Concurrency Theory

Lecture 20: McMillan Prefixes

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http://moves.rwth-aachen.de/teaching/ws-1718/ct

January 8, 2018

Overview

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- Partial-order semantics of Petri nets = set of distributed runs
  - a distributed run is an acyclic (causal) net which contains no choices
  - a distributed run is a partial ordering of transition occurrences
- Today: the set of all distributed runs can be represented by a finite prefix of the unfolding of the net.

Branching process: preamble

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Let $N = (P, T, F, M)$ be a net. Nodes x_1 and x_2 are in conflict, denoted $x_1 \# x_2$, if there exist distinct transitions $t_1, t_2 \in T$ such that ${}^\bullet t_1 \cap {}^\bullet t_2 \neq \varnothing$ and $(t_1, x_1) \in F^*$ and $(t_2, x_2) \in F^*$.

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Note that in a causal net $\# = \varnothing$ as ${}^\bullet t_1 \cap {}^\bullet t_2 = \varnothing$ for any two distinct transitions t_1 and t_2.

Occurrence net

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A net $K = (Q, V, G, M)$ is an occurrence net iff:

1. for each $q \in Q$, $|{}^{\bullet}q| \leqslant 1$
2. the transitive closure G^+ of G is irreflexive
3. for each node $x \in Q \cup V$ we have $\{ y \mid (y, x) \in G^+ \}$ is finite
4. no transition $v \in V$ is in self-conflict
5. $M_0 = {}^{\circ}K \ = \ \{ q \in Q \mid \ {}^{\bullet}q = \varnothing \}$.

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Remark

Since $\# = \varnothing$ in a causal net, and each causal net fulfils the remaining conditions, every causal net is an occurrence net.

Branching process

Branching process [Engelfriet 1991]

A branching process of net N is a pair (K, h) where $K = (Q, V, G, M)$ is an occurrence net and h a net homomorphism from K to N such that:

$$\forall v, v' \in Q.\ ({}^{\bullet}v = {}^{\bullet}v' \text{ and } h(v) = h(v') \quad \text{implies} \quad v = v')\,.$$

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Examples

On the black board.

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Relating branching processes

Homomorphisms and isomorphisms between branching processes

Let $B_1 = (K_1, h_1)$ and $B_2 = (K_2, h_2)$ be two branching processes of net N. A homomorphism from B_1 to B_2 is a homomorphism h from K_1 to K_2 such that $h_2 \circ h = h_1$.[2]

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Being isomorphic is an equivalence relation. Its equivalence classes are called isomorphism classes.

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Approximation of branching processes

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Intuition

B_1 approximates B_2 whenever every (partial) distributed run in B_1 is also contained in B_2. In other words, B_1 is isomorphic to an initial part of B_2. Being an approximation on branching processes is the analogue of being a prefix on sequences.

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Lemma

Approximation is preserved by isomorphism: if B'_i is isomorphic to B_i (for $i = 1, 2$), then $B_1 \sqsubseteq B_2$ implies $B'_1 \sqsubseteq B'_2$. Thus, $\sqsubseteq$ can be extended to a partial order on isomorphism classes (of branching processes).

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Lemma

Approximation is preserved by isomorphism: if B_i' is isomorphic to B_i (for $i = 1, 2$), then $B_1 \sqsubseteq B_2$ implies $B_1' \sqsubseteq B_2'$. Thus, $\sqsubseteq$ can be extended to a partial order on isomorphism classes (of branching processes).

Proof.

Home exercise. Basically juggling with homomorphisms. □

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Complete lattice

Recall that a complete lattice is a partial order $(\mathbb{B}, \sqsubseteq)$ such that all subsets of $\mathbb{B}$ have LUBs and GLBs.

The true concurrency semantics of a net

Corollary: the unfolding of a net

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We denote by $B_{\mathsf{max}} = ((P_{\mathsf{max}}, T_{\mathsf{max}}, F_{\mathsf{max}}), h_{\mathsf{max}})$ a representative of the isomorphism class of the maximal branching process of N.

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Example

On the black board.

The true concurrency semantics of Petri nets

The true concurrency semantics of a Petri net is given by its unfolding.

Recall: The interleaving semantics of a Petri net is given by its marking graph.

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Finite prefix existence theorem [McMillan, 1992]

For every finite one-bounded net N, there exists a finite prefix B_{fin} of B_{max} that covers all reachable markings of N. The size of the finite prefix can maximally be exponential in the size of N.

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Proof.

Follows directly from two facts:

1. Every reachable marking is represented by some cut of B_{max}, and
2. The set of reachable markings of a finite one-bounded net is finite.

□

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The set $C \subseteq V$ is a configuration of K whenever:

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A configuration can be seen as the set of transitions that have occurred so far in a distributed run. Examples on the black board.

Fact

For configuration C of $B_{\max}$ (of net N), and $x_1 \ldots x_n$ a linearisation of the transitions in C (respecting $\preceq$), the sequence $h_{\max}(x_1) \ldots h_{\max}(t_n)$ is a sequential run of the original net N.

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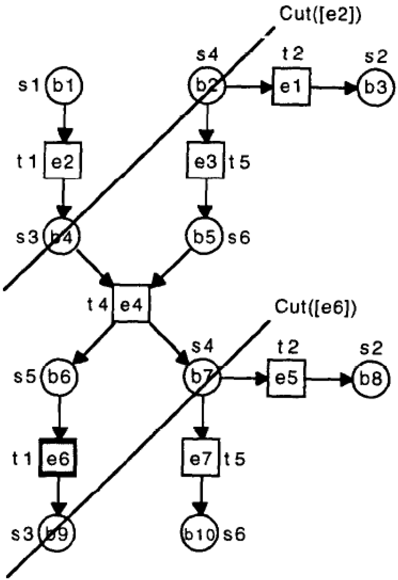
Intuition

Cuts correspond to markings reached by firing all transitions in a given finite configuration.

Example


Cut([e2])
s4
t2
s2
s1
b1
b2
e1
b3
t1
e2
e3
t5
s3
b4
b5
s6
t4
e4
Cut([e6])
s4
t2
s2
s5
b6
b7
e5
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t1
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b9
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s6

Transition causes

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Let $K = (Q, V, G)$ be an occurrence net and $v \in V$. The set $[v]$ of causes of v is defined by:

$$[v] \;=\; \{\, v' \in V \mid v' \preceq v \,\}.$$

(Recall that $\preceq$ denotes G^*, the reflexive and transitive closure of G.)

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Facts

1. For each v, $[v]$ is a finite configuration.
2. For every configuration C of K, either $v \notin C$ or $[v] \subseteq C$.

Cut-off event

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Let $B_{\max} = ((P_{\max}, T_{\max}, G_{\max}), h_{\max})$. Transition $t \in T_{\max}$ is a cut-off transition if there exists a transition $t' \in T_{\max} \cup \{\bot\}$ such that:

$$\big|[t']\big| < \big|[t]\big| \text{ and } M([t]) = M([t']).$$

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Dummy transition

Remark: $\perp$ is a dummy transition having no input places and ${}^\circ B_{\max}$ as output places, for which we let $[\perp] = \varnothing$. This yields that if $M([t]) = M_0$, then t is a cut-off transition.

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Remark: $\bot$ is a dummy transition having no input places and $^\circ B_{\max}$ as output places, for which we let $[\bot] = \varnothing$. This yields that if $M([t]) = M_0$, then t is a cut-off transition.

Fact

If $\big|[t']\big| < \big|[t]\big|$ and $M([t]) = M([t'])$, then the "continuations" of $B_{\max}$ from $Cut([t])$ and $Cut([t'])$ are isomorphic.

McMillan prefix

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The McMillan prefix of one-bounded net N is the branching process B_{fin}, the unique prefix of $B_{\max}$ having T_{fin} as set of transitions satisfying for each $t \in T_{\max}$::

$$t \in T_{\text{fin}} \quad \text{iff} \quad \text{no transition } t' \prec t \text{ is a cut-off transition.}$$

Computing the McMillan prefix

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5. Terminate when no further transitions can be added.

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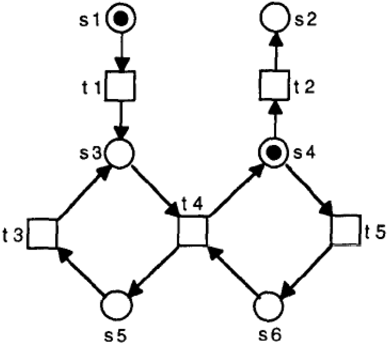
Remark

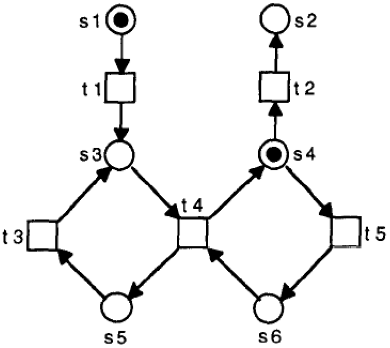
Termination is ensured by the finiteness of the number of reachable markings on N, as N is one-bounded.

Example net and one of its branching processes



A sample one-bounded elementary system net

Example net and one of its branching processes



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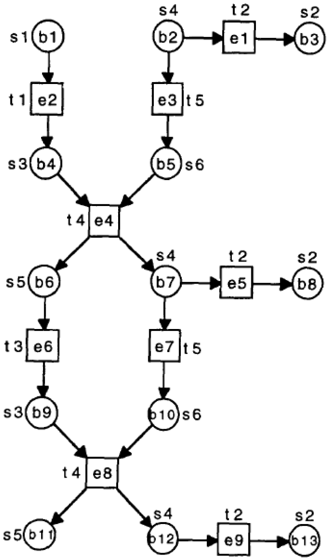
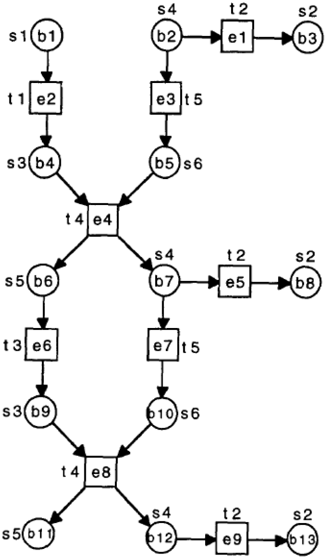


Its McMillan prefix



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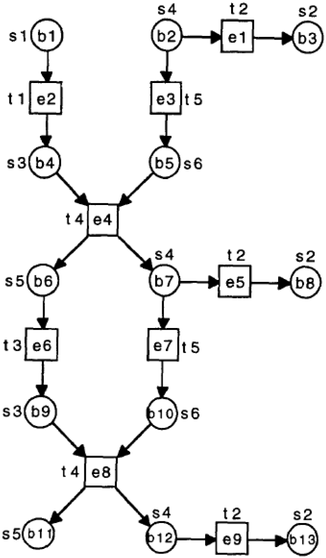
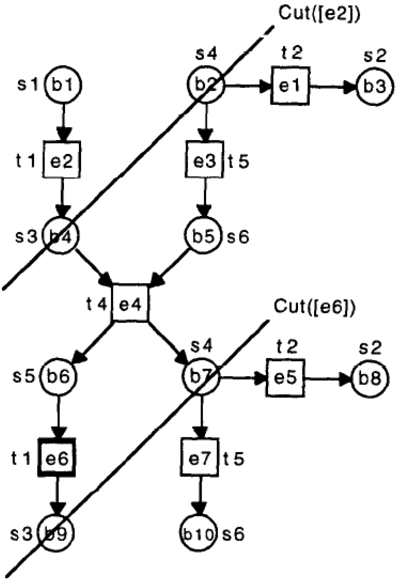
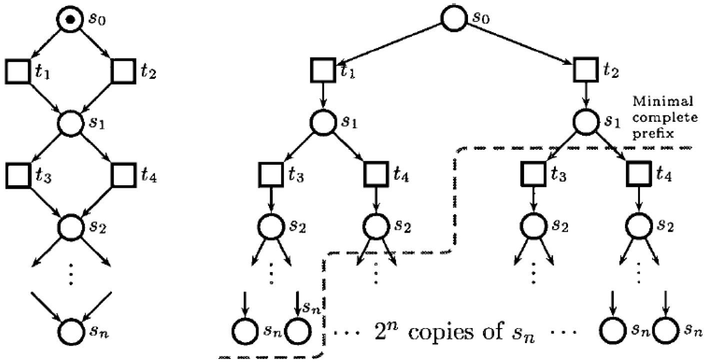




An exponentially-sized McMillan prefix



For every marking M all the configurations $[t]$ satisfying $M([t]) = M$ have the same size, and therefore there exist no cut-off events [Kishinevsky and Taubin]

Overview

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