Concurrency Theory

Winter Semester 2017/18

Lecture 10: Trace Equivalence

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Outline of Lecture 10

Introduction

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- This gives rise to the natural question: when are two CCS processes behaving the same?
- As there are many different interpretations of “behaving the same”, different behavioural equivalences have emerged.

Behavioural Equivalence

Implementation

$$CM = coin.\overline{coffee}.CM$$
$$CS = \overline{pub}.\overline{coin}.coffee.CS$$
$$Uni = (CM \parallel CS) \setminus \{coin, coffee\}$$

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$$Uni = (CM \parallel CS) \setminus \{coin, coffee\}$$

Specification

$$Spec = \overline{pub}.Spec$$

Question

Are the specification *Spec* and implementation *Uni* behaviourally equivalent:

$$Spec \stackrel{?}{\equiv} Uni$$

Outline of Lecture 10

Equivalence Relations

Some reasonable required properties

- Reflexivity: $P \equiv P$ for every process P
- Symmetry: $P \equiv Q$ if and only if $Q \equiv P$
- Transitivity: $Spec_0 \equiv \ldots \equiv Spec_n \equiv Impl$ implies that $Spec_0 \equiv Impl$

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Definition 10.1 (Equivalence)

A binary relation $\equiv \subseteq S \times S$ over a set S is an equivalence if

- it is reflexive: $s \equiv s$ for every $s \in S$,
- it is symmetric: $s \equiv t$ implies $t \equiv s$ for every $s, t \in S$,
- it is transitive: $s \equiv t$ and $t \equiv u$ implies $s \equiv u$ for every $s, t, u \in S$.

Isomorphism: An Example Behavioural Equivalence

Definition 10.2 (LTS isomorphism)

Two LTSs $T_1 = (S_1, Act_1, \longrightarrow_1)$ and $T_2 = (S_2, Act_2, \longrightarrow_2)$ are isomorphic, denoted $T_1 \equiv_{iso} T_2$, if there exists a bijection $f : S_1 \to S_2$ such that

$$s \xrightarrow{\alpha}_1 t \quad \text{if and only if} \quad f(s) \xrightarrow{\alpha}_2 f(t).$$

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Example 10.3 (Abelian monoid laws for $+$ and $\|$)

For all CCS processes $P, Q \in Prc$,

1. $LTS(P + Q) \equiv_{iso} LTS(Q + P)$, $LTS(P \parallel Q) \equiv_{iso} LTS(Q \parallel P)$
2. $LTS((P + Q) + R) \equiv_{iso} LTS(P + (Q + R))$, $LTS((P \parallel Q) \parallel R) \equiv_{iso} LTS(P \parallel (Q \parallel R))$
3. $LTS(P + \text{nil}) \equiv_{iso} LTS(P \parallel \text{nil}) \equiv_{iso} LTS(P)$

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Assumption

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Caveat

But: isomorphism is very distinctive. For instance,

$$X = a.X \quad \text{and} \quad Y = a.a.Y$$

are distinguished although both can (only) execute infinitely many a-actions and should thus be considered equivalent.

Outline of Lecture 10

The Wish List for Behavioural Equivalences

1. Less distinctive than isomorphism: an equivalence should distinguish less processes than LTS isomorphism does, i.e., $\equiv$ should be coarser than LTS isomorphism:

$$LTS(P) \equiv_{iso} LTS(Q) \implies P \equiv Q.$$

[1] Later, we will enlarge this to a set of properties that can be expressed in a logic.

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4. Deadlock preservation: equivalent processes should have the same deadlock behaviour, i.e., equivalent process can either both deadlock, or both cannot.[1]
5. Optional: the coarsest possible equivalence: there should be no less discriminating equivalence satisfying all these requirements.

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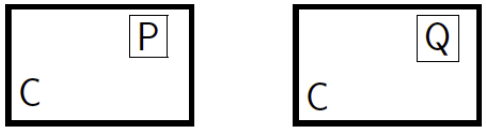
Requirements on Behavioural Equivalences

What is a Congruence?



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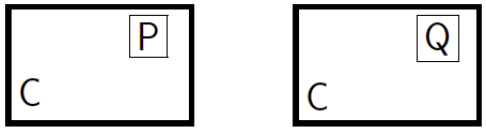

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A CCS context is a CCS process fragment with a “hole” in it (examples on the board).

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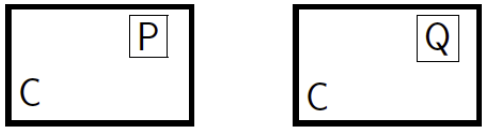

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CCS congruences informally

Relation $\equiv$ is a CCS congruence whenever $P \equiv Q$ implies $C(P) \equiv C(Q)$ for every CCS context C.

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Example 10.4 (Congruence)

Let $a \equiv b$ for $a, b \in \mathbb{Z}$ whenever $a \bmod k = b \bmod k$, for some $k \in \mathbb{N}_+$.
Equivalence relation $\equiv$ is a congruence for addition and multiplication.

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Important motivations of requiring $\equiv$ to be a congruence on processes:

1. Model-based development through refinement: replacing an abstract model *Spec* by a more detailed model *Impl*
2. Abstraction/optimisation: replacing a large (concrete) model *Impl* by a smaller (more abstract) model *Spec*.

Remark: congruences induce quotient structures with equivalence classes as elements

CCS Congruences Formally

Definition 10.5 (CCS congruence)

An equivalence relation $\equiv \subseteq Prc \times Prc$ is a CCS congruence if it is preserved by all CCS constructs, i.e., if $P, Q \in Prc$ with $P \equiv Q$ then:

$$
\begin{aligned}
\alpha.P &\equiv \alpha.Q && \text{for every } \alpha \in Act \\
P + R &\equiv Q + R && \text{for every } R \in Prc \\
P \parallel R &\equiv Q \parallel R && \text{for every } R \in Prc \\
P \setminus L &\equiv Q \setminus L && \text{for every } L \subseteq A \\
P[f] &\equiv Q[f] && \text{for every } f : A \to A
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Thus, a CCS congruence is substitutive for all possible CCS contexts.

Deadlocks

Definition 10.6 (Deadlock)

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$P = a.b.\text{nil} + a.\text{nil}$ has an a-deadlock, whereas $Q = a.b.\text{nil}$ has not.

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Definition 10.8 (Deadlock sensitivity)

Relation $\equiv \subseteq Prc \times Prc$ is deadlock sensitive whenever:

$$P \equiv Q \text{ implies } \left(\forall w \in Act^*.\ P \text{ has a } w\text{-deadlock iff } Q \text{ has a } w\text{-deadlock}\right).$$

Outline of Lecture 10

Trace Equivalence

Trace language (Definition 3.2)

The trace language of $P \in Prc$ is defined by:

$$Tr(P) := \{w \in Act^* \mid \exists P' \in Prc.\, P \xrightarrow{w} P'\}.$$

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Trace equivalence is evidently an equivalence relation and is less discriminative than isomorphism.

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For the other CCS constructs, the proof goes along similar lines. Exercise: do the proof for $\|$. □

Trace Equivalence Revisited

Two coffee machines

Example 10.10

Consider the coffee/tea machines CTM and its variant CTM':

$$CTM = coin.\left(\overline{coffee}.CTM + \overline{tea}.CTM\right)$$

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Why? $C(CTM')$ may yield a deadlock, but $C(CTM)$ does not.

Checking Trace Equivalence

Traces by automata

For finite-state P, the trace language $Tr(P)$ of process P is accepted by the (non-deterministic) finite automaton obtained from the LTS of P with initial state P and making all states accepting (final).

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Proof.

Checking whether $Tr(P) = Tr(Q)$, for finite-state P and Q, boils down to deciding whether their non-deterministic automata accept the same language. As this problem in automata theory is PSPACE-complete, it follows that checking $Tr(P) = Tr(Q)$ is PSPACE-complete. □

Traces and Deadlocks

Example 10.12 (Traces and deadlocks)

Traces and deadlocks are independent in the following sense:

same traces
different deadlocks

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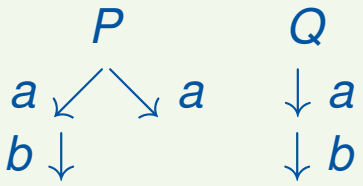

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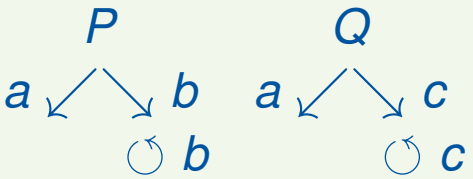

different traces
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But: processes with finite trace sets and identical deadlocks are trace equivalent (since every trace is a prefix of some deadlock).

Summary: Trace Equivalence

1. Trace equivalence equates processes that have the same traces, i.e., action sequences
2. Isomorphism implies trace equivalence
3. Trace equivalence trivially implies trace equivalence
4. Trace equivalence is a CCS congruence
5. Trace equivalence is not deadlock sensitive.
6. Checking trace equivalence is PSPACE-complete

Outline of Lecture 10

Completed Trace Equivalence

Definition 10.13 (Completed traces)

A completed trace of $P \in Prc$ is a sequence $w \in Act^*$ such that:

$$P \xrightarrow{w} Q \quad \text{and} \quad Q \not\longrightarrow$$

for some $Q \in Prc$.

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The completed traces of process P may be seen as capturing its deadlock behaviour, as they are precisely the action sequences that could lead to a process from which no transition is possible (i.e., is a deadlock).

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Exercise

Check whether completed trace equivalence is a congruence for restriction.

Further Variations of Trace Equivalence

Definition 10.14 (Ready trace equivalence) (Baeten et al.)

A sequence $A_0\alpha_0A_1\alpha_1 \ldots \alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a ready trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i = \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\}$.

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Definition 10.15 (Failure trace equivalence) (Reed and Roscoe)

A sequence $A_0\alpha_0A_1\alpha_1 \ldots \alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a failure trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i \cap \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\} = \emptyset$.

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A sequence $A_0\alpha_0A_1\alpha_1 \ldots \alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a failure trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i \cap \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\} = \emptyset$. Processes P and Q are failure-trace equivalent if they have exactly the same set of failure traces.

Further Variations of Trace Equivalence

Definition 10.14 (Ready trace equivalence) (Baeten et al.)

A sequence $A_0\alpha_0A_1\alpha_1\ldots\alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a ready trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i = \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\}$. Processes P and Q are ready-trace equivalent if they have exactly the same set of ready traces.

Definition 10.15 (Failure trace equivalence) (Reed and Roscoe)

A sequence $A_0\alpha_0A_1\alpha_1\ldots\alpha_nA_n$ with $A_i \subseteq Act$ and $\alpha_i \in Act$ ($i \in \mathbb{N}$) is a failure trace of process P if $P = P_0 \xrightarrow{\alpha_0} P_1 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} P_n$ such that $A_i \cap \{\alpha \in Act \mid P_i \xrightarrow{\alpha}\} = \emptyset$. Processes P and Q are failure-trace equivalent if they have exactly the same set of failure traces.

Example 10.16

$\alpha.P + \alpha.Q$ and $\alpha.P + \alpha.Q + \alpha.(P + Q)$ are failure-trace equivalent for every $P, Q \in Prc$ and $\alpha \in Act$, but not ready-trace equivalent

Outline of Lecture 10

Summary

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   ii. more distinctive than trace equivalence
   iii. a CCS congruence
   iv. deadlock sensitive

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3. Variations: completed, ready, and failure traces