Exercise Sheet 7

Due date: December 13th. You can hand in your solutions at the start of the exercise class.

Remark: We started uploading solutions to previous exercises. The password to access the solutions on our website will be announced in the next exercise class.

Hint: Notation is as in the lecture. That is, c is a program, b a Boolean expression, σ a program state, etc.

Task 1: Partial Correctness (2+2 Points)

Recall that Goldbach's conjecture states that every even natural number $n \in \mathbb{N}$ can be written as the sum of two primes $p, q \in \mathbb{N}$. Such a pair (p, q) is called a *Goldbach partition* of n.

(a) Write a program c that searches for a Goldbach partition of variable n and stores it in variables p and q. If that Goldbach partition does not exist, the program should not terminate. You may assume that you can test for primality, i.e. statements of the form `if` y prime `then` ... are allowed.

(b) Prove in Hoare logic that the program c you found in (a) will compute a Goldbach partition for *every* input n, if the Goldbach conjecture is true.

Task 2: Total Correctness (2+2+2 Points)

Consider the following program c:

```
while x ≠ 0 do
    x := x − 1
end
```

(a) Prove or disprove in Hoare logic: $\{\mathsf{true}\}\ c\ \{\mathsf{true}\}$

(b) Prove or disprove in Hoare logic: $\{x > 0\}\ c\ \{\Downarrow\mathsf{true}\}$

(c) Prove or disprove in Hoare logic: $\{\mathsf{true}\}\ c\ \{\Downarrow\mathsf{true}\}$