

# Theoretical Foundations of the UML

## Lecture 19: Introduction to Statecharts

Joost-Pieter Katoen

Lehrstuhl für Informatik 2  
Software Modeling and Verification Group

[moves.rwth-aachen.de/teaching/ws-1718/fuml/](http://moves.rwth-aachen.de/teaching/ws-1718/fuml/)

14. Januar 2018

- 1 Background
- 2 Ingredients of Statecharts
  - Mealy Machines
  - State Hierarchy
  - Orthogonality
  - Broadcast Communication
  - Some Small Examples
  - Priority and Nondeterminism
- 3 Semantics of Statecharts
- 4 Formal Definition of UML Statecharts

## 1 Background

## 2 Ingredients of Statecharts

- Mealy Machines
- State Hierarchy
- Orthogonality
- Broadcast Communication
- Some Small Examples
- Priority and Nondeterminism

## 3 Semantics of Statecharts

## 4 Formal Definition of UML Statecharts

- MSCs are a visual modelling formalism for requirements
- Statecharts is a visual modelling formalism for describing the behaviour of discrete-event systems
  - automata + hierarchy + communication + concurrency
- Developed by David Harel in 1987
  - professor at Weizmann Institute (Israel); co-founder of I-Logix Inc.
- Extensively used in embedded systems, automotive and avionics
- Variants: UML Statecharts, Stateflow, hierarchical state machines
  - supported by Statemate toolset, and Matlab/Simulink

## 1 Background

## 2 Ingredients of Statecharts

- Mealy Machines
- State Hierarchy
- Orthogonality
- Broadcast Communication
- Some Small Examples
- Priority and Nondeterminism

## 3 Semantics of Statecharts

## 4 Formal Definition of UML Statecharts

# What are Statecharts?

Statecharts constitute a **visual** formalism for:

[Harel, 1987]

- Describing states and transitions in a **modular** way
- Enabling **clustering** of states
- **Orthogonality**, i.e., concurrency
- **Refinement**, and
- Encouraging “**zoom**“ **capabilities** for moving easily back and forth between levels of abstraction

# What are Statecharts?

Statecharts := Mealy machines

- + State hierarchy
- + Broadcast communication
- + Orthogonality

## Definition (Mealy machine)

A **Mealy machine**  $\mathcal{A} = (Q, q_0, \Sigma, \Gamma, \delta, \omega)$  with:

- $Q$  is a finite set of states with initial state  $q_0 \in Q$
- $\Sigma$  is the input alphabet
- $\Gamma$  is the output alphabet
- $\delta : Q \times \Sigma \rightarrow Q$  is the deterministic (input) transition function, and
- $\omega : Q \times \Sigma \rightarrow \Gamma$  is the output function

## Intuition

A Mealy machine (or: finite-state transducer) is a finite-state automaton that produces **output** on a transition, based on current input and state.

## Moore machines

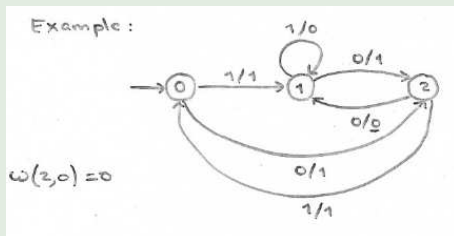
In a Moore machine  $\omega : Q \rightarrow \Gamma$ , output is purely state-based.



## Mealy machines

- No final (accepting) states
  - Transitions produce output
  - Deterministic input transition function
- ⇒ Acceptance of input words is not important, but the **generation of output words** from input words is important

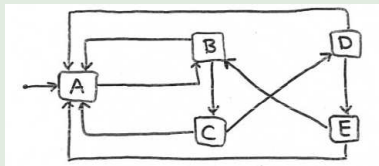
## Example



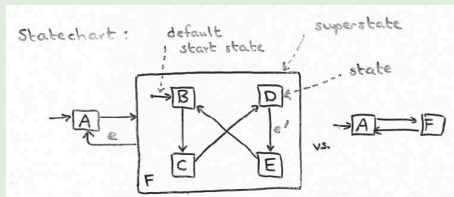
# Limitations of Mealy machines

- No support for **hierarchy**
  - all states are arranged in a flat fashion
  - no notion of substates
- Realistic systems require complex transition structure and huge number of states
  - scalability problems yields unstructured state diagrams
- No notion of concurrency
  - need for modeling independent components
- No notion of **communication** between automata.

## A bit unstructured Mealy machine



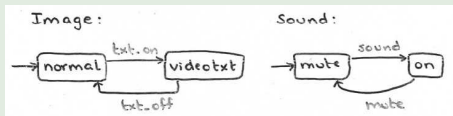
## An equivalent statechart



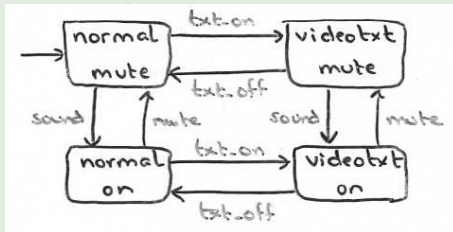
State hierarchy yields modular, hierarchical and structured models.

# Orthogonality

## Two independent components



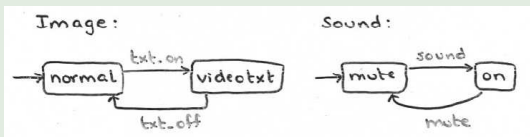
## Mealy machine for *Image* || *Sound*



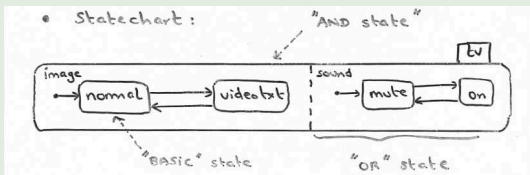
Number of states is exponential in size of concurrent components

# Orthogonality

## Two independent components



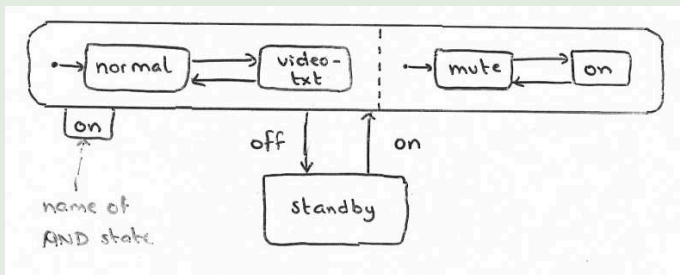
## Statechart for *Image* || *Sound*



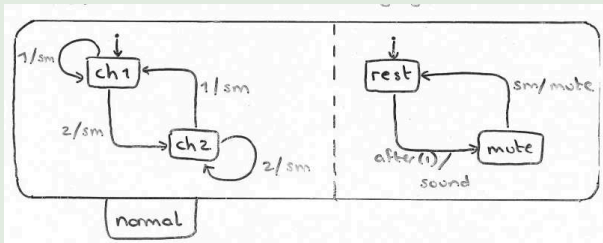
Concurrency modeled by independence

# Combined with state hierarchy

## Switching on and off the television

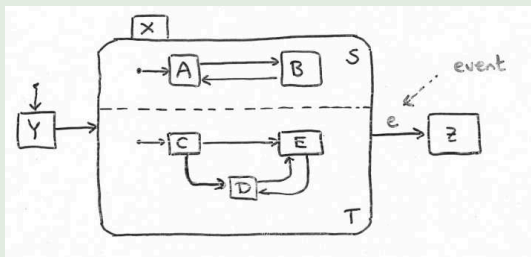


## Turn off sound on switching a tv channel



- Output is broadcast that can be received by any other component
- When pushing button 1, channel switches to its state channel 1, while generating signal  $sm$  on which component  $SM$  switches off the sound.

## Example concurrency in statecharts

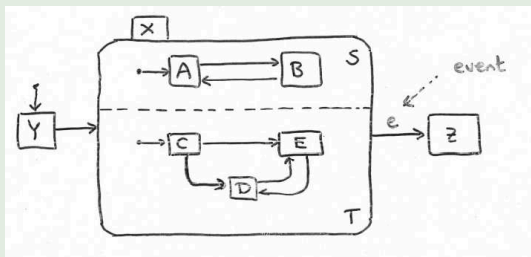


### Active

- As long as node  $X$  is **active**, nodes  $S$  and  $T$  are active
- Node  $S$  is active when either node  $A$  or  $B$  is active
- Node  $T$  is active if one of  $C$ ,  $D$  or  $E$  is active



## Example concurrency in statecharts

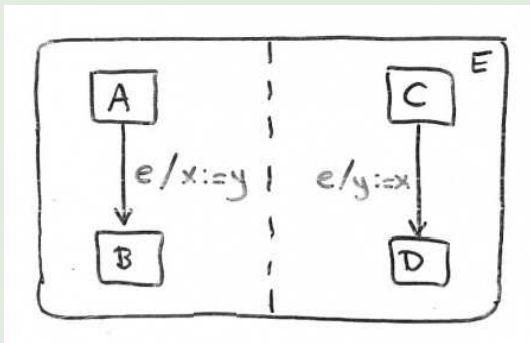


## Exit behaviour

- When node  $X$  exits, both nodes  $S$  and  $T$  exit
- When  $Y$  exits,  $X$  starts,  $S$  starts in  $A$ , and  $T$  starts in  $C$
- On the occurrence of event  $e$ , node  $X$  exits (regardless of current state in  $S$  or  $T$ )

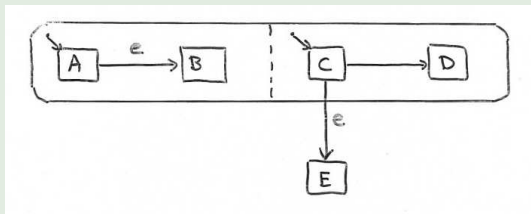
# Swapping two variables

## Swapping the value of variables $x$ and $y$



- If nodes  $A$  and  $C$  are active, assume  $x = 1, y = 2$
  - On occurrence of event  $e$ ,  $B$  and  $D$  are active, and  $x = 2, y = 1$
- ⇒ In Harel's statecharts, memory is shared, i.e., concurrent components have access to shared variables.

What if event  $e$  occurs when  $A$  and  $C$  are active?



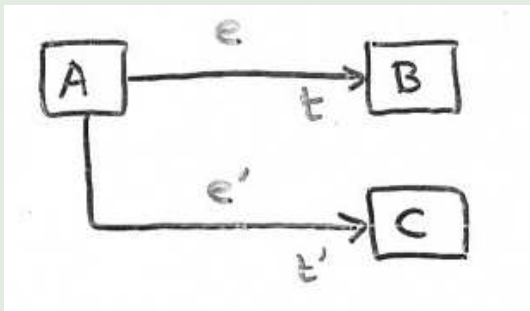
Solution:

Add a **priority** mechanism that decides whether:

- **inter-level** transitions (such as  $C \rightarrow E$ ), or
- **intra-level** transitions (such as  $A \rightarrow B$ )

prevail in case both are enabled.

What if event  $e$  and  $e'$  occur in  $A$ ?



**Solution:**

Choice is resolved nondeterministically, i.e., the next state is either  $B$  or  $C$ , but not both.

## 1 Background

## 2 Ingredients of Statecharts

- Mealy Machines
- State Hierarchy
- Orthogonality
- Broadcast Communication
- Some Small Examples
- Priority and Nondeterminism

## 3 Semantics of Statecharts

## 4 Formal Definition of UML Statecharts

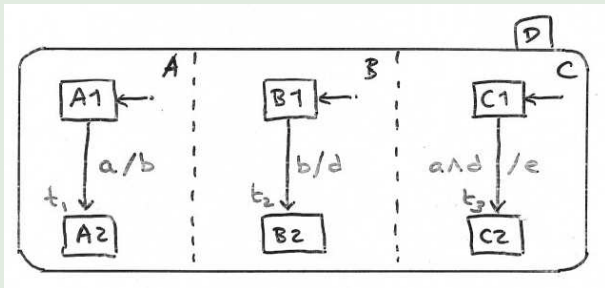
- Synchrony hypothesis (or: zero response time)
- Self-triggering
- Transition effect is contradicting its cause

Note: [von der Beeck, 1994]

Due to all these problems, hundred(s) (!) of different semantics for Statecharts have been defined in the literature.

# Synchrony hypothesis

## Event may yield chain of reactions



### Note:

- If A1, B1 and C1 are active and event *a* occurs, a **chain** of reactions occurs: transition  $t_1$  triggers  $t_2$ , and  $t_2$  triggers  $t_3$
- But transitions  $t_1$ ,  $t_2$ ,  $t_3$  occur at the same time as events do not take time (except for *after(d)* events with real *d*)

- 1 No shared variables
- 2 No negated and no compound events (like  $e \wedge e'$ )
- 3 Two-party communication rather than broadcast
- 4 No synchrony hypothesis:
  - events generated in step  $i$  can only be consumed in step  $i+1$ ,
  - and die otherwise, i.e., when they are not consumed in step  $i+1$ , events disappear



## 1 Background

## 2 Ingredients of Statecharts

- Mealy Machines
- State Hierarchy
- Orthogonality
- Broadcast Communication
- Some Small Examples
- Priority and Nondeterminism

## 3 Semantics of Statecharts

## 4 Formal Definition of UML Statecharts

## Definition (Statecharts)

A **statechart**  $SC$  is a triple  $(N, E, Edges)$  with:

- 1  $N$  is a set of **nodes** (or: states) structured in a **tree**
- 2  $E$  is a set of **events**
  - pseudo-event  $after(d)$  denotes a delay of  $d \in \mathbb{R}_{\geq 0}$  time units
  - $\perp \notin E$  stands for “no event available”
- 3  $Edges$  is a set of (hyper-) **edges**, defined later on.

## Definition (System)

A **system** is described by a finite collection of statecharts  $(SC_1, \dots, SC_k)$ .

*this is an elementary form; the UML allows more constructs  
that can be defined in terms of these basic elements*

- Deferred events simulate by regeneration
- Parametrised events simulate by set of parameter-less events
- Activities that take time simulate by start and end event
- Dynamic choice points simulate by intermediate state
- Synchronization states use a hyperedge with a counter
- History states (re)define an entry point

# Tree structure

## Function *children*

Nodes obey a **tree structure** defined by function  $children : N \rightarrow 2^N$  where  $x \in children(y)$  means that  $x$  is a child of  $y$ , or equivalently,  $y$  is the parent of  $x$ .

## Partial order $\trianglelefteq$

The partial order  $\trianglelefteq \subseteq N \times N$  is defined by:

- $\forall x \in N. x \trianglelefteq x$
- $\forall x, y \in N. x \trianglelefteq y$  if  $x \in children(y)$
- $\forall x, y, z \in N. x \trianglelefteq y \wedge y \trianglelefteq z \Rightarrow x \trianglelefteq z$

$x \trianglelefteq y$  means that  $x$  is a **descendant** of  $y$ , or equivalently,  $y$  is an **ancestor** of  $x$ . If  $x \trianglelefteq y$  or  $y \trianglelefteq x$ , nodes  $x$  and  $y$  are ancestrally related.

## Root node

There is a unique **root** with no ancestors, and  $\forall x \in N. x \trianglelefteq \text{root}$ .

## The type of nodes

Nodes are **typed**,  $type(x) \in \{ \text{BASIC}, \text{AND}, \text{OR} \}$  such that for  $x \in N$ :

- $type(\text{root}) = \text{OR}$
- $type(x) = \text{BASIC}$  iff  $children(x) = \emptyset$ , i.e.,  $x$  is a leaf
- $type(x) = \text{AND}$  implies  $(\forall y \in children(x). type(y) = \text{OR})$

## Default nodes

$default : N \rightarrow N$  is a partial function on domain

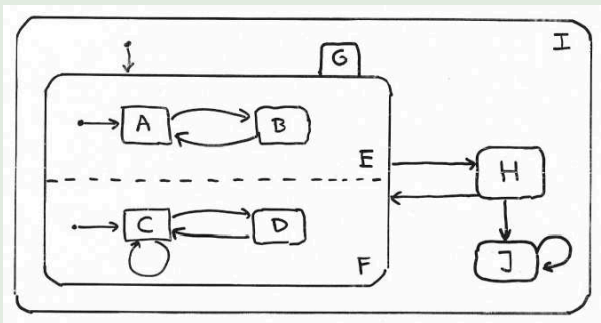
$\{ x \in N \mid type(x) = \text{OR} \}$  such that

$$default(x) = y \quad \text{implies} \quad y \in children(x).$$

The function  $default$  assigns to each OR-node  $x$  one of its children as **default** node that becomes active once  $x$  becomes active.

# Example

## Example statechart



## Definition (Edges)

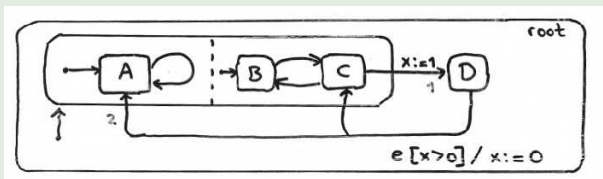
An **edge** is a quintuple  $(X, e, g, A, Y)$ , denoted  $X \xrightarrow{e[g]/A} Y$  with:

- $X \subseteq N$  is a set of **source** nodes with  $X \neq \emptyset$
- $e \in E \cup \{\perp\}$  is the **trigger** event
- $A \subseteq Act$  is a set of **actions**
  - such as  $v := \text{expr}$  or local variable  $v$  and expression  $\text{expr}$
  - or *send*  $j.e$ , i.e., send event  $e$  to statechart  $SC_j$
- **Guard**  $g$  is a Boolean expression over all variables in  $(SC_1, \dots, SC_k)$
- $Y \subseteq N$  is a set of **target** nodes with  $Y \neq \emptyset$

The sets  $X$  and  $Y$  may contain nodes at different depth in the node tree.

# Example (1)

## Example statechart



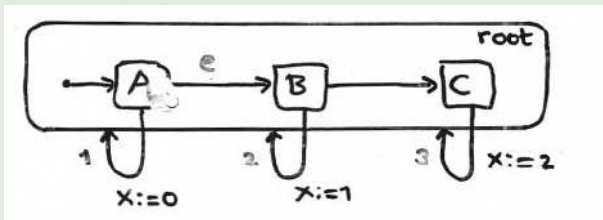
edge 1:  $\{ C \} \xrightarrow{\perp[true]/\{x:=1\}} \{ D \}$

edge 2:  $\{ D \} \xrightarrow{e[x>0]/\{x:=0\}} \{ A, C \}$



# Example (2)

## Example statechart

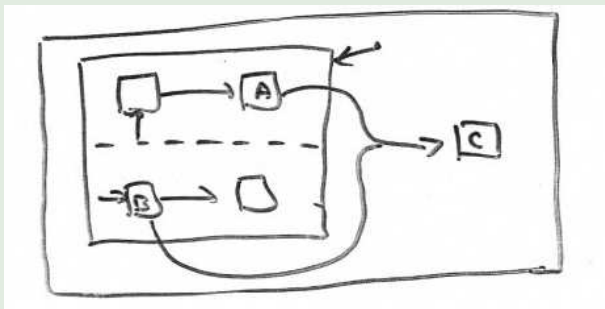


edge 1:  $\{ A \} \xrightarrow{e[true]/\emptyset} \{ B \}$

edge 2:  $\{ B \} \xrightarrow{\perp[true]/\{x:=1\}} \{ \text{root} \}$

# Example (3)

## Example statechart



edge :  $\{ A, B \} \dashrightarrow \{ C \}$