Concurrency Theory

Winter Semester 2017/18

Lecture 11: Strong Bisimulation

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Outline of Lecture 11

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- Our (serious) attempt today: Milner's strong bisimulation.

Robin Milner (1934–2010)

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Three versions will be considered in this course:

1. Strong bisimulation: ignore the special function of τ-actions
2. Weak bisimulation: treat τ-actions as invisible
3. Simulation relations: unidirectional versions of bisimulation

Strong Bisimulation I

Definition 11.1 (Strong bisimulation) (Park 1981, Milner 1989)

A binary relation $\rho \subseteq \mathit{Prc} \times \mathit{Prc}$ is a strong bisimulation whenever for every $(P, Q) \in \rho$ and $\alpha \in \mathit{Act}$:

1. if $P \xrightarrow{\alpha} P'$, then there exists $Q' \in \mathit{Prc}$ such that $Q \xrightarrow{\alpha} Q'$ and $(P', Q') \in \rho$, and
2. if $Q \xrightarrow{\alpha} Q'$, then there exists $P' \in \mathit{Prc}$ such that $P \xrightarrow{\alpha} P'$ and $(P', Q') \in \rho$.

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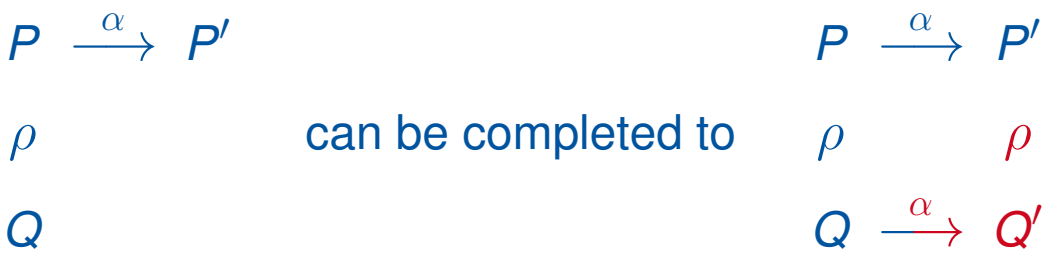
Relation $\sim$ is called strong bisimilarity.

Strong Bisimulation II



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$$\begin{array}{ccc} P & \xrightarrow{\alpha} & P' \\ \rho & & \\ Q & & \end{array} \quad \text{can be completed to} \quad \begin{array}{ccc} P & \xrightarrow{\alpha} & P' \\ \rho & & \rho \\ Q & \xrightarrow{\alpha} & Q' \end{array}$$

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Examples

Example 11.3 (A first example)

Claim: $P \sim Q$ where $P = a.P_1 + a.P_2$ $\quad Q = a.Q_1$

$P_1 = b.P_2$ $\quad Q_1 = b.Q_1$

$P_2 = b.P_2$

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Proof: $\rho = \{(P, Q), (P_1, Q_1), (P_2, Q_1)\}$ is a strong bisimulation

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Example 11.5 (Counterexample; cf. Example 10.10)

Show on board that $CTM \not\sim CTM'$ where

$$CTM = coin.\left(\overline{coffee}.CTM + \overline{tea}.CTM\right)$$
$$CTM' = coin.\overline{coffee}.CTM' + coin.\overline{tea}.CTM'.$$

Properties of Strong Bisimilarity

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Proof.

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Bisimulation on Paths

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Whenever we have:

$$P_0 \xrightarrow{\alpha_1} P_1 \xrightarrow{\alpha_2} P_2 \xrightarrow{\alpha_3} P_3 \xrightarrow{\alpha_4} P_4 \ldots\ldots$$

$$\rho$$

$$Q_0$$

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Proof.

by induction on the length of the path □

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But: $P \not\sim Q$, as there is no state in the LTS of Q that is bisimilar to P_1.
Why? No state in Q can perform both b and c. □

Deterministic Transition Systems

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$P \in Prc$ is deterministic whenever for every of its states s it holds:

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Left as an exercise. In fact, for deterministic processes, trace equivalence, complete trace, failure trace, and ready trace equivalence all coincide. □