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Theoretical Foundations of the UML WS 17/18
— Exercise Sheet 7 —

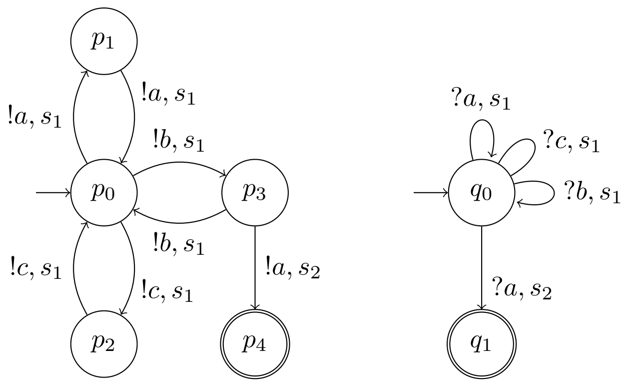
Hand in until December 19th before the exercise class.

General Remarks

- The exercises should be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 15:30 or by dropping them into the "TFUML" box at our chair. Do *not* hand in your solutions via L2P.
- There is no exercise class on 12th of December. The next exercise class in on 19th December.

Exercise 1 ($1 + 3 + 2 = 6$ Points)

a) Is the following CFM $\mathcal{A}_1$ deterministic? Justify your answer.



In the following we want to show that deterministic CFMs are strictly weaker than CFMs.

Consider the processes $\mathcal{P} = \{p_0, p_1, p_2, p_3, p_4\}$. We only send messages m from process p_0 to p_1 and p_2, from p_1 to p_3 and from p_2 to p_4. Consider the following language $\mathcal{L}_0$ which consists of all MSCs $\mathcal{M}$ such that

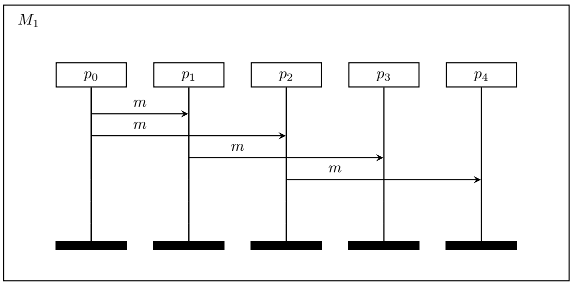
- $M \upharpoonright p_0 \in \Big(!(p_0, p_1, m)!(p_0, p_2, m)\Big)^*$
- $M \upharpoonright p_1 \in \Big(?(p_1, p_0, m)\ \big(!(p_1, p_3, m)+!(p_1, p_3, m)!(p_1, p_3, m)\big)\Big)^*$
- $M \upharpoonright p_2 \in \Big(?(p_2, p_0, m)\ \big(!(p_2, p_4, m)+!(p_2, p_4, m)!(p_2, p_4, m)\big)\Big)^*$
- $M \upharpoonright p_3 \in \Big(?(p_3, p_1, m)\Big)^*$
- $M \upharpoonright p_4 \in \Big(?(p_4, p_2, m)\Big)^*$

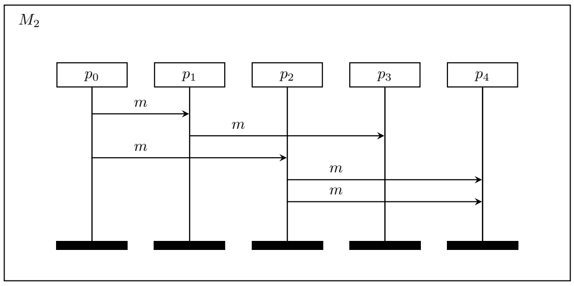
Intuitively process p_0 sends alternately to p_1 and p_2. Process p_1 (p_2) will receive an action from p_0 and then either send one or two messages to p_3 (p_4). Process p_3 (p_4) will just receive messages from p_1 (p_2).

Next we define the mapping $\phi : Act^* \to Act^*$ by renaming p_2 to p_1 and p_4 to p_3. For example $\phi\big(!(p_2, p_4, m)\big) =!(p_1, p_3, m)$.

We define the language $\mathcal{L}_1 \subseteq \mathcal{L}_0$ as the set of all MSCs from $\mathcal{L}_0$ where the sequence of actions of processes p_1 and p_2 are the same modulo ϕ, i.e., $M \upharpoonright p_1 = \phi\big(M \upharpoonright p_2\big)$.

As an example consider the MSCs M_1 and M_2 depicted below. It is $M_1 \in \mathcal{L}_1$ but $M_2 \notin \mathcal{L}_1$.



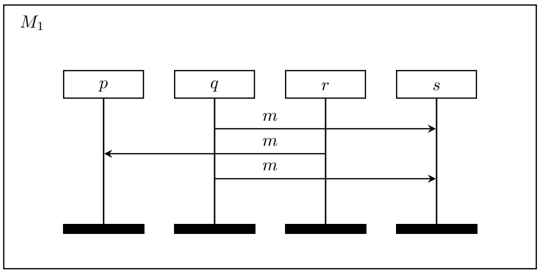


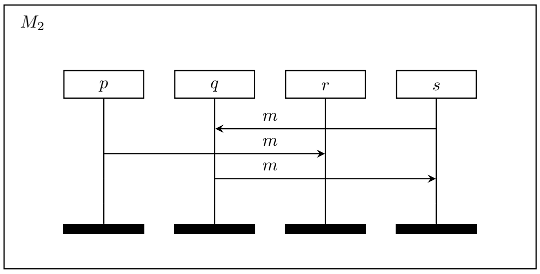
b) Give a CFM $\mathcal{A}$ which accepts the language $\mathcal{L}_1$, i.e., $\mathcal{L}(\mathcal{A}) = \mathcal{L}_1$.

c) Argue intuitively why there does not exist a *deterministic* CFM $\mathcal{A}'$ which accepts $\mathcal{L}_1$, i.e., $\mathcal{L}(\mathcal{A}') = \mathcal{L}_1$.

Exercise 2 (2 + 2 + 2 + 2 = 8 Points)

a) Two MSCs M_1 and M_2 are given as follows:

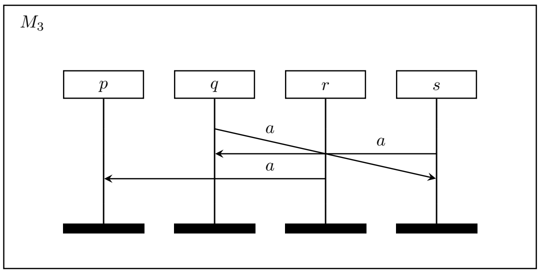


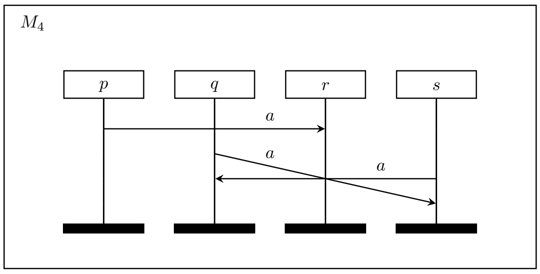


Show that the language $L := Lin(M_1) \cup Lin(M_2)$ is not closed under $\models$.

b) Modify M_1 or M_2 by adding only a pair of events, such that $Lin(M_1) \cup Lin(M_2)$ is closed under $\models$. Justify why it is closed under $\models$.

c) Two MSCs M_3 and M_4 are given as follows:





Is the language $L' := Lin(M_3) \cup Lin(M_4)$ closed under $\models$? Justify your answer.

d) Is the language $L' := Lin(M_3) \cup Lin(M_4)$ closed under $\models^{df}$? Justify your answer.

Exercise 3 (2 + 3 + 1 = 6 Points)

We consider the relation $\models^{df}_{alt}$ which is defined for a well-formed language $L \subseteq Act^*$ and proper word $w \in Act^*$ as follows:

$$L \models^{df}_{alt} w \text{ iff } (\forall p \in \mathcal{P}.\, \exists v \in \mathit{pref}(L).\, w \upharpoonright p = v \upharpoonright p)$$

a) Prove or disprove whether this definition is equivalent to the definition of the (deadlock-free) inference relation $\models^{df}$ as given in the lecture (Lecture 13, slide 12). More precisely, prove or disprove whether

$$L \models^{df}_{alt} w \text{ iff } L \models^{df} w$$

holds for any proper w.

b) Let $L \subseteq Act^*$ be well-formed and closed under $\models^{df}$. Does this imply that L is also closed under $\models$? Prove or disprove.

c) Let $L \subseteq Act^*$ be well-formed, closed under $\models^{df}$ and weakly closed under $\models$. Does this imply that L is also closed under $\models$? Prove or disprove.