotes the syntactic replacement of statement c by c' in P.

Prove or disprove that all `ParWHILE` programs have the same property.

Task 2: Fairness in CSP (2 Points)

Prove or disprove: Every strongly unfair execution is weakly unfair.

Task 3: Axiomatic Semantics of Nondeterminism (6 Points)

Consider an extension of the `WHILE` programming language with a nondeterministic operator $c_1 \Box c_2$ as in the lecture. There are two possible interpretations of this operator. In the *demonic model* to establish a postcondition Q one requires that every possible program execution (induced by its nondeterministic choices) establishes Q. In the *angelic model* one requires that at least one execution establishes Q.

a) Extend the Hoare logic proof system by a rule for *demonic* nondeterminism.

b) Give an inductive definition of $wp\,(c_1 \Box c_2, Q)$ in the demonic model.[1]

c) Prove or disprove for a demonic model of nondeterminism:

$$wp\,(c, Q_1 \vee Q_2) = wp\,(c, Q_1) \vee wp\,(c, Q_2)$$

d) Extend the Hoare logic proof system by a rule for *angelic* nondeterminism.

e) Give an inductive definition of $wp\,(c_1 \Box c_2, Q)$ in the angelic model.

f) Prove or disprove for an angelic model of nondeterminism:

$$wp\,(c, Q_1 \wedge Q_2) = wp\,(c, Q_1) \wedge wp\,(c, Q_2)$$

[1] You may of course reuse the definition of weakest preconditions for ordinary `WHILE` programs.