Concurrency Theory

Winter Semester 2017/18

Lecture 12: Properties of Strong Bisimulation

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Outline of Lecture 12

Strong Bisimulation I

Definition (Strong bisimulation) (Park 1981, Milner 1989)

A binary relation $\rho \subseteq Prc \times Prc$ is a strong bisimulation whenever for every $(P, Q) \in \rho$ and $\alpha \in Act$:

1. if $P \xrightarrow{\alpha} P'$, then there exists $Q' \in Prc$ such that $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \rho$, and
2. if $Q \xrightarrow{\alpha} Q'$, then there exists $P' \in Prc$ such that $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \rho$.

Definition (Strong bisimilarity)

Processes P and Q are strongly bisimilar, denoted $P \sim Q$, iff there is a strong bisimulation ρ with $(P, Q) \in \rho$. Thus,

$$\sim = \bigcup \{\rho \mid \rho \text{ is a strong bisimulation}\}.$$

Relation $\sim$ is called strong bisimilarity.

Recap: Strong Bisimulation

Summary So Far

1. $\sim$ is an equivalence relation.
2. $\sim$ is less distinctive than LTS isomorphism.
3. $P \sim Q$ implies that P and Q are trace equivalent.
4. For deterministic P and Q: $P \sim Q$ iff $Tr(P) = Tr(Q)$.

Remaining interesting properties:

- $\sim$ is a CCS congruence.
- $\sim$ preserves deadlocks.

Outline of Lecture 12

Recap: Strong Bisimulation

Congruence and Deadlock Sensitivity

Buffers Revisited

Epilogue

Congruence and Deadlock Sensitivity

Congruence

Theorem 12.1 (CCS congruence property of $\sim$)

Strong bisimilarity $\sim$ is a CCS congruence, that is, whenever $P, Q \in Prc$ such that $P \sim Q$,

$$\begin{array}{rcll} \alpha.P & \sim & \alpha.Q & \text{for every action } \alpha \\ P + R & \sim & Q + R & \text{for every process } R \\ P \parallel R & \sim & Q \parallel R & \text{for every process } R \\ P \setminus L & \sim & Q \setminus L & \text{for every set } L \subseteq A \\ P[f] & \sim & Q[f] & \text{for every relabelling } f : A \to A \end{array}$$

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Proof.

- for $\parallel$: on the board
- for other CCS operators: left as an exercise □

Deadlock Sensitivity of $\sim$

Definition (Deadlock; cf. Definition 10.6)

Let $P, Q \in Prc$ and $w \in Act^*$ such that $P \xrightarrow{w} Q$ and $Q \not\longrightarrow$. Then Q is called a w-**deadlock** of P.

Congruence and Deadlock Sensitivity

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Definition (Deadlock sensitivity; cf. Definition 10.8)

Relation $\equiv \subseteq Prc \times Prc$ is deadlock sensitive whenever:

$$P \equiv Q \text{ implies } \left(\forall w \in Act^*.\ P \text{ has a } w\text{-deadlock iff } Q \text{ has a } w\text{-deadlock}\right).$$

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on the board $\square$

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Two Buffers

Example 12.3 (One-place buffer)

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Example 12.4 (Two-place buffer)

$$\begin{aligned} B_0^2 &= in.B_1^2 \\ B_1^2 &= in.B_2^2 + \overline{out}.B_0^2 \\ B_2^2 &= \overline{out}.B_1^2. \end{aligned}$$

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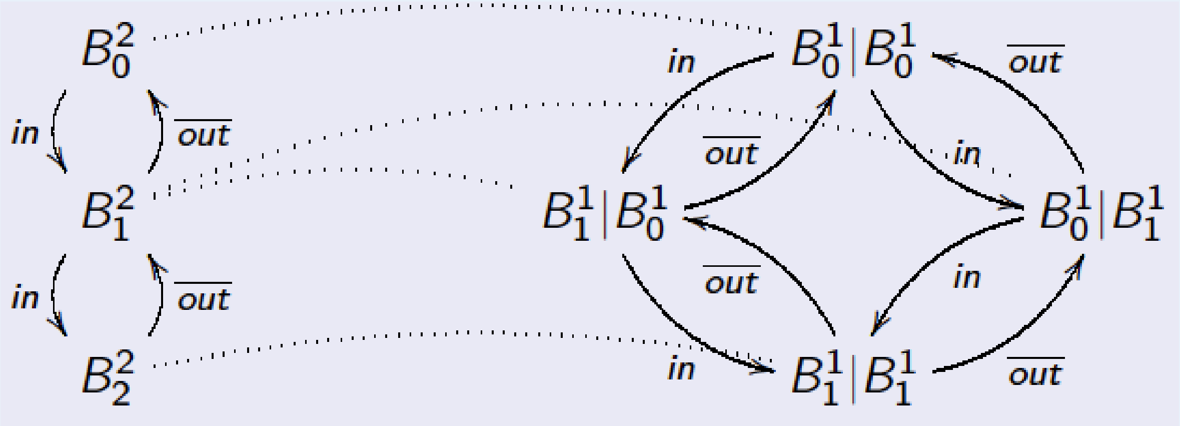
$B^2_0 \sim B^1_0 \parallel B^1_0$



Semaphores: A Generalisation

Example 12.5 (An n-ary semaphore)

Let S_i^n stand for a semaphore for n resources i of which are taken:

$$\begin{aligned} S_0^n &= get.S_1^n \\ S_i^n &= get.S_{i+1}^n + put.S_{i-1}^n \quad \text{for } 0 < i < n \\ S_n^n &= put.S_{n-1}^n \end{aligned}$$

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This process is strongly bisimilar to n parallel binary semaphores:

Lemma 12.6

For every $n \in \mathbb{N}_+$, we have: $S_0^n \sim \underbrace{S_0^1 \parallel \cdots \parallel S_0^1}_{n \text{ times}}$.

Semaphores II

Lemma

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Proof.

Consider the following binary relation where $i_1, i_2, \ldots, i_n \in \{0, 1\}$:

$$\rho = \left\{ \left(S_i^n, S_{i_1}^1 \parallel \cdots \parallel S_{i_n}^1\right) \;\middle|\; \sum_{j=1}^{n} i_j = i \right\}$$

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Then: ρ is a strong bisimulation and $(S_0^n, \underbrace{S_0^1 \parallel \cdots \parallel S_0^1}_{n \text{ times}}) \in \rho$. □

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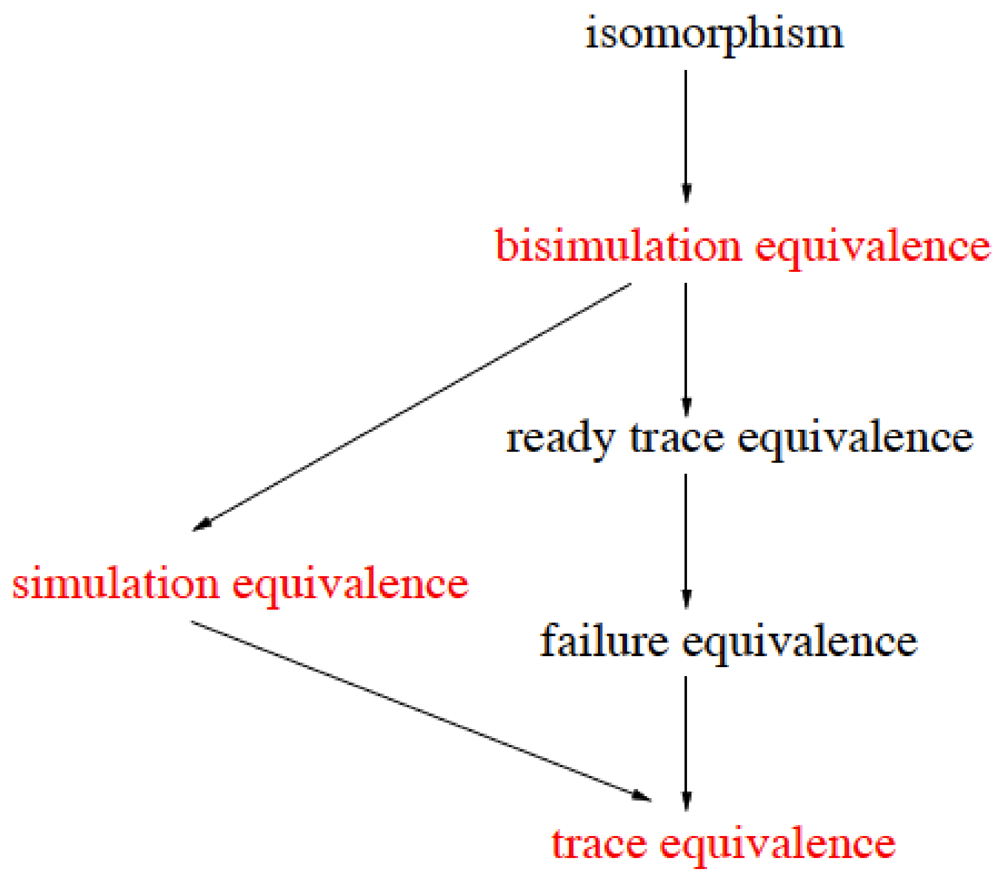
Overview of Some Behavioural Equivalences



Summary

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- Strong bisimilarity $\sim$:
  1. is the largest strong bisimulation
  2. is an equivalence
  3. is a CCS congruence
  4. is strictly finer than trace equivalence
  5. is deadlock sensitive