Lehrstuhl für Informatik 2 Softwaremodellierung und Verifikation | RWTH AACHEN UNIVERSITY

Exercise Sheet 5

Due date: November 29$^{\text{th}}$. You can hand in your solutions at the start of the exercise class.

Hint: Notation is as in the lecture. That is, c is a program, b a Boolean expression, σ a program state, etc.

Task 1: Fusion Lemma (3+2 points)

Prove or disprove:

(a) Let $(D_1, \sqsubseteq_1)$ and $(D_2, \sqsubseteq_2)$ be two CCPOS with least elements $\bot_1$ and $\bot_2$ respectively and let $F\colon D_1 \to D_1$, $G\colon D_2 \to D_2$, and $H\colon D_1 \to D_2$ be continuous functions, such that H is strict, i.e.

$$H(\bot_1) = \bot_2 \ ,$$

and $H \circ F = G \circ H$, i.e.

$$\forall\, d \in D_1\colon\ \ H(F(d)) = G(H(d)) \ .$$

Then

$$\mathsf{fix}(G) = H(\mathsf{fix}(F)) \ .$$

(b) (a) holds if strictness of H is dropped.

Task 2: Tarski–Kantorovich Principle (3 points)

Prove or disprove: Let $(D, \sqsubseteq)$ be a CCPO and let $F\colon D \to D$ be continuous. Moreover, let $d \in D$, such that $d \sqsubseteq F(d)$.

Then F has at least one fixpoint larger than d and the least of those fixpoints is given by

$$\bigsqcup \{F^n(d) \mid n \in \mathbb{N}\} \ .$$

Task 3: Complete Lattices (2 points)

A *complete lattice* is a pair $(D, \sqsubseteq)$, such that *every subset* of D (as opposed to every chain of D) has a supremum with respect to $\sqsubseteq$.

Prove or disprove: The Tarski–Knaster Fixpoint Theorem applies to complete lattices as well, i.e. if $F\colon D \to D$ is a continuous self–map on a complete lattice $(D, \sqsubseteq)$, then F has a least fixpoint with respect to $\sqsubseteq$ and this least fixpoint is given by

$$\bigsqcup \{F^n(\sqcup \emptyset) \mid n \in \mathbb{N}\} \ .$$