



# Concurrency Theory WS 2017/2018

## — Series 6 —

Hand in until November 24th before the exercise class.

### Exercise 1 (Structural Congruence) (3 Points)

Prove that  $P \rightarrow Q$  implies that there exists a derivation of this reduction in which the (Struct) rule (see Definition 8.8) is applied, if at all, only at the root of the derivation tree.

### Exercise 2 (Reaction Relation) (4 Points)

Let

$$S = \text{new } x( \\
 (x(u) . u(y) . u(z) . \bar{y}\langle z \rangle . \text{nil} \\
 || x(t) . t(w) . t(v) . \bar{v}\langle w \rangle . \text{nil} \\
 || !\text{new } s(\bar{x}\langle s \rangle . \bar{s}\langle a \rangle . \bar{s}\langle b \rangle . \text{nil}) \\
 ).$$

Show that

$$S \longrightarrow^{\leq 12} (\bar{a}\langle b \rangle . \text{nil} || \bar{b}\langle a \rangle . \text{nil}) || \text{new } x(!\text{new } s(\bar{x}\langle s \rangle . \bar{s}\langle a \rangle . \bar{s}\langle b \rangle . \text{nil}))$$

where  $\longrightarrow^{\leq 12}$  denotes at most 12 applications of the reaction relation.

### Exercise 3 (Recursive Definitions) (3 Points)

Consider again the buffer from Example 9.3, split into two processes:

$$B(l, r) = l(x) . C\langle x, l, r \rangle \\
 C(x, l, r) = \bar{r}\langle x \rangle . B\langle l, r \rangle$$

Let the right hand sides be  $Q_B$  and  $Q_C$ , respectively. Follow the definitions in lecture 9, section *Adding recursive process calls* and write down  $\hat{Q}_B$  and  $\hat{Q}_C$ . Let

$$P = \bar{l}\langle x_1 \rangle . \bar{l}\langle x_2 \rangle . \bar{l}\langle x_3 \rangle || B_2 || r(y_1) . r(y_2) . r(y_3)$$

where  $B_2$  is a two-place buffer defined by

$$B_2 = \text{new } m(B\langle l, m \rangle || B\langle m, r \rangle).$$

Following part (3) of the definition in “Recursive Process Calls II”, write down  $P'$  and draw the corresponding LTS.