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Theoretical Foundations of the UML WS 17/18

— Exercise Sheet 3 —

Hand in until November 14th before the exercise class.

General Remarks

- The exercises should be solved in groups of *three* students.
- You may hand in your solutions for the exercises just before the exercise class starts at 15:30 or by dropping them into the "TFUML" box at our chair. Do *not* hand in your solutions via L2P.

Exercise 1 (6 Points)

Consider the following sets U and W as input of the Post correspondence problem:

$$U = \{\underbrace{a}_{u_1}, \underbrace{aba}_{u_2}, \underbrace{ba}_{u_3}\} \qquad W = \{\underbrace{aa}_{w_1}, \underbrace{aab}_{w_2}, \underbrace{bab}_{w_3}\}$$

a) Apply the reduction to the emptiness problem of the intersection of MSG languages, i.e., draw the corresponding MSGs $\mathcal{G}_U$ and $\mathcal{G}_W$ as presented in the lecture (cf. Lecture 5 from October 30th).

b) Does $\mathcal{L}(\mathcal{G}_U) \cap \mathcal{L}(\mathcal{G}_W) = \emptyset$ hold? Justify your answer.

Exercise 2 (8 Points)

Given a nondeterministic finite-state automaton (NFA) P and an MSG G, let $L(P)$ be the language of P and $Lin(L(G))$ be the word language of G. Our goal in this exercise is to show that the decision problem whether $L(P) \cap Lin(L(G)) = \emptyset$ is undecidable. To this end, we provide a reduction from the Post correspondence problem (PCP) to the emptiness problem of the intersection of $L(P)$ and $Lin(L(G))$ as follows:

Let (U, W) be an arbitrary input for the PCP with $U = \{u_1, \ldots, u_n\}$ and $W = \{w_1, \ldots, w_n\}$. We consider the MSG G_U as presented in the lecture (cf. Exercise 1).

Tasks:

a) Provide a (computable) construction of an NFA P_W from the set W such that

$$L(P_W) \cap Lin(L(G_U)) \neq \emptyset \text{ if and only if there is a solution for the PCP instance } (U, W) \qquad (*)$$

Sketch a proof that (*) indeed holds, i.e.,

b) Show that if $L(P_W) \cap Lin(L(G_U)) \neq \emptyset$ then the PCP instance (U, W) has a solution.

c) Show that if the PCP instance (U, W) has a solution then $L(P_W) \cap Lin(L(G_U)) \neq \emptyset$.

Exercise 3 (3 Points)

Prove or disprove: There exists a CMSC M_1 with process set $\mathcal{P}_1 = \{p_1, p_2\}$, such that for all CMSC M_2 which satisfy the following side conditions, it holds that $M_1 \bullet M_2$ violates the FIFO property.

The side conditions are:

- For the process set $\mathcal{P}_2$ of M_2 it holds that $\mathcal{P}_2 = \mathcal{P}_1$, and
- M_2 contains an unmatched receive event of the form "p_2 receives message content a from p_1".

Exercise 4 (3 Points)

Give three CMSCs M_1, M_2, and M_3, such that

$$(M_1 \bullet M_2) \bullet M_3 \;\neq\; M_1 \bullet (M_2 \bullet M_3)\ ,$$

even though both $(M_1 \bullet M_2) \bullet M_3$ and $M_1 \bullet (M_2 \bullet M_3)$ are defined, i.e., they are valid CSMCs which satisfy the FIFO property.